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Superluminal radiation by uniformly moving charges

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Abstract

The emission of superluminal quanta (tachyons) by freely propagating particles is scrutinized. Estimates are derived for spontaneous superluminal radiation from electrons moving close to the speed of the Galaxy in the microwave background. This is the threshold velocity for tachyon radiation to occur, a lower bound. Quantitative estimates are also given for the opposite limit, tachyon radiation emitted by ultra-relativistic electrons in linear colliders and supernova shock waves. The superluminal energy flux is studied and the spectral energy density of the radiation is derived, classically as well as in second quantization. There is a transversal bosonic and a longitudinal fermionic component of the radiation. We calculate the power radiated, its angular dependence, the mean energy of the radiated quanta, absorption and emission rates, as well as tachyonic number counts. We explain how the symmetry of the Einstein *A*-coefficients connects to time-symmetric wave propagation and to the Wheeler–Feynman absorber theory. A relation between the tachyon mass and the velocity of the Local Group of galaxies is suggested.

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1. Introduction

We will explore the spontaneous emission of tachyons by uniformly moving sources. In a relativistic setting such as electrodynamics, freely moving charges do not radiate and radiating particles slow down by radiation losses. (We will consider point charges without an internal structure.) Some explanations as to the context are therefore in order.

When considering superluminal signals, we have to give up relativity or causality, as Lorentz boosts can change the time order of spacelike connections [1–5]. We will maintain causality, and model superluminal signals in an absolute spacetime as defined by the expanding galaxy grid, the rest frame of the microwave background. We may try a wave theory or a particle picture as the starting point. The latter has been studied for quite some time but did not result in viable interactions of tachyons with matter [6–9]. So we suggest to model tachyons as wave fields with negative mass-square, coupled by minimal substitution to subluminal particles.

Whatever the specifics of the superluminal wave equation, there is only one Green function supported outside the light cone; it is time symmetric, half-retarded, half-advanced. To achieve fully retarded wave propagation, an absorber is needed, capable of turning advanced modes into retarded ones [10–15]. A causal theory of superluminal signals requires an absolute space, quite independently of the actual mechanism of signal transfer. On this basis we can identify space itself as the absorber medium, the ether, the medium of wave propagation [16].

Having settled for a wave theory, we have to define the interaction of the superluminal modes with matter. This is the crucial point; after all, what else can one expect from a theory of tachyons other than suggestions as to where to search for them? We will maintain the best established interaction mechanism, minimal substitution, by treating tachyons as a sort of photons with negative mass-square, a real Proca field minimally coupled to subluminal particles [17,18]. Although great care is taken to maintain the analogy to electrodynamics, there are some basic differences. There is no gauge freedom but there is longitudinal radiation, even more pronounced than the transversal counterpart, due to the mass term in the wave equation. More importantly, this is not only a theory of superluminal wave motion, but also a theory of the absolute cosmic spacetime, this cannot be disentangled. The universal frame of reference is generated by the galaxy grid; it is the rest frame of the ether, the absorber medium, as well as the rest frame of the cosmic background radiations [19,20]. Uniform motion and rest are distinguishable states, and in this context we will show that freely moving charges can radiate superluminal quanta. They even do so without slowing down, as the radiated energy is drained from the absorber, from the oscillators of the ether. Superluminal radiation by inertial charges is but a manifestation of the absolute nature of space.

In Section 2 we will derive the superluminal power radiated by a classical point charge in arbitrary motion. We will discuss transversal and longitudinal radiation, its angular dependence, time symmetry outside the lightcone, the absorber field, retardation, and tachyonic Liénard–Wiechert potentials [21,22]. In Section 3, we specialize to uniformly moving charges and calculate the transversal and longitudinal spectral densities. In Section 4 these densities are quantized, and we discuss their asymptotic limits with respect to the speed of the radiating charge. We find a threshold velocity, a lower bound on the speed of the source, for tachyon radiation to occur. This is a pure quantum effect absent in the classical theory. This threshold happens to numerically coincide with the speed of the Galaxy in the microwave background, which suggests a connection between the tachyon mass and the velocity of the Local Group of galaxies in the ether,

$$\frac{v_{\rm LG}}{c} \approx \frac{1}{2}\frac{m_{\rm t}}{m} . \tag{1.1}$$

Here, $v_{\rm LG}/c \approx 2.10 \times 10^{-3}$ is inferred from the temperature dipole anisotropy of the microwave background [23], and the electron–tachyon mass ratio $m_{\rm t}/m_e \approx \frac{1}{238}$ is derived from Lamb shifts in hydrogenic ions [18]. At the end of Section 4, we derive estimates for superluminal radiation (spectral range, power, tachyonic mean energy and number counts, spectral maxima) by electrons in linear colliders and supernova shocks; in this way illustrating the three asymptotic regimes, that is, the ultra-relativistic limit, the non-relativistic limit, and the extreme non-relativistic limit close to the threshold velocity (1.1). In Section 5, we calculate the tachyonic emission rates for freely moving electrons in second quantization, in particular the Einstein A-coefficients for spontaneous emission [24]. The symmetry of the A-coefficients is linked to the spontaneous absorption of absorber quanta balancing the spontaneous tachyon emission. In Section 6, the conclusion, we further discuss radiation by inertial charges, the underlying spacetime concept, the ether, the absorber theory, and compare with the relativistic spacetime view.

