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Tachyonic synchrotron radiation from γ-ray pulsars

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Abstract

Superluminal radiation emitted by electrons orbiting in strong magnetic fields is investigated. We show that electrons gyrating in the surface fields of rotation-powered neutron stars can radiate superluminal quanta (tachyons) via the synchrotron mechanism. The tachyonic luminosity of γ-ray pulsars is inferred from COMPTEL and EGRET observations, and so is the magnetospheric electron population generating this radiation. In the surface fields, electromagnetic synchrotron radiation in the γ-ray band is suppressed by a quantum cutoff, but not so tachyonic γ-radiation. This provides an exceptional opportunity to search for tachyon radiation, unspoiled by electromagnetic emission. Estimates of the superluminal power radiated and the tachyonic count rates are obtained for each of the seven established γ-ray pulsars, the Crab and Vela pulsars, as well as PSR B1706−44, Geminga, PSR B1055−52, B1951+32, and B1509−58. Detection mechanisms such as tachyonic ionization and Compton scattering are analyzed with regard to superluminal γ-rays.

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1. Introduction

When considering superluminal quanta, we may try a wave theory or a particle picture as starting point. The latter has been studied for quite some time, but did not result in viable interactions with matter [1–6]. Here, tachyons will be modeled as wave fields with negative mass-square, coupled by minimal substitution to subluminal particles. Interaction with matter is indeed the crucial point, we will maintain the best established interaction mechanism, minimal substitution, by treating tachyons like photons with negative mass square, a real Proca field minimally coupled to subluminal matter [7,8].

We will work out a specific example, tachyonic synchrotron emission from electrons gyrating in the surface fields of γ-ray pulsars. The detectors used to collect the γ-ray fluxes operated in adjacent or partly overlapping energy bands with very

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different mechanisms, ionization [9], Compton scattering [10], and electron-positron pair creation [11], and we will discuss the respective cross-sections with regard to tachyonic γ-rays. We will calculate the energy stored in the electron population gyrating in the surface magnetic fields, and relate the slope of the electronic power-law distribution to the frequency scaling of the tachyon flux. We will conclude from the break frequencies of the observed fluxes that the gyration energies can reach the low TeV range, and comment on cosmic ray acceleration in the magnetospheres.

In Section 2, we study superluminal synchrotron radiation from electrons subjected to high magnetic field strengths. We will introduce the tachyonic counterparts of certain quantities familiar from electromagnetic synchrotron radiation, such as critical frequencies, break frequencies and critical field strengths, and explain how they relate to the spectral densities, the tachyon mass, and the electronic Lorentz factor. In Section 3, we study the synchrotron power radiated in tachyonic γ-rays by the magnetospheric electron populations of γ-ray pulsars. We calculate the synchrotron power in the high-magnetic-field limit as well as the tachyonic count rates and average the tachyonic spectral densities with electronic power-law distributions. The tachyonic γ-ray luminosity is inferred from the measured fluxes and all that goes with it, such as the electronic power-law indices and the energy range of the gyrating electrons. In Section 4, we present our conclusions.

2. Tachyonic synchrotron radiation in strong magnetic fields

The superluminal spectral densities can be readily assembled from the general formalism of tachyonic radiation theory [12,13], which will not be repeated here, but we restate the densities in (2.1)–(2.8). There is a low-and a high-frequency regime, separated by the break frequency $\omega_b = \gamma m_t c$, where $\gamma \gg 1$ is the Lorentz factor of the radiating ultra-relativistic source, typically a circularly orbiting electron, and m_t is the tachyon mass. The latter has the dimension of an inverse length, being a shortcut for $m_t c/\hbar$. The subsequent spectral densities only apply in the range $\omega \leqslant \omega_b$. There is also high-frequency radiation beyond this break frequency, but this needs second quantization, as the high magnetic field strengths considered here lead to an exponential cutoff of the classical radiation. In the low-frequency regime, however, quantum effects are perturbative, cf. after (2.10).

The transversal radiation can be split into two linear polarization components,

$$p^{\mathrm{T}}(\omega) = p^{\mathrm{T}}_{\parallel}(\omega) + p^{\mathrm{T}}_{\perp}(\omega), \tag{2.1}$$

$$\begin{aligned} p^{\mathrm{T}}_{\parallel,\perp}(\omega) &= \frac{1}{2}\frac{q^2}{4\pi}\frac{m_t^2 c\omega}{\omega^2+m_t^2c^2}\left(1-\frac{\omega^2}{\omega_b^2}\right) \\ &\quad\times(1+F_0(\xi)\mp G_0(\xi)) \\ &= \frac{1}{2}\frac{q^2}{4\pi}\frac{m_t^2 c}{\kappa^{2/3}\omega_b^{4/3}}\frac{\omega^{7/3}}{\omega^2+m_t^2c^2}\xi^{2/3} \\ &\quad\times(1+F_0(\xi)\mp G_0(\xi)). \end{aligned} \tag{2.2}$$

The lower plus-sign in (2.2) refers to the ⊥-polarization, and q is the tachyonic charge of the electron. The tachyon mass, $m_t \approx 2.15\ \mathrm{keV/c^2}$, and the ratio $q^2/e^2 \approx 1.4\times10^{-11}$ of tachyonic and electric fine structure constants can be inferred from Lamb shifts in hydrogenic systems [7]. The spectral functions can be traced back to Airy functions,

$$\begin{aligned} F_0(\xi) &:= -\frac{1}{3}\int_\xi^\infty (J_{-5/3}(x)+J_{5/3}(x))\,\mathrm{d}x, \\ G_0(\xi) &:= \frac{1}{3}(J_{-2/3}(\xi)-J_{2/3}(\xi)), \\ F_0(\xi) &= 2G_0(\xi)-L_0(\xi), \\ L_0(\xi) &:= \frac{1}{3}\int_\xi^\infty (J_{-1/3}(x)+J_{1/3}(x))\,\mathrm{d}x, \end{aligned} \tag{2.3}$$

where the argument ξ is a shortcut for

$$\xi(\omega) := \kappa\frac{\omega_b^2}{\omega^2}(1-\omega^2/\omega_b^2)^{3/2}, \quad \kappa := \frac{2}{3}\frac{m_t c}{\gamma^2\omega_B}, \tag{2.4}$$

and $\omega_b = eB/(\gamma mc)$ is the gyrofrequency, cf. after (2.9). As mentioned, we restrict to frequencies $\omega \leqslant \omega_b$. The parameter κ will turn out to be the expansion parameter for the power radiated, cf. Section 3. We will consider the $\kappa \to 0$ asymptotics, attained in high magnetic fields. The spectral density of the longitudinal radiation reads,

$$p^{\mathrm{L}}(\omega) = \frac{q^2}{4\pi}\frac{m_{\mathrm{t}}^2 c\omega}{\omega^2 + m_{\mathrm{t}}^2 c^2}(1 - L_0(\xi)), \tag{2.5}$$

with the spectral function $L_0(\xi)$ defined in (2.3) and (2.4). The ascending series and asymptotic limits of F_0, G_0 and L_0 can be found in Ref. [13].

We split the densities (2.2) and (2.5) into a linear and a curvature component,

$$p^{\mathrm{T}}_{\|,\perp}(\omega) = p^{\mathrm{T,lin}}/2 - p^{\mathrm{T,curv}}_{\|,\perp}, \quad p^{\mathrm{L}}(\omega) = p^{\mathrm{L,lin}} - p^{\mathrm{L,curv}}, \tag{2.6}$$

where the densities $p^{\mathrm{T,lin}}$ and $p^{\mathrm{L,lin}}$ stand for the tachyon radiation generated by a charge in linear uniform motion (in the ultra-relativistic limit),

$$p^{\mathrm{T,lin}}(\omega) := \frac{q^2}{4\pi}\frac{m_{\mathrm{t}}^2 c\omega}{\omega^2 + m_{\mathrm{t}}^2 c^2}\left(1 - \frac{\omega^2}{\omega_{\mathrm{b}}^2}\right),$$
$$p^{\mathrm{L,lin}}(\omega) := \frac{q^2}{4\pi}\frac{m_{\mathrm{t}}^2 c\omega}{\omega^2 + m_{\mathrm{t}}^2 c^2}. \tag{2.7}$$

The curvature radiation subtracted in (2.6) can be read off from (2.2) and (2.5),

$$p^{\mathrm{T,curv}}_{\|,\perp}(\omega) := -(p^{\mathrm{T,lin}}/2)(F_0 \mp G_0),$$
$$p^{\mathrm{L,curv}}(\omega) := p^{\mathrm{L,lin}} L_0, \tag{2.8}$$

where the upper minus-sign refers to $p^{\mathrm{T,curv}}_{\|}$. The Lorentz factor of the radiating charge enters the transversal linear density $p^{\mathrm{T,lin}}$ via ω_{b}. It also enters in $p^{\mathrm{L,lin}}$, again by ω_{b}, which is the cutoff frequency for uniform motion; a uniformly moving ultra-relativistic charge can only radiate frequencies $\omega \leqslant \omega_{\mathrm{b}}$, cf. Ref. [12]. The three densities $p^{\mathrm{T,L}}_{\|,\perp}$ in (2.6) are safely positive definite, but not so the terms $p^{\mathrm{T,curv}}_{\|,\perp}$ and $p^{\mathrm{L,curv}}$ in (2.8), which are generated by the orbital curvature and oscillate for large ξ.

