Topologically induced chaos in the open Universe

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Abstract - An elementary account on chaos, its origins, and its physical impact in an infinite and multiply connected space-time is given. The anisotropy of the microwave background and the violation of the space-reflection symmetry (parity) by topological self-interference are reviewed in this context.

I. THE CENTER OF THE UNIVERSE

It is largely agreed today that the rough overall evolution of the universe should be described by Riemannian geometry, but virtually nothing is known about the topological structure of space-time. Though allusions to this question emerge in the literature for a long time, cf. Refs. 1 and 2, neither philosophers nor scientists have been much inspired by it.

In Refs. 3-6 we suggested a practical way to gain more insight into the global topological structure of the universe. We assumed a given topological scenario, and tried to find physical effects of the topology. Do particles, rays, fields, and galaxies behave qualitatively differently in infinite universes of different connectivity?

Our basic assumption is that the Universe is open (infinite) with spacelike slices that are topologically multiply connected, and locally negatively curved (extended Robertson-Walker cosmologies). We need not make further specifications, since the phenomena which we outline here do not depend qualitatively on the concrete degree of connectivity, or the local structure of the metric.

There are two fundamental differences between an extended RW cosmology and the traditional textbook examples of cosmological models. Firstly, extended RW cosmologies have a chaotic nucleus, a domain of finite size in the infinite 3-space, in which world lines have the mixing property. This is the center of the Universe, in which the galaxies are nearly equidistributed because of this mixing mechanism. Secondly, the 3-space of extended RW cosmologies can undergo global metrical deformations, cf. Figs. 1 and 2, without changing locally the curvature, in strong contrast to simply connected models. This topologically induced evolution of the Universe is unpredicted and undetermined by Einstein's equations, which relate only the local structure of space-time to the energy-momentum tensor. This local structure, the curvature, is left unchanged by such deformations.

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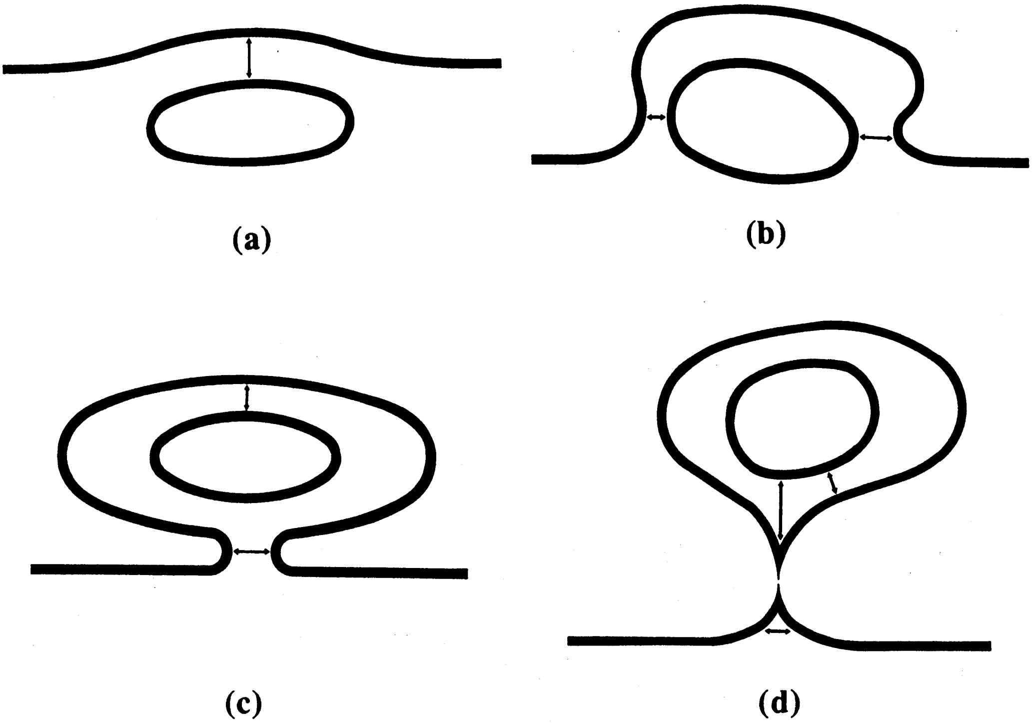


Fig. 1 a-d. Global metrical deformations of the 3-space do not change the local curvature. The depicted boundaries are at infinity, unattainable within a finite time for a particle or ray moving with a uniformly bounded speed. In particular the distances indicated by double arrows are infinite. In Fig. 1d a bubble (of infinite volume) peels off, leaving behind a cusp singularity.