2. Superluminal radiation fields, their energy, and the power radiated

The Proca equation [17] with negative mass-square, $F^{\mu\nu}{}_{,\nu} - m_{\rm t}^2 A^\mu = c^{-1} j^\mu$, can equivalently be written as $(\Box + m_{\rm t}^2)A_\mu = -c^{-1} j_\mu$, subject to the Lorentz condition $A^\mu_{,\mu} = 0$. The sign conventions for tachyon mass and field tensor are $m_{\rm t} > 0$ and $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$, for metric and d'Alembertian, $\eta_{\mu\nu} = {\rm diag}(-c^2, 1, 1, 1)$ and $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$, respectively. The tachyon mass $m_{\rm t}$ has the dimension of an inverse length, being a shortcut for $m_{\rm t}c/\hbar$. We find $m_{\rm t}/m_{\rm e} \approx \frac{1}{238}$, estimated from Lamb shifts in hydrogenic systems [18]. The Lagrangian and the energy–momentum tensor of the free Proca field read

$$L_{\rm P} = -\tfrac{1}{4} F_{\alpha\beta}F^{\alpha\beta} + \tfrac{1}{2} m_{\rm t}^2 A_\alpha A^\alpha, \quad T_\mu{}^\nu = -F_{\alpha\mu}F^{\alpha\nu} + m_{\rm t}^2 A_\mu A^\nu - \delta_\mu^\nu L_{\rm P} , \tag{2.1}$$

and the above field equations follow from $L = L_{\rm P} + c^{-1} A_\alpha j^\alpha$. The tachyonic $\mathbf{E}$ and $\mathbf{B}$ fields are related to the vector potential by

$$E_i = c^{-1} F_{i0} = c^{-1}(\nabla A_0 - \partial \mathbf{A}/t), \quad F_{ij} = \varepsilon_{ijk} B^k ,$$
$$B^k = (1/2)\varepsilon^{kij} F_{ij} = {\rm rot}\, \mathbf{A}, \quad A_\alpha = (A_0, \mathbf{A}) , \tag{2.2}$$

so that $F_{\alpha\beta}F^{\alpha\beta} = -2(\mathbf{E}^2 - \mathbf{B}^2)$. The field equations decompose into

$$\operatorname{div}\mathbf{B} = 0, \quad \operatorname{rot}\mathbf{E} + c^{-1}\partial\mathbf{B}/\partial t = 0 ,$$
$$\operatorname{div}\mathbf{E} = \rho - c^{-1}m_t^2 A_0, \quad \operatorname{rot}\mathbf{B} - c^{-1}\partial\mathbf{E}/\partial t = c^{-1}\mathbf{j} + m_t^2\mathbf{A} , \tag{2.3}$$

where we identified $j^\mu = (\rho, \mathbf{j})$. The Lorentz condition $c^{-2}\partial A_0/\partial t = \operatorname{div}\mathbf{A}$ apparently follows from the field equations and current conservation, $\partial\rho/\partial t + \operatorname{div}\mathbf{j} = 0$. The vector potential is completely determined by the current and the **E** and **B** fields, there is no gauge freedom due to the tachyon mass.

We represent the spatial component of the vector potential as $\mathbf{A}(\mathbf{x},t) = (2\pi)^{-1}\int_{-\infty}^{+\infty}\hat{\mathbf{A}}(\mathbf{x},\omega)\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}\omega$, $\hat{\mathbf{A}}^*(\mathbf{x},\omega) = \hat{\mathbf{A}}(\mathbf{x},-\omega)$, and the same relations hold for the time component, the charge and current densities, and the **E** and **B** fields. We consider tachyonic charges, by definition subluminal, located in the vicinity of the coordinate origin. The charges should be confined to a bounded region, so that we can use their asymptotic fields when calculating the energy flux radiated through a large sphere centered at the origin. In the subsequent example of uniformly moving charges, cf. Section 3, we will show how to circumvent this restraint by time averaging. The asymptotic radiation fields can be decomposed into transversally and longitudinally polarized components $\hat{\mathbf{A}}^{\mathrm{T,L}}$. To this end, we define $\hat{\mathbf{j}}^{\mathrm{T}}(\mathbf{x}',\mathbf{x},\omega) := \hat{\mathbf{j}}(\mathbf{x}',\omega) - \mathbf{n}(\mathbf{n}\cdot\hat{\mathbf{j}}(\mathbf{x}',\omega))$ and $\hat{\mathbf{j}}^{\mathrm{L}}(\mathbf{x}',\mathbf{x},\omega) := \mathbf{n}(\mathbf{n}\cdot\hat{\mathbf{j}}(\mathbf{x}',\omega))$, with $\mathbf{n} = \mathbf{x}/r$, and find