We consider $\kappa \ll 1$, cf. (2.4), but still large enough that $\kappa\gamma^2 \gg 1$. Since $\gamma \gg 1$ in the ultra-relativistic limit, we can assume $m_{\mathrm{t}}c \ll \sqrt{\kappa}\omega_{\mathrm{b}}$, so that the mass term in the denominator of the spectral densities can be dropped for frequencies $\omega/\omega_{\mathrm{b}} \geqslant \sqrt{\kappa}$. Moreover, $0 \leqslant \xi(\omega) \leqslant 1$ in the interval $\sqrt{\kappa} \leqslant \omega/\omega_{\mathrm{b}} \leqslant 1$, cf. (2.4), so that $\xi(\sqrt{\kappa}\omega_{\mathrm{b}}) = 1$ and $\xi(\omega_{\mathrm{b}}) = 0$. Within this interval, the spectral functions $\xi^{2/3}F_0$, $\xi^{2/3}G_0$ and L_0 are of O(1) . Outside this range, we find $\xi \gg 1$ for $\omega/\omega_{\mathrm{b}} \ll \sqrt{\kappa}$, so that the spectral functions can be replaced by their asymptotic limits. There are three frequencies helpful to understand the qualitative behavior of the spectral densities,

$$p^{\mathrm{T}}_{\|,\perp}(\omega = m_{\mathrm{t}}c, \sqrt{\kappa}\omega_{\mathrm{b}}, \omega_{\mathrm{b}}) \propto (\gamma, \kappa^{-1/2}, \kappa^{-2/3}),$$
$$p^{\mathrm{L}}(\omega = m_{\mathrm{t}}c, \sqrt{\kappa}\omega_{\mathrm{b}}, \omega_{\mathrm{b}}) \propto (\gamma, \kappa^{-1/2}, 1), \tag{2.9}$$

where the three proportionality constants pertinent to the indicated choices of ω are of the same order. The densities decay $\propto \omega$ for $\omega \ll m_{\mathrm{t}}$c. The longitudinal density attains its maximum at $\omega \approx m_{\mathrm{t}}$c. The transversal density also has its maximum at $\omega \approx m_{\mathrm{t}}$c, followed by a minimum at about $\omega_{\min} = \sqrt{\kappa}\omega_{\mathrm{b}}$. The spectral functions (2.3) are composed of (anti-) derivatives of Airy functions, and admit straightforward analytic continuation [13]. The argument $\xi(\omega)$ in the spectral functions can still be used in the high-frequency regime, $\omega \geqslant \omega_{\mathrm{b}}$, if we replace the parentheses in (2.4) by an absolute value. We then find $\xi(\omega_{\mathrm{b}}/\kappa) = 1$, where the ratio $\omega_{\mathrm{b}}/\kappa$ is independent of the tachyon mass and coincides with the critical photon frequency, $\omega_{\mathrm{c}} = (3/2)\omega_{\mathrm{B}}\gamma^3$. At about this frequency, the analytically continued transversal spectral density attains a second maximum, like the electromagnetic counterpart, before the exponential decay sets in. In actual fact, the classical densities are exponentially cut off before this high-frequency maximum is reached. This is a quantum effect arising in the $\kappa \ll 1$ limit, due to the high magnetic field strength.

We shortly summarize the notation and the constants. Gyroradius and gyrofrequency relate as $R \approx c/\omega_{\mathrm{B}}$, where $\omega_{\mathrm{B}} = eB/(\gamma mc)$, and $1\mathrm{G} \cdot e \approx 2.998 \times 10^{-7}\ \mathrm{GeV\,cm^{-1}}$. We write $\gamma = E/mc^2$, so that $eB \approx E/R$, where E and m denote energy and mass of the gyrating electron or positron, other possible source particles like protons and heavier nuclei will not be considered. We use the Heaviside–Lorentz system, so that $e^2/(4\pi\hbar c) =: \alpha_{\mathrm{e}} \approx 1/137$ and $q^2/(4\pi\hbar c) =: \alpha_q \approx 1.0 \times 10^{-13}$ are the electric and tachyonic fine structure constants. We restore the mass unit in the above densities, $m_{\mathrm{t}} \to m_{\mathrm{t}}c/\hbar \approx 1.09 \times 10^8\ \mathrm{cm}^{-1}$, so that the break frequency reads $\omega_{\mathrm{b}} = \gamma m_{\mathrm{t}}c^2/\hbar$. Finally, $\alpha_q/\alpha_e \approx 1.4 \times 10^{-11}$ and $m_{\mathrm{t}} \approx m/238 \approx 2.15\ \mathrm{keV/c^2}$.

When calculating the power radiated, cf. Section 3, we will use κ in (2.4) as expansion parameter,

$$\kappa = \frac{\omega_{\mathrm{b}}}{\omega_{\mathrm{c}}} \approx \frac{2}{3}\frac{m_{\mathrm{t}}c^2}{\hbar}\frac{mc}{eB}\frac{1}{\gamma}. \tag{2.10}$$

In the surface magnetic fields of γ-ray pulsars, $\kappa \ll 1$ as well as $\kappa\gamma^2 \gg 1$ usually apply, cf. Table 2. The opposite limit, $\kappa \gg 1$, is realized in planetary magnetospheres and supernova remnants [13]. The surface fields are quite comparable to the critical field, $B_{\mathrm{c}} = m^2c^3/(e\hbar) \approx 4.414 \times 10^{13}$ G, cf. Table 1. The gyrofrequency and the ratio of critical and break energy relate to the magnetic field ratio as

$$\omega_B = \frac{mc^2}{\hbar}\frac{1}{\gamma}\frac{B}{B_{\mathrm{c}}}, \quad \frac{\omega_c}{\omega_{\mathrm{b}}} = \frac{3}{2}\frac{m}{m_{\mathrm{t}}}\gamma\frac{B}{B_{\mathrm{c}}}. \tag{2.11}$$

This notation is kept close to electromagnetic synchrotron radiation [14–17], as is the whole formalism, of course. However, the spectral densities (2.2) applicable below the break frequency do not have an electromagnetic counterpart, only the upper frequency range has, but the high-frequency radiation requires second quantization in strong fields. Quantum corrections [18] are negligible for frequencies much smaller than the electron energy, which is the case below the break frequency. In the opposite limit, $\hbar\omega/E \gg 1$, the classical spectral peak near the critical frequency is wiped out by an exponential cutoff, as is the electromagnetic synchrotron radiation [19]. Quantum effects also lead to an attenuation of the classical radiation in the cyclotron limit, for slowly gyrating source particles. The quantization of tachyonic cyclotron and synchrotron radiation will be discussed elsewhere.