In Ref. 9 we showed how a wave track composed of positive frequencies receives an admixture of negative frequencies during a deformation: the evolution of the Universe induced by global topological deformations gives rise to particle creation.

II. CLASSICAL AND QUANTUM DISPERSION

One of the most remarkable features of RW cosmologies of negative spatial curvature is the instability of the classical geodesic trajectories, the probabilistic character of world lines.

In Ref. 3 we developed a probabilistic description of this instability, similar to the Liouville equation, but in a manifestly covariant, non-Hamiltonian form. To achieve this we introduced the concept of a horospherical geodesic flow of expanding bundles of parallel world lines. We constructed an invariant measure and a covariant evolution equation for the probability density on which this flow acts. The orthogonal surfaces to these bundles of trajectories are horospheres, closed surfaces in 3-space, touching the boundary at infinity of hyperbolic space, where the flow lines emerge, cf. Fig. 3.

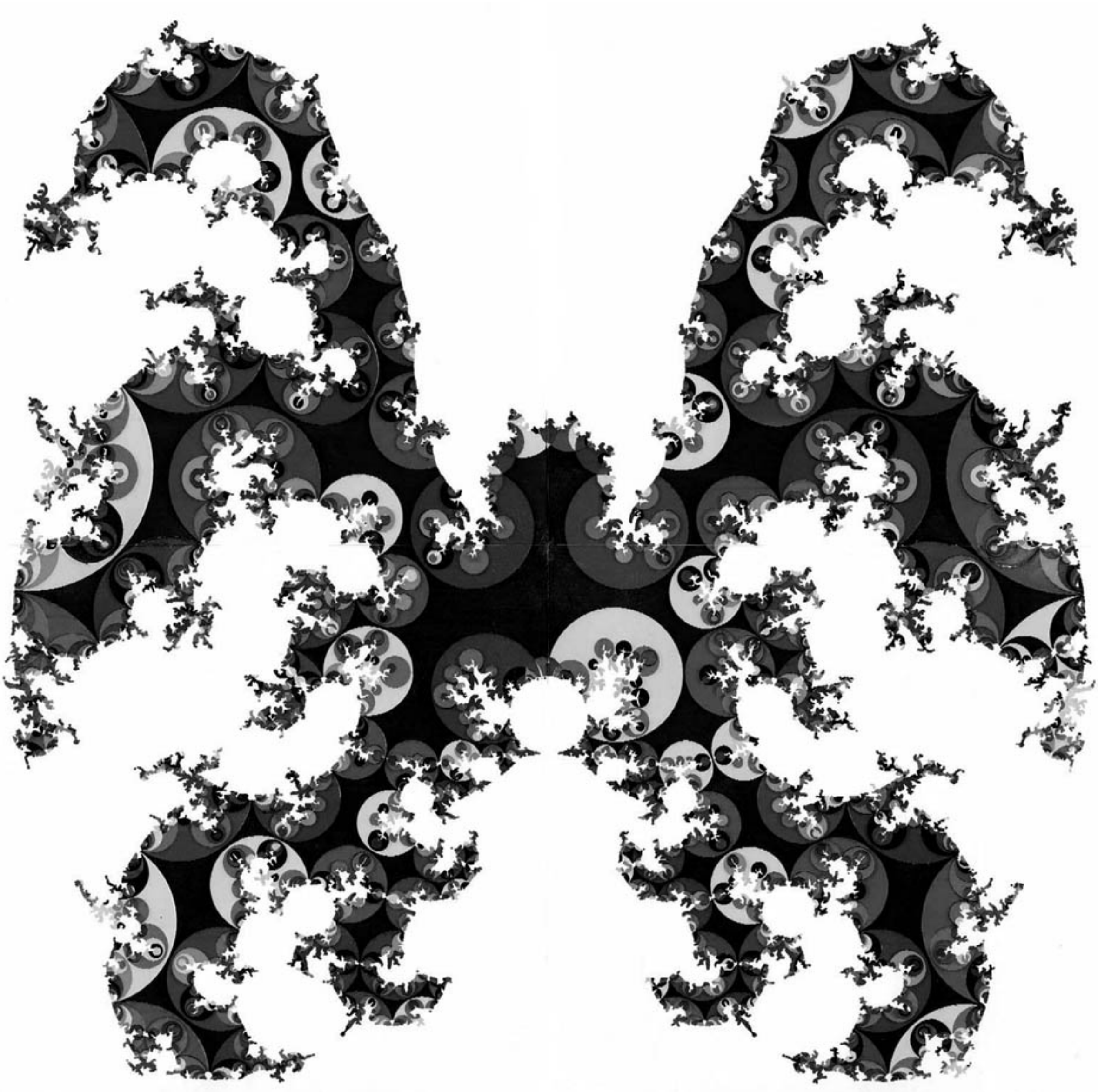


Fig. 2a.

Caption for Figs. 2a, b. The horizon at infinity of the Poincaré half-space H^3. A spacelike slice (F, Γ) is realized in H^3 as a polyhedron F with a face-identification. The identifying transformations generate a discrete group Γ which, applied to the polyhedron, gives a tessellation $\Gamma(F)$ of H^3 with polyhedral images. This tessellation induces by continuity also a tiling on the boundary of H^3, which is depicted here. The qualitative structure of the singular set depends on the covering group Γ, for quasi-Fuchsian groups like here it is a Jordan curve, for Schottky groups a Cantor set, cf. Refs. 7 and 8. The spacelike slices can undergo global metrical deformations. The depicted tilings correspond to 3-slices that are non-isometric, but have the same topology ($(F, \Gamma) \approx I \times S$, S a Riemann surface, $g(S) = 49$, $\delta(\Lambda) \approx 1.6$) and curvature. These tilings make deformations as schematized in Fig. 1 quantitative. One can easily determine from them the chaotic or nearly chaotic trajectories, which shadow each other over long times. Their lifts have initial and terminal points in or close to the singular set, cf. Ref. 10. Projecting them into the manifold, one obtains the chaotic nucleus.