$$\hat{\mathbf{A}}^{\mathrm{T,L}}(\mathbf{x},\omega) \sim \frac{1}{4\pi c r}\exp(\mathrm{i}k(\omega)r)\hat{\mathbf{J}}^{\mathrm{T,L}}(\mathbf{x},\omega), \quad k(\omega) := \operatorname{sign}(\omega)\sqrt{\omega^2/c^2 + m_t^2} , \tag{2.4}$$

$$\hat{\mathbf{J}}^{\mathrm{T,L}}(\mathbf{x},\omega) := \int \mathrm{d}\mathbf{x}'\hat{\mathbf{j}}^{\mathrm{T,L}}(\mathbf{x}',\mathbf{x},\omega)\exp(-\mathrm{i}k(\omega)\mathbf{n}\cdot\mathbf{x}') . \tag{2.5}$$

This is completely general, there are no specific assumptions on the current, other than being localized in the vicinity of the coordinate origin, a bounded domain, that is. A discussion of superluminal Green functions and the derivation of (2.4) is given in Ref. [16]. The only classical Green function outside the lightcone is time-symmetric, half-retarded, half-advanced. Its convolution with the current results in a time-symmetric vector field $\hat{\mathbf{A}}^{\mathrm{sym}} = \frac{1}{2}\hat{\mathbf{A}}^{\mathrm{ret}} + \frac{1}{2}\hat{\mathbf{A}}^{\mathrm{adv}}$, where $\hat{\mathbf{A}}^{\mathrm{ret}}$ stands for $\hat{\mathbf{A}}^{\mathrm{T}}$ or $\hat{\mathbf{A}}^{\mathrm{L}}$, and the advanced field $\hat{\mathbf{A}}^{\mathrm{adv}}$ is likewise given by (2.4) with the substitution $k(\omega) \to -k(\omega)$. An absorber medium, the ether, is needed to cancel the advanced component of $\hat{\mathbf{A}}^{\mathrm{sym}}$ and to supply the missing half of the retarded field [12]. The oscillators of the ether [16,20] generate the absorber field, $\hat{\mathbf{A}}^{\mathrm{abs}} = \frac{1}{2}\hat{\mathbf{A}}^{\mathrm{ret}} - \frac{1}{2}\hat{\mathbf{A}}^{\mathrm{adv}}$, which, when added to $\hat{\mathbf{A}}^{\mathrm{sym}}$, results in the fully retarded $\hat{\mathbf{A}}^{\mathrm{T,L}}$ in (2.4). In short, the retarded potential is a superposition of the time-symmetric field of the radiating particle and the absorber field. This is a crucial difference to electromagnetic radiation based on a retarded Green function. There is no radiation damping resulting from spontaneous tachyon radiation, since the energy balance for the time-symmetric field is zero; every outgoing mode has an incoming counterpart. The radiated energy stems from the absorber field, from the oscillators of the ether. The Lorentz force of the absorber field may be compared to inertia, and the derivation of the absorber field from the oscillators of the ether reminds us of the Mach principle, the attempt to extract the inertial force from the galaxy background. In both cases, the result is known beforehand, whatever the derivation.

The Fourier transforms of the field strengths and the time component of the 4-potential are readily calculated by making use of (2.2), (2.4) and the Lorentz condition, $i\omega\hat{A}_0(\mathbf{x},\omega) = -c^2 \,\mathrm{div}\, \hat{\mathbf{A}}(\mathbf{x},\omega)$. The polarized components read in leading order

$$\hat{\mathbf{E}}^{\mathrm{T}}(\mathbf{x},\omega) \sim \frac{i}{4\pi r c^2}\,\omega \exp(ik(\omega)r)\hat{\mathbf{J}}^{\mathrm{T}}(\mathbf{x},\omega)\,,$$

$$\hat{\mathbf{B}}^{\mathrm{T}}(\mathbf{x},\omega) \sim \frac{i}{4\pi r c}\,k(\omega) \exp(ik(\omega)r)\mathbf{n}\times\hat{\mathbf{J}}^{\mathrm{T}}(\mathbf{x},\omega), \quad \hat{A}_0^{\mathrm{T}}(\mathbf{x},\omega) = \mathrm{O}(r^{-2})\,,$$