To get an overview regarding orders of magnitude, we define the shortcuts $E_0 = E$[GeV] and $B_0 = B$[G], and write the preceding formulas as scaling relations in these dimensionless numbers. For instance, $R[\mathrm{cm}] \approx 3.336 \times 10^6 E_0 B_0^{-1}$ and $\omega_{\mathrm{B}}[\mathrm{GHz}] \approx 29.98/R[\mathrm{cm}]$. Lorentz factor and expansion parameter scale as $\gamma \approx 1957E_0$ and $\kappa \approx 6.33\times 10^7 E_0^{-1}B_0^{-1}$, respectively. We also note $\kappa\gamma \approx 1.24\times 10^{11}B_0^{-1}$ and $\kappa\gamma^2 \approx 2.42 \times 10^{14}E_0/B_0$. The energies attached to break and critical frequency and to the transversal spectral minimum as defined after (2.9), $\varepsilon_{\mathrm{b},c,\min} = \hbar\omega_{\mathrm{b},c,\min}$, scale as

$$\begin{aligned}
&\varepsilon_{\mathrm{b}}[\mathrm{keV}] \approx 4.21 \times 10^3 E_0,\\
&\varepsilon_{\mathrm{c}}[\mathrm{keV}] \approx 6.65 \times 10^{-5} E_0^2 B_0,\\
&\varepsilon_{\mathrm{c}}/E \approx 6.65 \times 10^{-11} E_0 B_0,\\
&\varepsilon_{\min}[\mathrm{keV}] \approx 3.35 \times 10^7 E_0^{1/2} B_0^{-1/2}.
\end{aligned} \tag{2.12}$$

Some other numerical relations are needed to connect the foregoing to synchrotron fluxes from pulsars, cf. Section 3. The surface magnetic fields are inferred from the period and period derivative via $B[\mathrm{G}] \approx 3.2 \times 10^{19}(P[s]\dot{P})^{1/2}$. The light cylinder field is obtained by a rescaling of the surface field, $B_{\mathrm{lc}} = (R_{\mathrm{ns}}/R_{\mathrm{lc}})^3 B$. Here, $R_{\mathrm{ns}} \approx 10^6$ cm and $R_{\mathrm{lc}} = cP/(2\pi)$

Table 1
Entries as defined in Section 2: electron energy $E(\mathrm{GeV}) = 10^k$ (input), surface field B (input), gyroradius R, tachyonic break energy $\varepsilon_b(\mathrm{keV}) \approx 4.2 \times 10^{3+k}$, critical energy ε_c

	B (10^{12}G)	R (cm)	ε_{c} (GeV)	$\varepsilon_{\min}$ (keV)	P (ms)	B_{lc} (G)
Crab	3.8	$8.8 \times 10^{-7+k}$	$2.5 \times 10^{2k+2}$	$1.7 \times 10^{k/2+1}$	33.40	9.4×10^5
Vela	3.4	$9.8 \times 10^{-7+k}$	$2.3 \times 10^{2k+2}$	$1.8 \times 10^{k/2+1}$	89.29	4.4×10^4
B1706−44	3.1	$1.1 \times 10^{-6+k}$	$2.1 \times 10^{2k+2}$	$1.9 \times 10^{k/2+1}$	102.45	2.7×10^4
Geminga	1.6	$2.1 \times 10^{-6+k}$	$1.1 \times 10^{2k+2}$	$2.65 \times 10^{k/2+1}$	237.09	1.1×10^3
B1055−52	1.1	$3.0 \times 10^{-6+k}$	$7.3 \times 10^{2k+1}$	$3.2 \times 10^{k/2+1}$	197.10	1.3×10^3
B1951+32	0.49	$6.8 \times 10^{-6+k}$	$3.3 \times 10^{2k+1}$	$4.8 \times 10^{k/2+1}$	39.53	7.3×10^4
B1509−58	15.5	$2.15 \times 10^{-7+k}$	$1.0 \times 10^{2k+3}$	$8.5 \times 10^{k/2}$	150.66	4.2×10^4

One may envisage $-2 < k < 4$ as typical range for the continuous parameter k labeling the electron energy. The transversal and longitudinal spectral peaks are located at the tachyon mass (2.15 keV), the minimum of the transversal tachyonic energy density is at $\varepsilon_{\min}$. The surface fields B and periods P are taken from Ref. [26], and B_{lc} is the field strength at the light cylinder. With one exception, the listed pulsars are detected in hard γ-rays, with power-law spectra extending beyond 1 GeV: Crab (PSR B0531+21), Vela (PSR B0833−45), PSR B1706−44, Geminga (PSR B0633+17), PSR B1055−52, and PSR B1951+32. The γ-ray spectrum of the high-magnetic-field pulsar PSR B1509−58 is soft, cut off at about 10 MeV. The γ-ray fluxes of these pulsars are quoted in Section 3.

Table 2
Setting the scales: break energy, critical energy and electronic Lorentz factor determining the shape of the tachyonic spectral densities, cf. (2.9)

	κ	$\kappa\gamma$	$\kappa\gamma^2$	ε_c/E	κ_{lc}	D (kpc)
Crab	$1.67\times10^{-5-k}$	0.0326	$6.37\times10^{1+k}$	$2.5\times10^{2+k}$	$6.7\times10^{1-k}$	2.0
Vela	$1.86\times10^{-5-k}$	0.0365	$7.12\times10^{1+k}$	$2.3\times10^{2+k}$	$1.4\times10^{3-k}$	0.3
B1706−44	$2.04\times10^{-5-k}$	0.0400	$7.81\times10^{1+k}$	$2.1\times10^{2+k}$	$2.3\times10^{3-k}$	1.82
Geminga	$3.96\times10^{-5-k}$	0.0775	$1.51\times10^{2+k}$	$1.1\times10^{2+k}$	$5.75\times10^{4-k}$	0.16
B1055−52	$5.75\times10^{-5-k}$	0.113	$2.20\times10^{2+k}$	$7.3\times10^{1+k}$	$4.9\times10^{4-k}$	1.53
B1951+32	$1.29\times10^{-4-k}$	0.253	$4.94\times10^{2+k}$	$3.3\times10^{1+k}$	$8.7\times10^{2-k}$	2.50
B1509−58	$4.08\times10^{-6-k}$	8.00×10^{-3}	$1.56\times10^{1+k}$	$1.0\times10^{3+k}$	$1.5\times10^{3-k}$	4.3

Input parameters as in Table 1. The expansion parameter for the tachyonic powers and count rates is the ratio $\varepsilon_b/\varepsilon_c$ of break and critical energy, denoted by κ, cf. (2.4) and (2.10). The κ-expansions in Section 3 apply for $\kappa \ll 1$ and $\kappa\gamma^2 \gg 1$, which defines constraints on the electronic Lorentz factor, $\gamma \approx 1.96\times10^{3+k}$. The product $\kappa\gamma$ occurring in the averaged tachyonic spectral densities (3.12) is independent of the electron energy. A large ratio ε_c/E of critical and electron energy indicates that electromagnetic synchrotron radiation and the resulting radiation damping is suppressed by a quantum cutoff. κ_{lc} is the expansion parameter for gyration in the light cylinder fields, cf. Table 1. For κ_{lc} to be small, very high gyration and TeV γ-ray energies are required, which are not observed in the pulsed emission. These light cylinder fields and the surface fields of millisecond pulsars exemplify the crossover between the asymptotic regimes studied here and in Ref. [13]. The pulsar distance D(kpc) is taken from Ref. [26].

are the neutron star and light cylinder radii, respectively, so that $R_{lc}/R_{ns} \approx 4.77P$ [ms]. Some of these quantities are listed in Tables 1 and 2. There is a voltage gap, $\Delta U = B_{lc}R_{lc}$, or $\Delta U[\mathrm{V}] \approx 1.43\times10^9 B_{lc}[\mathrm{G}]P[\mathrm{ms}]$, between surface and light cylinder. This potential drop is an order of magnitude estimate, essentially on dimensional grounds (1G $\approx$ 299.8 V/cm), which may even be substantially reduced by pair production. ΔU is of the order of 10^{14}–10^{16} V for the known γ-ray pulsars [20–22]. This gap is not related to synchrotron radiation, but there may be an important implication for cosmic ray acceleration [23]. Electrons cycling in the surface fields can acquire energies in the low TeV range, cf. Section 3. When subjected to a voltage of the indicated magnitude or even a few orders less, they are accelerated into the 10^{21} eV region, unless slowed down by electromagnetic radiation loss. In the surface field, electromagnetic synchrotron radiation is suppressed by a quantum cutoff, which annihilates the classical radiation peak [19]. However, tachyonic synchrotron radiation below the break frequency (vanishing in the zero-mass limit) is not affected by this cutoff. When the synchrotron electrons or any other charged particles spiral toward the light cylinder, driven by the electric voltage, the synchrotron radii increase, but the electromagnetic radiation loss may still be much smaller than the classical estimates on curvature radiation along the magnetospheric electric field lines would suggest. There is no radiation damping by tachyon radiation, as the Green function outside the light cone is time symmetric and the radiated energy is drained from the absorber field [8,12].