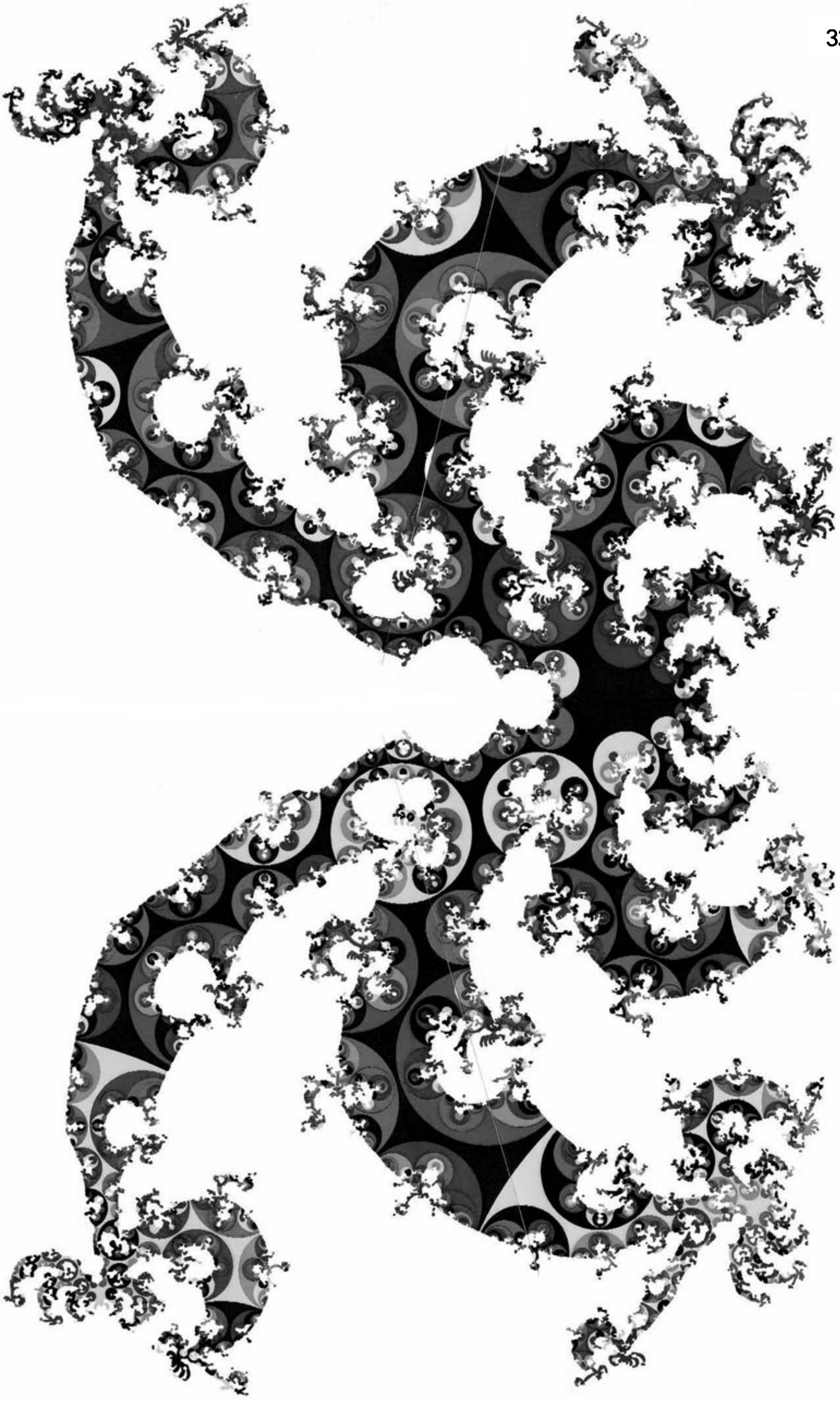
