TOPOLOGICALLY INDUCED CHAOS IN THE UNIVERSE

ROMAN TOMASCHITZ
Department of Physics, Hiroshima University,
Higashi-Hiroshima 724, JAPAN, and
Physics Department, University of the Witwatersrand,
Johannesburg, WITS 2050, SOUTH AFRICA

ABSTRACT
Some topological effects that arise in an infinite and multiply connected universe are pointed out: the appearance of a chaotic nucleus, topologically induced particle creation and *CP* violation, and the temperature anisotropy of the background radiation.

The chaotic center of the Universe

Our basic assumptions are that the Universe is open and that its spacelike slices are multiply connected and negatively curved (extended Robertson-Walker cosmologies). Under these conditions there exists a finite region in the infinite 3-space in which the world lines are chaotic. It is beyond any doubt that some mechanism to generate chaos is needed to achieve the remarkable uniformity of the galactic background. In these cosmologies it is the local instability of the world lines and the global topology which induce chaos in a finite domain, whose size scales with the expansion factor. Moreover there are regular trajectories which are shadowed over long times by chaotic ones. This could provide an explanation that perfect equidistribution has not really been attained[1,2].

The open and multiply connected 3-space can undergo global metrical deformations without changing locally its curvature. Different spacelike sections are non-isometric, even after a rescaling with the expansion factor, in contrast to the traditional simply connected RW cosmologies. This type of evolution by global deformations is evidently unpredicted and undetermined by Einstein's equations. In a universe that evolves by such deformations particle creation occurs, even in conformally coupled neutrino or electromagnetic fields[1,3,5].

The classical geodesic equations are still reflection invariant in a multiply connected universe, but the situation is quite different concerning quantum mechanics. A space-reflected wave packet can wrap around a tiny geodesic loop and overlap with itself. this gives rise to self-interference, and the unitarity of the parity operator is lost. In particular *CP* and *CPT* are already broken in the free Dirac equation[1,2,4].

Assume for the moment that the spacelike slices are just copies of hyperbolic space H^3. Because H^3 is homogeneous, it happens that the eikonal of geometric optics appears in the phase of the spectral horospherical elementary waves of the Maxwell equations. Geometric optics does not know the concept of momentum. However, because of this relation we attach to the rays a momentum via the Einstein relation, and obtain so a vector field $p_\mu(x,t;\eta)$ on H^3, describing the 4-momentum of a horospherical bundle of classical flow lines issuing from some point η at infinity of H^3. A spacelike slice we represent as a polyhedron F with face-identification in H^3. The covering group Γ is generated by the face-pairing transformations. We project the horospherical bundles together with the vector

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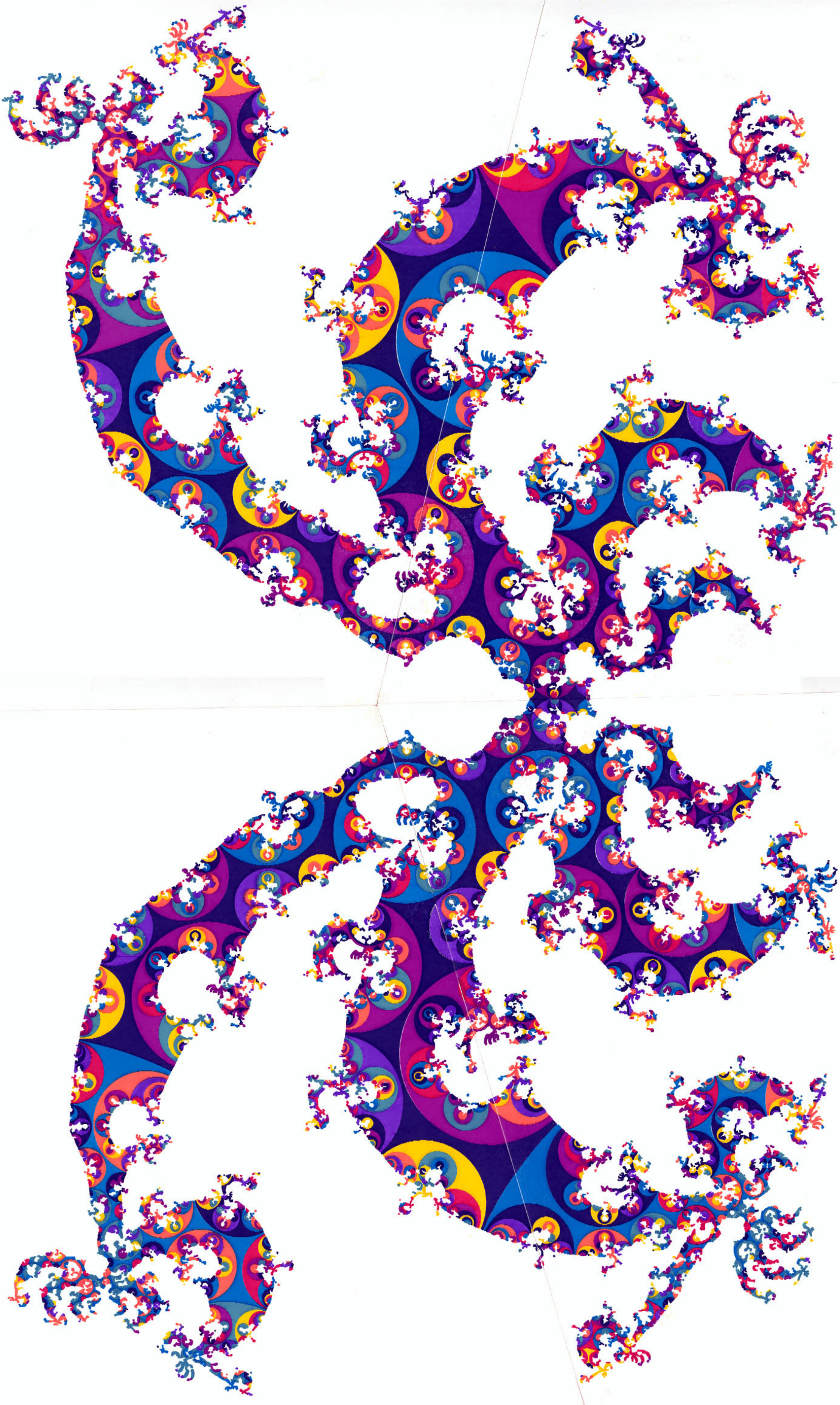