3. Tachyonic γ-rays from pulsar magnetospheres

We will study the tachyonic synchrotron power radiated by electrons gyrating in the surface magnetic fields of γ-ray pulsars, based on the spectral densities (2.1) and (2.5). The parameters of the individual pulsars are listed in Tables 1 and 2. The radiated power and the tachyonic count rates calculated in (3.1)–(3.10) are stated in Table 3 for each of the known γ-ray pulsars, parametrized by the gyration energy. In ((3.11)–(3.13)), we will average the tachyonic spectral densities with an electronic power-law distribution, and then proceed with a phenomenological discussion of the observed γ-ray fluxes, including the detection mechanisms of the γ-ray counters.

We start with the integration of the transversal spectral density (2.1). The power radiated by a single gyrating electron can be split into linear polarization components,

$$P^{\mathrm{T}} := P^{\mathrm{T}}_{\parallel} + P^{\mathrm{T}}_{\perp}, \quad P^{\mathrm{T}}_{\parallel,\perp} := \int_0^{\omega_b} p^{\mathrm{T}}_{\parallel,\perp}(\omega)\,\mathrm{d}\omega. \tag{3.1}$$

Table 3
Superluminal power and tachyonic count rates

	$P(\mathrm{GeV\ s^{-1}}) = P^{\mathrm{T}} + P^{\mathrm{L}}$	$N(10^5\ \mathrm{s^{-1}}) = N^{\mathrm{T}} + N^{\mathrm{L}}$
Crab	$P^{\mathrm{T}} \approx 3.2 \times 10^{2k/3+2} + 1.1k + 2.3$ $P^{\mathrm{L}} \approx 3.1 + 1.1k - 4.8 \times 10^{-2k/3-4}$	$N^{\mathrm{T}} \approx 5.1 + 3.05 \times 10^{-k/3} - 8.5 \times 10^{-k/2-1}$ $N^{\mathrm{L}} \approx 5.1 - 1.2 \times 10^{-k/2-1} + 1.1 \times 10^{-k-3}$
Vela	$P^{\mathrm{T}} \approx 3.0 \times 10^{2k/3+2} + 1.1k + 2.3$ $P^{\mathrm{L}} \approx 3.15 + 1.1k - 5.1 \times 10^{-2k/3-4}$	$N^{\mathrm{T}} \approx 5.1 + 2.8 \times 10^{-k/3} - 8.1 \times 10^{-k/2-1}$ $N^{\mathrm{L}} \approx 5.1 - 1.15 \times 10^{-k/2-1} + 1.1 \times 10^{-k-3}$
B1706−44	$P^{\mathrm{T}} \approx 2.8 \times 10^{2k/3+2} + 1.1k + 2.4$ $P^{\mathrm{L}} \approx 3.2 + 1.1k - 5.45 \times 10^{-2k/3-4}$	$N^{\mathrm{T}} \approx 5.1 + 2.7 \times 10^{-k/3} - 7.7 \times 10^{-k/2-1}$ $N^{\mathrm{L}} \approx 5.1 - 1.1 \times 10^{-k/2-1} + 1.1 \times 10^{-k-3}$
Geminga	$P^{\mathrm{T}} \approx 1.8 \times 10^{2k/3+2} + 1.1k + 2.5$ $P^{\mathrm{L}} \approx 3.3 + 1.1k - 8.5 \times 10^{-2k/3-4}$	$N^{\mathrm{T}} \approx 5.1 + 1.7 \times 10^{-k/3} - 5.5 \times 10^{-k/2-1}$ $N^{\mathrm{L}} \approx 5.1 - 7.9 \times 10^{-k/2-2} + 1.1 \times 10^{-k-3}$
B1055−52	$P^{\mathrm{T}} \approx 1.4 \times 10^{2k/3+2} + 1.1k + 2.6$ $P^{\mathrm{L}} \approx 3.4 + 1.1k - 1.1 \times 10^{-2k/3-3}$	$N^{\mathrm{T}} \approx 5.1 + 1.3 \times 10^{-k/3} - 4.6 \times 10^{-k/2-1}$ $N^{\mathrm{L}} \approx 5.1 - 6.5 \times 10^{-k/2-2} + 1.1 \times 10^{-k-3}$
B1951+32	$P^{\mathrm{T}} \approx 8.2 \times 10^{2k/3+1} + 1.1k + 2.8$ $P^{\mathrm{L}} \approx 3.6 + 1.1k - 1.9 \times 10^{-2k/3-3}$	$N^{\mathrm{T}} \approx 5.1 + 7.8 \times 10^{-k/3-1} - 3.1 \times 10^{-k/2-1}$ $N^{\mathrm{L}} \approx 5.1 - 4.4 \times 10^{-k/2-2} + 1.1 \times 10^{-k-3}$
B1509−58	$P^{\mathrm{T}} \approx 8.2 \times 10^{2k/3+2} + 1.1k + 2.0$ $P^{\mathrm{L}} \approx 2.8 + 1.1k - 1.9 \times 10^{-2k/3-4}$	$N^{\mathrm{T}} \approx 5.1 + 7.8 \times 10^{-k/3} - 1.7 \times 10^{-k/2}$ $N^{\mathrm{L}} \approx 5.1 - 2.45 \times 10^{-k/2-1} + 1.1 \times 10^{-k-3}$

Input parameters as in Tables 1 and 2, the parameter k refers to the electron energy, $E = 10^k$ GeV. The transversal (unpolarized) and longitudinal powers $P^{\mathrm{T,L}}(\mathrm{GeV\ s^{-1}})$ include linear and curvature radiation, cf. Section 3. The corresponding tachyonic count rates are denoted by $N^{\mathrm{T,L}}(10^5\ \mathrm{s^{-1}})$. For comparison, the superluminal power transversally and longitudinally radiated by an electron in straight uniform motion with the same Lorentz factor reads $P^{\mathrm{T,lin}}(\mathrm{GeV\ s^{-1}}) \approx 1.6k + 4.95$ and $P^{\mathrm{L,lin}}(\mathrm{GeV\ s^{-1}}) \approx 1.6k + 5.3$, respectively [13]. The curvature radiation is obtained by subtracting these linear powers from $P^{\mathrm{T,L}}$. The linear count rates $N^{\mathrm{T,L,lin}}$ approach a constant value of $5.1 \times 10^5\ \mathrm{s^{-1}}$ in the ultra-relativistic limit.

Alternatively, we may decompose P^{T} into a linear and a curvature component according to ((2.6)–(2.8)),

$$P^{\mathrm{T}} = P^{\mathrm{T,lin}} - P^{\mathrm{T,curv}}, \quad P^{\mathrm{T,curv}} := P_{\parallel}^{\mathrm{T,curv}} + P_{\perp}^{\mathrm{T,curv}},$$

$$P^{\mathrm{T,lin}} := \int_0^{\omega_{\mathrm{b}}} p^{\mathrm{T,lin}}(\omega)\,\mathrm{d}\omega,$$

$$P_{\parallel,\perp}^{\mathrm{T,curv}} := \int_0^{\omega_{\mathrm{b}}} p_{\parallel,\perp}^{\mathrm{T,curv}}(\omega)\,\mathrm{d}\omega, \tag{3.2}$$

so that $P_{\parallel,\perp}^{\mathrm{T}} = P^{\mathrm{T,lin}}/2 - P_{\parallel,\perp}^{\mathrm{T,curv}}$. The power stemming from the linear transversal density (2.7) is readily calculated,

$$P^{\mathrm{T,lin}} \sim \alpha_q \frac{m_{\mathrm{t}}^2 c^4}{\hbar}\left(\log\gamma - \frac{1}{2}\right),$$

$$\alpha_q \frac{m_{\mathrm{t}}^2 c^4}{\hbar} \approx 0.70\ \mathrm{GeV\,s^{-1}}, \tag{3.3}$$

which is the leading order in the ultra-relativistic $1/\gamma$-expansion of the first integral in (3.2), cf. Ref. [12]. We have restored the natural units, $m_{\mathrm{t}} \to m_{\mathrm{t}}c/\hbar$; the tachyonic fine structure constant α_q is defined before (2.10). The second integral in (3.2) gives the transversal curvature radiation,

$$P_{\parallel,\perp}^{\mathrm{T,curv}} \sim -\frac{1}{2}\frac{\alpha_q m_{\mathrm{t}}^2 c^4}{\hbar\kappa^{2/3}}\left(\frac{1}{2}\frac{2 \mp 1}{2^{1/3}\Gamma(1/3)} - \frac{\kappa^{2/3}}{3}\left(\log\frac{1}{\kappa} + \log 2 - \frac{3}{2}\log 3 - \gamma_E + 2 \mp \frac{3}{2}\right) + \mathrm{O}(\kappa^{4/3})\right),$$

$$P^{\mathrm{T,curv}}[\mathrm{GeV\ s^{-1}}] \approx -\frac{0.21}{\kappa^{2/3}} + 0.23\log\frac{1.6}{\kappa} + \cdots \tag{3.4}$$

The lower plus-sign in (3.4) refers to the $\perp$-polarization, and $P^{\mathrm{T,curv}}$ is the total transversally polarized curvature radiation, cf. (3.2). The expansion parameter κ, the ratio of break and critical frequency, is defined in (2.10), and $\gamma_{\mathrm{E}} \approx 0.5772$.