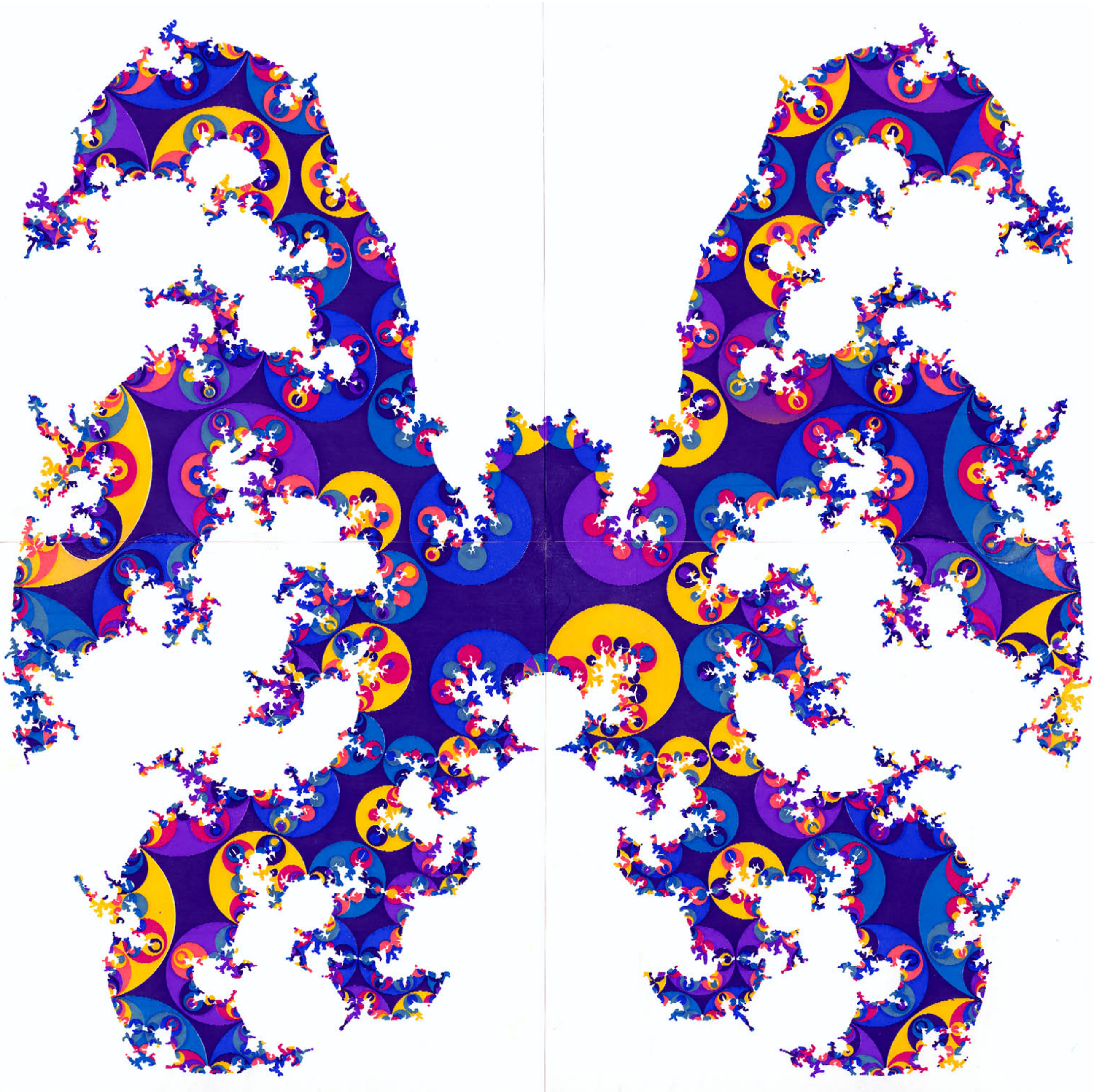