fields attached to them into F by means of the universal covering projection. A global deformation of the 3-space (F,Γ) is generated by adding a small Γ-periodic field $\tilde{g}_{ij}$ to the hyperbolic metric of H^3. The perturbed horospherical eikonal for rays issuing at η is $\tilde{\psi}^\Gamma = \psi^\Gamma + \chi(x,t;\eta)$, ψ^Γ denotes the eikonal on (F,Γ), and χ is a Γ-periodic scalar field. The perturbed frequencies are $\tilde{\nu} = \nu\left(1+\frac{1}{\omega}\frac{\partial\chi}{\partial t}\right)$, which means that we have to replace in the Planck distribution $\rho(h\nu/kT)d\nu$ the temperature by $\tilde{T} = T\left(1-\frac{1}{\omega}\frac{\partial\chi(x,t;\eta)}{\partial t}\right)$. This amounts to an angular variation η of the temperature[1,3,5].

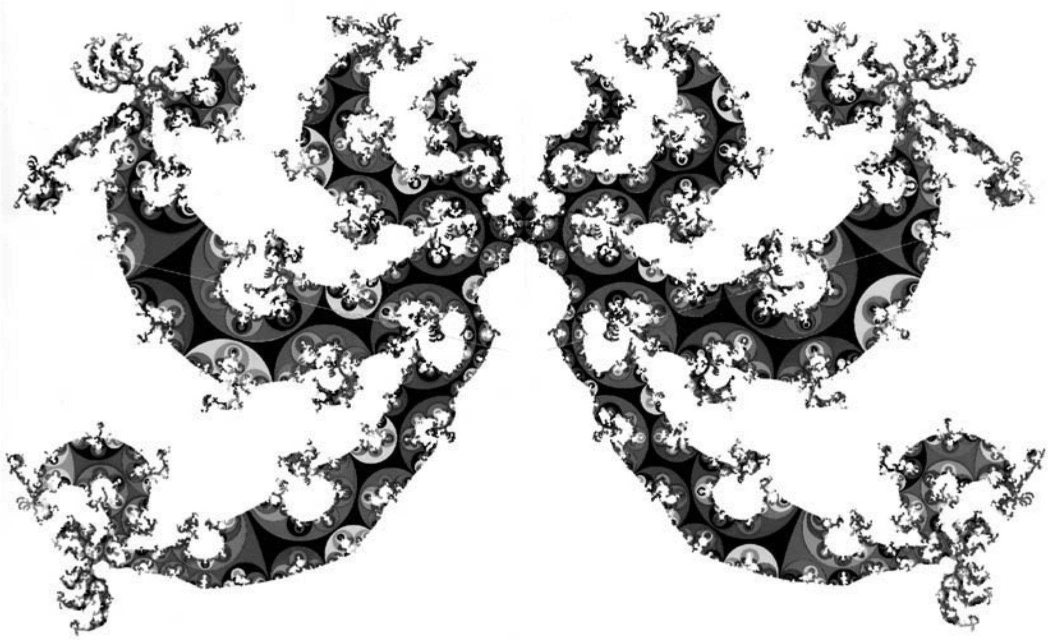

Fig. 1. The horizon at infinity of the Poincaré half-space H^3, the covering space of the spacelike slices (F, Γ). The polyhedral tiling $\Gamma(F)$ of H^3 induces a tiling at infinity which is depicted here. From the fractal limit set $\Lambda(\Gamma)$ one can easily determine the chaotic or nearly chaotic trajectories, which shadow each other over long times. Their lifts have initial and end points in or close to $\Lambda(\Gamma)$. Projecting them into the 3-space (F, Γ) one obtains the chaotic nucleus.

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