The longitudinal radiant power can likewise be decomposed into a linear and a curvature term, cf. (2.6),

$$P^{\mathrm{L}} := \int_0^{\omega_{\mathrm{b}}} p^{\mathrm{L}}(\omega)\,\mathrm{d}\omega = P^{\mathrm{L,lin}} - P^{\mathrm{L,curv}},$$

$$P^{\mathrm{L,lin}} := \int_0^{\omega_{\mathrm{b}}} p^{\mathrm{L,lin}}(\omega)\,\mathrm{d}\omega,$$
$$P^{\mathrm{L,curv}} := \int_0^{\omega_{\mathrm{b}}} p^{\mathrm{L,curv}}(\omega)\,\mathrm{d}\omega. \tag{3.5}$$

An elementary integration gives the linear power,

$$P^{\mathrm{L,lin}} \sim \alpha_q \frac{m_{\mathrm{t}}^2 c^4}{\hbar} \log\gamma, \tag{3.6}$$

up to $\mathrm{O}(\gamma^{-1})$ like in (3.3). The κ-expansion of the longitudinal curvature radiation reads,

$$P^{\mathrm{L,curv}} = \frac{\alpha_q m_{\mathrm{t}}^2 c^4}{\hbar}\frac{1}{3}\left(\log\frac{1}{\kappa} + \log 2 - \frac{3}{2}\log 3 - \gamma_E + \frac{27}{4}\frac{\kappa^{2/3}}{2^{2/3}\Gamma(2/3)} + \mathrm{O}(\kappa)\right),$$
$$P^{\mathrm{L,curv}}[\mathrm{GeV\,s^{-1}}] \approx 0.23\log\frac{0.216}{\kappa} + 0.73\kappa^{2/3} + \cdots \tag{3.7}$$

The total power radiated is $P = P^{\mathrm{T}}+P^{\mathrm{L}}$, with the polarized powers $P^{\mathrm{T,L}}$, cf. (3.2) and (3.5).

The count rates are composed like in (3.2) and (3.5), with the spectral densities divided by $\hbar\omega$; we write the transversal count as $N^{\mathrm{T}} = N^{\mathrm{T,lin}} - N^{\mathrm{T,curv}}$, where $N^{\mathrm{T,curv}} = N_{\parallel}^{\mathrm{T,curv}} + N_{\perp}^{\mathrm{T,curv}}$. The linear count is constant in leading order, in the ultra-relativistic regime that is,

$$N^{\mathrm{T,lin}} \sim \alpha_q \frac{m_{\mathrm{t}} c^2}{\hbar}\frac{\pi}{2}, \quad N^{\mathrm{T,lin}} \approx 5.1\times 10^5\ \mathrm{s^{-1}}. \tag{3.8}$$

The contribution of the orbital curvature to the transversal count is

$$N_{\parallel,\perp}^{\mathrm{T,curv}} = -\frac{1}{2}\frac{\alpha_q m_{\mathrm{t}}^2 c^4}{\hbar^2\omega_{\mathrm{b}}\kappa^{2/3}}\left(\frac{4\mp 2}{2^{1/3}\Gamma(1/3)} - (14\mp 6)\frac{1}{3^{3/4}}\frac{\Gamma(3/4)}{\Gamma(1/4)}\kappa^{1/6} + \mathrm{O}(\kappa^{2/3})\right),$$
$$N^{\mathrm{T,curv}}[10^5\ \mathrm{s^{-1}}] \approx -\frac{1}{\gamma}\left(\frac{3.9}{\kappa^{2/3}} - \frac{6.8}{\kappa^{1/2}} + \cdots\right), \tag{3.9}$$

where we used $\alpha_q m_{\mathrm{t}} c^2/\hbar \approx 3.27\times 10^5\ \mathrm{s^{-1}}$, cf. before (2.10). The longitudinal count rate, $N^{\mathrm{L}} = N^{\mathrm{L,lin}} - N^{\mathrm{L,curv}}$, is composed of the linear count, $N^{\mathrm{L,lin}} \sim N^{\mathrm{T,lin}}$, and the curvature term

$$N^{\mathrm{L,curv}} = \frac{\alpha_q m_{\mathrm{t}}^2 c^4}{\hbar^2\omega_{\mathrm{b}}\kappa^{1/2}}\frac{2}{3}\left(3^{1/4}\frac{\Gamma(3/4)}{\Gamma(1/4)} - \kappa^{1/2} + \mathrm{O}(\kappa)\right),$$
$$N^{\mathrm{L,curv}}[10^5\mathrm{s^{-1}}] \approx \frac{1}{\gamma}\left(\frac{0.97}{\kappa^{1/2}} - 2.2 + \cdots\right). \tag{3.10}$$

In Table 2, the expansion parameter κ is listed for the individual pulsars, parametrized by the electronic gyration energy. In Table 3, we list the above expansions parametrized accordingly, which gives an overview as to how the magnetic field strengths and electron energies impact the weight of the linear and curvature terms. The range of electron energies (k-interval) for these expansions to apply can be read off from Table 2 for each pulsar.

To connect to γ-ray fluxes from magnetospheric electron populations, we need to know the tachyonic spectral densities generated by a non-singular electron distribution. To this end, we average the densities (2.2) and (2.5) with an electronic power-law, $\mathrm{d}n(\gamma) = A\gamma^{-s}\mathrm{d}\gamma$, of index $s > 1$. The ultra-relativistic electronic Lorentz factors range in a finite interval, $\gamma_1 \leqslant \gamma \leqslant \gamma_2$, with $\gamma_1 \gg 1$; the normalization constant A relates to the electron count in this interval as stated in (3.12). The transversal density is averaged as

$$\langle p_{\parallel,\perp}^{\mathrm{T}}(\omega)\rangle_s = \int_{\gamma_1}^{\gamma_2} p_{\parallel,\perp}^{\mathrm{T}}(\omega,\gamma)\,\mathrm{d}n(\gamma), \tag{3.11}$$

and the longitudinal average $\langle p^{\mathrm{L}}(\omega)\rangle_s$ is defined in the same way. We will focus on the number densities $\langle n_{\parallel,\perp}^{\mathrm{T}}(\omega)\rangle_s = \langle p_{\parallel,\perp}^{\mathrm{T}}(\omega)\rangle_s/(\hbar\omega)$ and $\langle n^{\mathrm{L}}(\omega)\rangle_s = \langle p^{\mathrm{L}}(\omega)\rangle_s/(\hbar\omega)$, since scaling exponents for γ-rays are usually defined with regard to differential number counts. We restore the natural units, $m_{\mathrm{t}} \to m_{\mathrm{t}}c/\hbar$, and use the fine structure constant $\alpha_q = q^2/(4\pi\hbar c)$ and the scaling variable $\hat\omega := \hbar\omega/(m_{\mathrm{t}}c^2)$, to find

$$\langle n_{\parallel,\perp}^{\mathrm{T}}(\omega)\rangle_s \approx \frac{1}{2}\frac{(4\mp 2)}{2^{1/3}\Gamma(1/3)}\frac{A\alpha_q}{3s-1}\frac{\hat\omega^{-(1/3+s)}}{(\kappa\gamma)^{2/3}},$$
$$\langle n^{\mathrm{L}}(\omega)\rangle_s \approx \frac{1}{3}\frac{A\alpha_q}{s-1}\hat\omega^{-(1+s)}, \quad A = \frac{(s-1)n_{\mathrm{e}}\gamma_1^{s-1}}{1-(\gamma_1/\gamma_2)^{s-1}}. \tag{3.12}$$