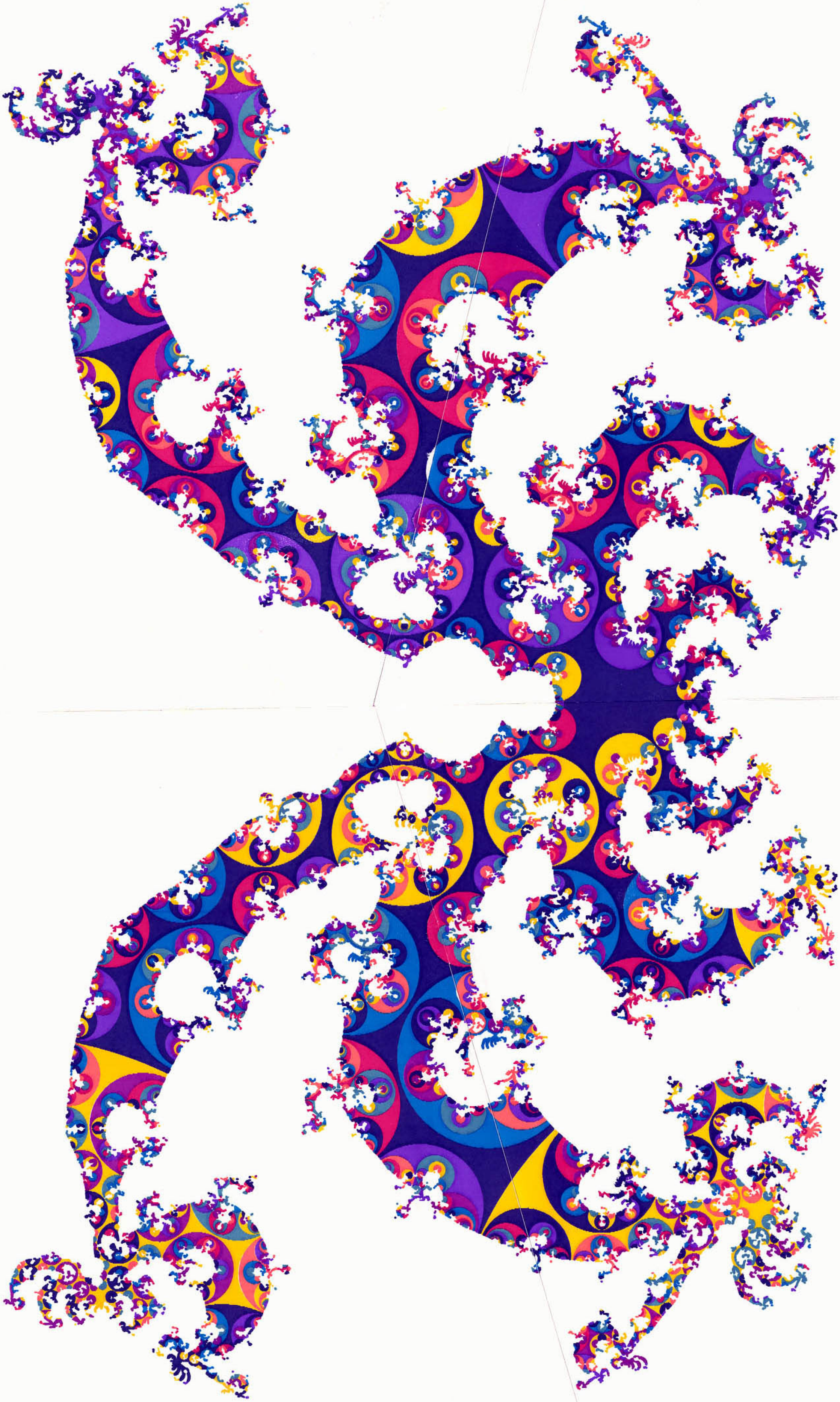


Fig. 2b.





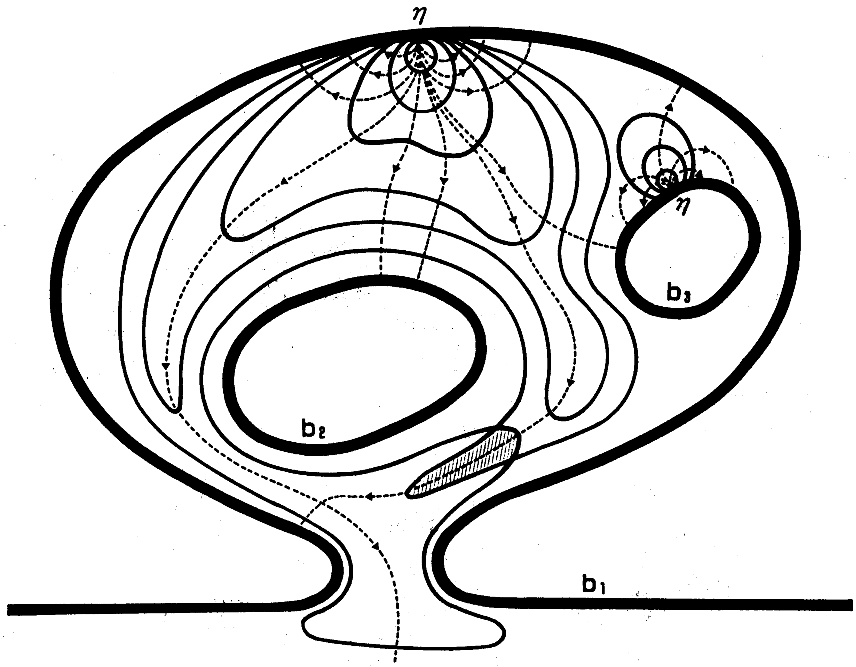


Fig. 3. Wave fronts of horospherical elementary waves are bent and deflected by the topology. The boundaries b_i of the 3-space are at infinity, the horizon, where the wave fronts originate (η). The expanding bundle of geodesics (dashed) with initial point at η is orthogonal to them. Wave fronts are generally self-intersecting (hatched), cf. Ref. 6.

These horospheres are the wave fronts of eigenfunctions of the relativistic wave equations on the 4-manifold. This fact suggests to compare the evolution of the quantum mechanical density with the classical one, on which expanding bundles of geodesic flow lines act, see also Ref. 11. We found asymptotic identity in the asymptotically flat region and in periods of adiabatic expansion, when no particle production processes occur. This led us furthermore to study the time behavior of the dispersion of the energy and the coordinates and of the energy-time uncertainty relation, and we found again identity in the late stage of the cosmic evolution. This identity can persist in the early phase of the expansion with a rapidly varying scale factor, provided the fields are conformally coupled to the curvature, and provided the time variation still permits to disentangle positive and negative frequencies, so that particles can emerge.

III. ANGULAR FLUCTUATIONS IN THE TEMPERATURE OF THE MICROWAVE BACKGROUND

Angular anisotropy in the Planck distribution is a possible consequence of global metrical deformations of the spacelike slices, cf. Ref. 4. Assume for the moment that the spacelike slices are just copies of hyperbolic space H^3. Because H^3 is homogeneous, it happens that the eikonal of geometric optics appears in the phase of the spectral horospherical elementary waves of the Maxwell equations.

Geometric optics does not know the concept of momentum. However, because of this relation we attach to the rays a momentum via the Einstein relation $p_\mu = \hbar k_\mu$. So we obtain a vector field $p_\mu(x,t;\eta)$ on H^3, describing the 4-momentum of a horospherical bundle of classical flow lines at time t issuing from some point η on the horizon (x denotes H^3 coordinates), cf. Fig. 3.