This frequency scaling, the leading order asymptotics of the integrals (3.11), is valid in the range $\gamma_1 \ll \hat{\omega} \ll \gamma_2$ and for electron indices $s > 1$. The amplitude A of the electronic power-law depends on the electronic source number n_e. The product $\kappa\gamma$ of expansion parameter and Lorentz factor is independent of γ, cf. (2.4) and (2.10); thus we may take both factors at γ_1 and conclude that the transversal number density overpowers the longitudinal one, since $\kappa\gamma/\hat{\omega} \ll 1$. In fact, the ratio of the longitudinal and transversal densities (3.12) reads,

$$\frac{\langle n^{\mathrm{L}}\rangle_s}{\langle n^{\mathrm{T}}\rangle_s} \approx 4.7 \times 10^{-3}\, \frac{3s-1}{s-1}\, \frac{(\kappa\gamma)^{2/3}}{E_{\mathrm{t}}^{2/3}}, \tag{3.13}$$

with $s > 1$. The constant $\kappa\gamma$ is listed in Table 2 for the respective pulsars, and the tachyon energy, $E_t = \hbar\omega$, is meant in MeV units. In the frequency bands studied in the subsequent examples of γ-ray pulsars, this ratio is well below 10^{-2}, so that we will focus on the transversal radiation only. The transversal densities $\langle n^{\mathrm{T}}_{\parallel,\perp}\rangle_s$ in (3.12) also apply to electron indices $1/3 < s < 1$, as one may expect, but this requires a further restriction on the scaling range, a limit on the upper cutoff of the electron density, $\gamma_2 \ll \gamma_1/\kappa(\gamma_1)$. For these indices, the radiation is likewise overwhelmingly transversal.

We denote the energy of the tachyonic γ-rays by E_t as in (3.13). The tachyon flux, $F = \mathrm{d}N/\mathrm{d}E_t$, is derived from the normalized differential number count $\mathrm{d}N = (4\pi D^2)^{-1}\langle n^{\mathrm{T}}(\omega)\rangle_s \mathrm{d}\omega$, where $n^{\mathrm{T}} = n^{\mathrm{T}}_{\parallel} + n^{\mathrm{T}}_{\perp}$ stands for the total (unpolarized) transversal radiation, cf. (3.12), and D is the distance to the pulsar, cf. Table 2. Concerning units, we choose $F[\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{MeV}^{-1}] \propto E_t^{-\alpha}$, where E_t is in MeV, and the measured proportionality factor and scaling exponent will be specified below for the enumerated pulsars. These units will be used throughout, mostly without mentioning. The scaling exponent relates to the electron index as $\alpha = 1/3 + s$, cf. (3.12), and the dimensionless number density can be retrieved from the flux and the distance, $\langle n^{\mathrm{T}}(\omega)\rangle_s = 4\pi D^2\hbar F$; we will infer the electronic source count n_e by equating $\langle n^{\mathrm{T}}_{\parallel} + n^{\mathrm{T}}_{\perp}\rangle_s$ in (3.12) with

$$\langle n^{\mathrm{T}}\rangle_s \approx 7.875 \times 10^{22} D^2[\mathrm{kpc}] F[\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{MeV}^{-1}]. \tag{3.14}$$

In the following case study of the seven established γ-ray pulsars, the observational input will be the amplitude and the scaling exponent of the measured flux, $F \propto E_t^{-\alpha}$, the energy band for a scaling exponent to apply, as well as the spectral breaks where different power-laws join. We will determine the electron distributions generating the radiation in the respective frequency bands, in particular the electronic source count, that is, the number of gyrating electrons generating the tachyon flux or a power-law component of it. When inferring the electronic source number from the measured fluxes, we have to take into account that only a fraction α_q/α_e (ratio of tachyonic and photonic fine structure constants, cf. before (2.10)) of the tachyon flux passing through the detectors is absorbed. This requires a rescaling of the electron count n_e by $\alpha_e/\alpha_q \approx 7.3 \times 10^{10}$, where n_e is based on the tachyon count in the detectors. That is, the electronic source count n_e is calculated, via (3.14) and (3.12), from the observed flux F, but the actual tachyon flux is by a factor of α_e/α_q larger than the observed one, so that the actual number of synchrotron electrons in the surface magnetic field is $N_e \approx n_e\alpha_e/\alpha_q$.

The three telescopes employed in measuring the fluxes in the various energy bands cited below operated with very different detection mechanisms, but the rescaling of the observed fluxes with α_e/α_q applies to all of them. In the OSSE detector [9], the basic interaction was ionization generating scintillations in NaI(Tl) crystals. COMPTEL [10] was a counter based on Compton scattering, and the EGRET telescope [11] counted γ-rays by conversion into electron-positron pairs. The crucial point here is, that the cross-sections for ultra-relativistic photoionization, photonic Compton scattering and photonic pair production also apply to transversal tachyons, with the mentioned rescaling by α_q/α_e. The tachyon mass drops out in the dispersion relation, $k^2 = \omega^2/c^2 + (m_t c/\hbar)^2$, at γ-ray energies, which only makes an overall rescaling of the respective photonic cross-sections [24] necessary. This rescaling also shows in the non-relativistic limit, derived for tachyonic ionization in Ref. [25], where we also studied the effect of the tachyon mass in the low-energy (soft X-ray) regime. The same rescaling of the Compton cross-section

(Klein–Nishina formula) can be traced back to classical Thomson scattering, that is, to the acceleration of an electron by the incoming tachyonic wave field, which triggers electromagnetic radiation [8]. Despite the fact that the photonic cross-sections of the three mentioned processes scale with higher powers of the electric fine structure constant, the tachyonic counterpart is obtained in all three cases by an α_q/α_e-rescaling, for transversal γ-rays at least; this is further discussed in the Conclusion.

Apart from the electronic source count and the total gyration energy, $E_e = mc^2 A \int_{\gamma_1}^{\gamma_2} \gamma^{1-s}\,d\gamma$, stored in these electrons, we will determine their tachyonic γ-ray luminosity, the power transversally radiated in a given frequency band,

$$L_t = (1\ \mathrm{MeV})^2 \hbar^{-1} \int_{E_{t,1}}^{E_{t,2}} \langle n^T \rangle_s E_t \,dE_t, \tag{3.15}$$

where the interval boundaries $E_{t,1,2}$ are in MeV units. The conversion factors used in the sequel are 1 TeV ≈ 1.602 erg and 1 kpc ≈ 3.086×10^{21} cm. The tachyon energy E_t is given in MeV and the electron energy E in GeV, parametrized by 10^k like in the tables, and the flux F is in units of $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{MeV}^{-1}$, cf. (3.14).