A spacelike slice we represent as a polyhedron F in H^3, see the caption of Fig. 2, and we project the horospherical bundles together with the vector fields attached to them into F by means of the universal covering projection, cf. Ref. 6.

A global deformation of the 3-space (F,Γ) is generated by adding a small Γ-periodic symmetric tensor field $\tilde{g}_{ij}$ to the hyperbolic metric of H^3, cf. Ref. 4. The perturbed horospherical eikonal for rays issuing at η is $\tilde{\psi}^\Gamma = \psi^\Gamma + \chi(x,t;\eta)$, ψ^Γ denotes the eikonal on (F,Γ), and χ is a Γ-periodic scalar field. The perturbed frequencies are $\tilde{\nu} = \nu\left(1+\frac{1}{\omega}\frac{\partial\chi}{\partial t}\right)$, which means that we have to replace in the Planck distribution $\rho(h\nu/kT)d\nu$ the temperature by $\tilde{T} = T\left(1-\frac{1}{\omega}\frac{\partial\chi(x,t;\eta)}{\partial t}\right)$. This amounts to an angular variation η of the temperature in the distribution, that remains otherwise unchanged. The adiabatic time dependence of the temperature reminds us that we have here only a first approximation to a non-equilibrium process.

IV. PARITY VIOLATION DUE TO SELF-INTERFERENCE OF SPACE-REFLECTED WAVES

The classical geodesic equations are still invariant under a space reflection (P) in an extended RW cosmology, but the situation is quite different concerning wave mechanics. Imagine a wave packet concentrated on a finite domain in a spacelike slice (F, Γ). If we apply P it can happen that the reflected wave wraps around a tiny geodesic loop, cf. Fig. 4. The wave packet starts to interfere with itself, and its norm is not preserved.

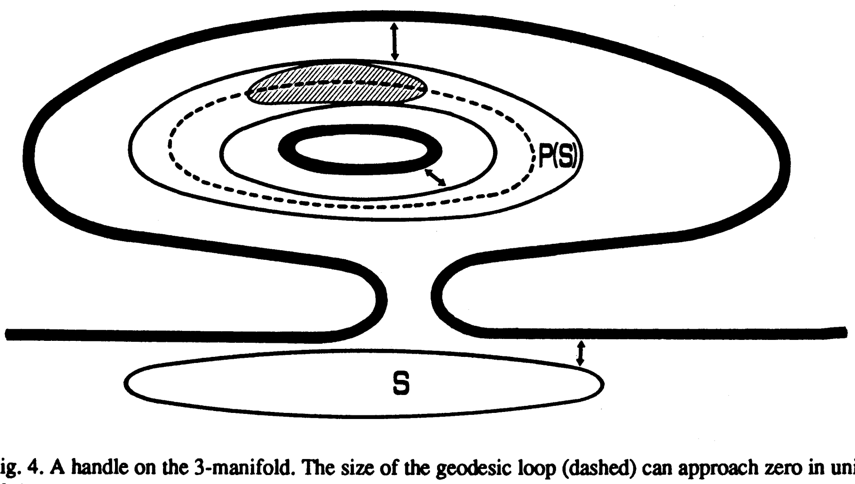


Fig. 4. A handle on the 3-manifold. The size of the geodesic loop (dashed) can approach zero in units of the curvature radius. The space reflection P (defined by means of the covering projection of the space reflection in H^3) maps the support S of a wave packet into the handle. $P(S)$ overlaps with itself, covering completely the geodesic loop.

Because of this self-interference the space reflection symmetry is already broken on the level of the free Dirac equation. This is remarkable, because usually one has to attach on purpose symmetry breaking interactions in order to achieve this. The time reversal symmetry is likewise broken, because of the time-dependent deformations of the 3-space, which can happen microscopically on very short time scales. The charge conjugation C is still a good symmetry, but all combinations of C, P, and T are broken.(A pictorial description of these symmetries can be found in Ref. 12). One can speculate if the baryon asymmetry is topologically generated. Clearly CP violation is a pure quantum effect, unthinkable in the classical context. It is interference, not so much dispersion, that distinguishes quantum mechanics from unstable classical dynamics, cf. Sec. II. Accordingly it seems perfectly natural to associate CP violation with an interference phenomenon.

V. CONCLUSION

The phenomena reviewed here indicate that from the investigation of the topological structure of spacetime further surprising consequences can be expected.

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