We start with the Crab pulsar. The flux in the 30–4000 MeV interval scales as $F \approx 2.9 \times 10^{-4} E_t^{-2.05}$, cf. Refs. [27,28], suggesting an electron index $s \approx 1.72$. As pointed out after (3.14), the observed flux has still to be rescaled by a factor of $\alpha_e/\alpha_q \approx 7.3 \times 10^{10}$, as most tachyons pass unnoticed through the detectors. The cited interval boundaries correspond to electron energies of $10^{0.85}$ and $10^{3.0}$ GeV, respectively. There is a spectral break at 30 MeV, followed by a second power-law, $F \approx 5.7 \times 10^{-4} E_t^{-2.25}$, in the 0.12–30 MeV band, cf. Refs. [29,30]. This flux component is generated by electrons gyrating with energies from $10^{-1.55}$ to $10^{0.85}$ GeV, with power-law index $s \approx 1.92$. The tachyonic spectral densities above and below the spectral break relate to the amplitudes of the respective electronic power-laws as $\langle n^T \rangle_{s=1.72} \approx 9.5 \times 10^{-6} A^{\alpha=2.05} \alpha_q E_t^{-2.05}$ and $\langle n^T \rangle_{s=1.92} \approx 2.4 \times 10^{-6} A^{\alpha=2.25} \alpha_q E_t^{-2.25}$, cf. (3.12). The normalization constants of the electron distributions in the adjacent intervals above and below the break energy (of $10^{0.85}$ GeV) read $A^{\alpha=2.05} \approx 690 n_e$ and $A^{\alpha=2.25} \approx 36.9 n_e$, respectively. We find $n_e \approx 1.4 \times 10^{35}$ in the upper interval (where $F \propto E_t^{-2.05}$) and $n_e \approx 2.0 \times 10^{37}$ below the break energy (where $F \propto E_t^{-2.25}$). These electron counts are calculated by equating flux and number density, cf. (3.14) and (3.12), and they still have to be renormalized; the actual count in the $10^{0.85}-10^{3.0}$ GeV range reads $N_e^{\alpha=2.05} \approx 1.0 \times 10^{46}$, and $N_e^{\alpha=2.25} \approx 1.5 \times 10^{48}$ is the count below the break energy, in the $10^{-1.55}-10^{0.85}$ GeV interval. The total electronic energy in the surface magnetic field is calculated as indicated before (3.15); we find $E_e^{s=1.72}[\mathrm{erg}] \approx 0.089 n_e^{\alpha=2.05}$ above the break energy, and $E_e^{s=1.92}[\mathrm{erg}] \approx 3.0 \times 10^{-4} n_e^{\alpha=2.25}$ below. The respective electron populations produce the tachyonic luminosity $L_t^{\alpha=2.05}[\mathrm{erg/s}] \approx 5.85 n_e^{\alpha=2.05}$ in the 30–4000 MeV band, and $L_t^{\alpha=2.25}[\mathrm{erg/s}] \approx 0.11 n_e^{\alpha=2.25}$ in the 0.12–30 MeV interval, cf. (3.15). These gyration energies and luminosities are based on the observed flux F; the actual tachyon flux is by a factor of α_e/α_q higher, so that we have to replace the n_e^α by the renormalized counts N_e^α.

Remarks. There is no superluminal radiation damping that could slow down the gyration, as the tachyonic Green function is time symmetric. The energy radiated stems from the oscillators of the cosmic absorber [25,31]. The absorber field breaks the time symmetry and renders the interaction causal and non-local [8]. The latter has implications on energy conservation, as the radiated energy is drained from the absorber. For this reason, the search for missing negative mass-squares suggested in Ref. [4] fails, where bubble-chamber events were reanalyzed, assuming a local but otherwise unspecified interaction of tachyons with matter, discounting the time symmetry of the Green function outside the light cone. Had there been detection in these experiments, in the relativistic framework in which they were interpreted, this would have been tantamount to causality violation. The radiation mechanism scrutinized here, based on a Proca field with negative mass-square minimally coupled to subluminal matter, is non-relativistic. It implies the absolute spacetime defined by the absorber [5,6], even though the Lagrangians are still covariant. Finally, electromagnetic synchrotron radiation would lead to radiation damping, but is suppressed by a quantum cutoff, cf. after (2.11).

There is a further spectral break at 0.12 MeV, where the spectrum flattens to $F \approx 2.0 \times 10^{-3} E_t^{-1.67}$, cf. Ref. [29], and then continues as an unbroken power-law even into the soft X-ray band [32,33]. The radiation below 0.12 MeV is not accessible in the ultra-relativistic limit because of the moderate electronic Lorentz factors; the spectral densities would have to be recalculated, with the Nicholson asymptotics replaced by Debye's approximation of large-order Bessel functions [34]. At the opposite end, above 4000 MeV, there is some indication that the radiation terminates in a steep spectral slope, and no pulsed emission is detected in the TeV region [28]. The fluxes used here and in the subsequent examples refer to the pulsed emission, and they are phase averages. Phase-resolved spectra exist for the Crab and Vela pulsars and Geminga. As for the Crab pulsar, the phase-dependent exponents α range from 1.42 to 2.65 in the 30–4000 MeV band [27], and from 2.16 to 2.41 in the 0.12–30 MeV interval, cf. Ref. [29]. The respective phase-resolved electron indices, $s = \alpha - 1/3$, are usually larger than 1, and they never fall below 1/3, cf. after (3.13).

The Vela pulsar radiates a flux of $F \approx 9.4 \times 10^{-5} E_t^{-1.62}$ in the 30–2000 MeV interval, cf. Refs. [27,35], generated by electrons in the $10^{0.85}$–$10^{2.7}$ GeV range with power-law index $s \approx 1.29$. The normalization of the electron distribution in this range is $A \approx 6.32 n_e$, and the electronic source count reads $n_e \approx 5.9 \times 10^{33}$ or $N_e \approx 4.3 \times 10^{44}$ if renormalized. These electrons store an energy of $E_e[\mathrm{erg}] \approx 0.125 N_e$ and generate the number density $\langle n^T \rangle_{s=1.29} \approx 1.8 \times 10^{-4} \; A \alpha_q E_t^{-1.62}$, which results in a tachyonic γ-ray luminosity of $L_t[\mathrm{erg/s}] \approx 10.4 N_e$ in the 30–2000 MeV band. The phase-resolved exponents of this pulsar range from 1.38 to 2.21, cf. Ref. [27]. Pulsed TeV radiation is not detectable [36].

γ-Radiation from PSR B1706−44 has been detected from 50 MeV up to 20 GeV, cf. Ref. [37]. A flux $F \approx 1.3 \times 10^{-6} E_t^{-1.27}$ is observed in the 50–1000 MeV interval, corresponding to electron energies from $10^{1.1}$ to $10^{2.4}$ GeV. This is the only example where the (phase-averaged) electron index drops below 1, $s \approx 0.94$. The electronic power-law normalization in the indicated GeV range is $A^{\alpha=1.27} \approx 0.160 n_e$, and the tachyonic number density scales as $\langle n^T \rangle_{s=0.94} \approx 2.3 \times 10^{-3} A^{\alpha=1.27} \alpha_q E_t^{-1.27}$ in the 50–1000 MeV interval. There is a spectral break at 1000 MeV, followed by a second flux component, $F \approx 1.15 \times 10^{-3} E_t^{-2.25}$, cf. Ref. [37], which applies up to 20 GeV, suggesting electron energies up to $10^{3.7}$ GeV in the surface field. This flux is generated by an electronic power-law of index $s \approx 1.92$ and amplitude $A^{\alpha=2.25} \approx 1.54 \times 10^5 n_e$ in the $10^{2.4}$–$10^{3.7}$ GeV range. The tachyonic number density in the 1–20 GeV interval reads $\langle n^T \rangle_{s=1.92} \approx 2.1 \times 10^{-6} A^{\alpha=2.25} \alpha_q E_t^{-2.25}$. By the way, the frequency scaling is obtained by substituting $\hbar\omega/1$ MeV for E_t. Both electronic power-laws admit the same source number, $n_e \approx 9.2 \times 10^{33}$, valid for $s \approx 0.94$ as well as $s \approx 1.92$; the renormalized source count for each of these slopes is accordingly $N_e \approx 6.7 \times 10^{44}$. These electron populations below and above the break energy of $10^{2.4}$ GeV contain a gyration energy of $E_e^{s=0.94}[\mathrm{erg}] \approx 0.13 N_e$ and $E_e^{s=1.92}[\mathrm{erg}] \approx 1.3 N_e$, respectively; they produce a tachyonic luminosity of $L_t^{\alpha=1.27}[\mathrm{erg/s}] \approx 17 N_e$ in the 50–1000 MeV band, and of $L_t^{\alpha=2.25}[\mathrm{erg/s}] \approx 29.5 N_e$ above the spectral break.

Geminga generates the flux $F \approx 1.2 \times 10^{-5} E_t^{-1.42}$ in the 30–2000 MeV band, corresponding to electron energies between $10^{0.85}$ and $10^{2.7}$ GeV, cf. Refs. [27,38,39]. This flux allows us to infer the electron index, $s \approx 1.09$, and the tachyonic number density, $\langle n^T \rangle_{s=1.09} \approx 4.7 \times 10^{-4} A \alpha_q E_t^{-1.42}$. The electronic power-law normalization in the mentioned GeV interval is $A \approx 0.649 n_e$, with source count $n_e \approx 7.9 \times 10^{32}$. We find the renormalized electron count, $N_e \approx 5.8 \times 10^{43}$, the total energy of these electrons, $E_e[\mathrm{erg}] \approx 0.16 N_e$, as well as their tachyonic γ-ray luminosity in the 30–2000 MeV interval, $L_t[\mathrm{erg/s}] \approx 9.6 N_e$. The power-law indices of the phase-resolved flux range from 1.27 to 1.89, cf. Ref. [27].

PSR B1055−52 radiates the flux $F \approx 1.8 \times 10^{-6} E_t^{-1.58}$ in the 70–1000 MeV band [40], generated by electrons cycling with energies from $10^{1.2}$ to $10^{2.4}$ GeV. In this range, we infer an electronic power-law distribution of index $s \approx 1.25$, normalization $A^{\alpha=1.58} \approx 6.65 n_e$, and source number $n_e \approx 4.4 \times 10^{33}$, which produces the tachyonic number density $\langle n^T \rangle_{s=1.25} \approx 1.1 \times 10^{-4} A^{\alpha=1.58}$

$\alpha_q E_t^{-1.58}$ in the 70–1000 MeV interval. Above the spectral break at 1000 MeV, the flux scales as $F \approx 4.2 \times 10^{-5} E_t^{-2.04}$ up to at least 4000 MeV, cf. Ref. [40]. This means electron energies up to $10^{3.0}$ GeV in the surface field, an electronic power-law index $s \approx 1.71$ above the electronic break energy of $10^{2.4}$ GeV, and a tachyonic number density of $\langle n^{\mathrm{T}} \rangle_{s=1.71} \approx 4.45 \times 10^{-6} A^{\alpha=2.04} \alpha_q E_t^{-2.04}$ in the 1000–4000 MeV interval. The electronic power-law normalization in the $10^{2.4}-10^{3.0}$ GeV range reads $A^{\alpha=2.04} \approx 1.15 \times 10^4 n_e$, with $n_e \approx 1.5 \times 10^{33}$. Below the electronic break energy, we count $N_e^{\alpha=1.58} \approx 3.2 \times 10^{44}$ electrons, which store a total energy of $E_e^{s=1.25}[\mathrm{erg}] \approx 0.12 N_e^{\alpha=1.58}$ and radiate a power of $L_t^{\alpha=1.58}[\mathrm{erg/s}] \approx 5.2 N_e^{\alpha=1.58}$ in tachyonic γ-rays in the 70–1000 MeV band. Above the break energy, there are $N_e^{\alpha=2.04} \approx 1.1 \times 10^{44}$ electrons gyrating in the $10^{2.4}$–$10^{3.0}$ GeV range, which add up to an electronic energy of $E_e^{s=1.71}[\mathrm{erg}] \approx 0.74 N_e^{\alpha=2.04}$ and produce a tachyonic luminosity of $L_t^{\alpha=2.04}[\mathrm{erg/s}] \approx 13 N_e^{\alpha=2.04}$ in the 1000–4000 MeV interval.

γ-radiation from PSR B1951+32 has been detected in a wide band, from 10 MeV to 20 GeV, cf. Refs. [41,42], generated by electrons gyrating in the $10^{0.38}$–$10^{3.7}$ GeV range. The flux scales as $F \approx 3.5 \times 10^{-6} E_t^{-1.74}$, cf. Ref. [41], seemingly without a spectral break in this band. We find the electron index $s \approx 1.41$, the electronic power-law normalization $A \approx 13.2 n_e$, and the tachyonic number density $\langle n^{\mathrm{T}} \rangle_{s=1.41} \approx 2.1 \times 10^{-5} A \alpha_q E_t^{-1.74}$, from which we infer the electronic source count $n_e \approx 6.2 \times 10^{34}$ or $N_e \approx 4.5 \times 10^{45}$ when renormalized. The gyration energy of these electrons is $E_e[\mathrm{erg}] \approx 0.25 N_e$, and their tachyonic radiation power in the 10 MeV–20 GeV interval amounts to $L_t[\mathrm{erg/s}] \approx 2.9 N_e$. A spectral break must occur below 10 MeV, according to the upper flux limits derived in Refs. [43,44]; no pulsed TeV radiation has been found [45].

PSR B1509−58 is not detected in hard γ-rays, possibly due to a quantum cutoff inflicted by its high magnetic field, but this needs further investigation in second quantization, cf. after (2.11). The observed flux, $F \approx 8.7 \times 10^{-5} E_t^{-1.68}$, extends from 0.2 to 10 MeV, cf. Refs. [46,47], and is produced by electrons with energies in the $10^{-1.3}$–$10^{0.38}$ GeV range. We infer an electronic power-law of index $s \approx 1.35$ and normalization $A \approx 2.25 n_e$, where $n_e \approx 1.75 \times 10^{36}$, as well as a tachyonic number density of $\langle n^{\mathrm{T}} \rangle_{s=1.35} \approx 3.2 \times 10^{-4} A \alpha_q E_t^{-1.68}$ in the mentioned MeV band. We note the electron count in the surface field, $N_e \approx 1.3 \times 10^{47}$, the energy stored in these electrons, $E_e[\mathrm{erg}] \approx 6.5 \times 10^{-4} N_e$, and their tachyonic luminosity, $L_t[\mathrm{erg/s}] \approx 0.82 N_e$, in the 0.2–10 MeV interval. Above 10 MeV, there is a steep spectral break [47], and no pulsed TeV radiation is observed [48]. At the lower end, at 0.2 MeV, there is likewise a spectral break, where the spectrum hardens, admitting a spectral index of about $\alpha \approx 1.3$ in the hard X-ray band [49,50], but this radiation is not accessible in the ultra-relativistic synchrotron limit [34].

4. Conclusion

We have given a quantitative and in part phenomenological discussion of superluminal synchrotron radiation in strong magnetic fields. We will further comment on the interaction of tachyons with matter, on the basis of the three detectors used to infer the magnetospheric electron populations and their tachyonic γ-ray luminosity. The ionization in the scintillation crystals of the OSSE counter [9] happens in the relativistic regime. OSSE operated above a 50 keV threshold, so that the tachyon mass of 2 keV can be neglected in the dispersion relation, given that only the squared energies enter, cf. before (3.15). Therefore, the relativistic photonic ionization cross-section [24] can be used for transversal tachyons, if properly rescaled. The only change necessary for tachyonic X-rays or γ-rays above 50 keV is a rescaling with the ratio of tachyonic and electric fine structure constant. This rescaling can be traced back to the non-relativistic limit, which depends on the tachyon mass and applies in the soft and hard X-ray bands [25]. The longitudinal relativistic cross-section has to be calculated from scratch without reference to a zero-mass limit [51]. The tachyon flux generated by synchrotron radiation in the surface fields of γ-ray pulsars is overwhelmingly transversally polarized, cf. (3.13), in contrast to the low-magnetic-field limit [13], and therefore we have not attempted a quantitative study of the longitudinal fraction beyond some basic estimates.

Compton scattering and pair creation were employed in the COMPTEL and EGRET detectors [10,11]. The tachyonic Thomson cross-section, the non-relativistic classical limit of Compton scattering, was derived in Ref. [8], but a quantum mechanical version is still lacking, especially if the incident tachyonic X-rays have energies close to the tachyon mass. In the relativistic regime, for γ-rays above 0.75 MeV (the threshold of the COMPTEL counter), the transversal cross-section is obtained by a rescaling of the Klein–Nishina formula with α_q/α_e, the same rescaling that applies to the Thomson and ionization cross-sections. The EGRET detector was a spark chamber recording electron-positron pairs produced by γ-rays scattered in tantalum foils. The conversion of tachyons into electron-positron pairs has not been studied in any limit and context as yet. However, one can reckon from the foregoing that the cross-section for the conversion of transversal γ-rays is just the Bethe–Heitler formula rescaled with α_q/α_e; the fluxes collected by EGRET were in the 20 MeV–30 GeV range, so that the tachyon mass is negligible in the dispersion relation. This rescaling of the EGRET fluxes is also suggested by cross-calibration with the COMPTEL detector, to avoid discontinuities in the band overlap.

Tachyonic synchrotron radiation above the break frequency has not been discussed here, cf. after (2.9) and Ref. [13]. This requires second quantization in strong magnetic fields, as quantum corrections in the high-frequency regime cannot be treated perturbatively. Above the break frequency, the classical radiation theory is severely modified by a quantum cutoff, which results in exponential decay of the spectral densities at the critical energy, where the classical spectral functions are peaked. This quantum cutoff may well be the reason that no pulsed TeV radiation is detected from the known γ-ray pulsars.

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