$$\hat{\mathbf{E}}^{\mathrm{L}}(\mathbf{x},\omega) \sim \frac{-i}{4\pi r}\frac{m_{\mathrm{t}}^2}{\omega} \exp(ik(\omega)r)\hat{\mathbf{J}}^{\mathrm{L}}(\mathbf{x},\omega), \quad \hat{\mathbf{B}}^{\mathrm{L}}(\mathbf{x},\omega) = \mathrm{O}(r^{-2})\,,$$

$$\hat{A}_0^{\mathrm{L}}(\mathbf{x},\omega) \sim -\frac{c}{4\pi r}\frac{k(\omega)}{\omega} \exp(ik(\omega)r)\mathbf{n}\cdot\hat{\mathbf{J}}^{\mathrm{L}}(\mathbf{x},\omega)\,. \tag{2.6}$$

The real-time field strengths $\mathbf{E}^{\mathrm{T,L}}(\mathbf{x},t)$ and $\mathbf{B}^{\mathrm{T,L}}(\mathbf{x},t)$ relate to these Fourier transforms as defined after (2.3), and so does the zero component of the 4-potential, $A_0^{\mathrm{T,L}}(\mathbf{x},t)$.

To illustrate the meaning of the integral transform $\hat{\mathbf{J}}^{\mathrm{T,L}}$ defined in (2.5), we consider a subluminal particle $\mathbf{x}_0(t)$, $\mathbf{v} = \dot{\mathbf{x}}_0$, arbitrarily moving in the vicinity of the coordinate origin. The particle carries a tachyonic charge q, resulting in the current density

$$j^0 = \rho = q\delta(\mathbf{x}-\mathbf{x}_0(t)), \quad \mathbf{j} = q\mathbf{v}\delta(\mathbf{x}-\mathbf{x}_0(t))\,,$$

$$\hat{\rho}(\mathbf{x},\omega) = q\int_{-\infty}^{+\infty} \delta(\mathbf{x}-\mathbf{x}_0(t))e^{i\omega t}\,dt, \quad \hat{\mathbf{j}}(\mathbf{x},\omega) = q\int_{-\infty}^{+\infty} \mathbf{v}(t)\delta(\mathbf{x}-\mathbf{x}_0(t))e^{i\omega t}\,dt\,. \tag{2.7}$$

We use the shortcuts $\mathbf{v}^{\mathrm{T}}(\mathbf{x},t) := \mathbf{v} - \mathbf{n}(\mathbf{n}\cdot\mathbf{v})$ and $\mathbf{v}^{\mathrm{L}}(\mathbf{x},t) := \mathbf{n}(\mathbf{n}\cdot\mathbf{v})$, and write (2.5) as

$$\hat{\mathbf{J}}^{\mathrm{T,L}}(\mathbf{x},\omega) = q\int_{-\infty}^{+\infty} dt\,\mathbf{v}^{\mathrm{T,L}}(\mathbf{x},t)\exp[i(\omega t - k(\omega)\mathbf{n}\cdot\mathbf{x}_0(t))]\,. \tag{2.8}$$

The asymptotic Liénard–Wiechert potentials and the corresponding field strengths are given by (2.4) and (2.6) with this $\hat{\mathbf{J}}^{\mathrm{T,L}}$ inserted.

We turn to the energy density and the flux vector, which can be read off from (2.1) and (2.2),

$$T_0{}^0 = (1/2)(\mathbf{E}^2+\mathbf{B}^2) - (m_{\mathrm{t}}^2/2)(c^{-2}A_0^2+\mathbf{A}^2)\,,$$

$$T_0{}^m = c\mathbf{E}\times\mathbf{B} + m_{\mathrm{t}}^2 A_0\mathbf{A}\,. \tag{2.9}$$

Thus we find the transversal and longitudinal densities and the corresponding energy flux as

$$\rho_{\mathrm{E}}^{\mathrm{T}}(\mathbf{x},t) \sim (1/2)(\mathbf{E}^{\mathrm{T}^2}+\mathbf{B}^{\mathrm{T}^2} - m_{\mathrm{t}}^2\mathbf{A}^{\mathrm{T}^2}), \quad \mathbf{S}^{\mathrm{T}} \sim c\mathbf{E}^{\mathrm{T}}\times\mathbf{B}^{\mathrm{T}}\,, \tag{2.10}$$

$$\rho_{\mathrm{E}}^{\mathrm{L}} \sim \frac{m_{\mathrm{t}}^2}{2}(A_0^{\mathrm{L}^2} + \mathbf{A}^{\mathrm{L}^2}) - \frac{1}{2}\mathbf{E}^{\mathrm{L}^2}, \quad \mathbf{S}^{\mathrm{L}} \sim -m_{\mathrm{t}}^2 A_0^{\mathrm{L}}\mathbf{A}^{\mathrm{L}} , \tag{2.11}$$

with the asymptotic fields (2.4) and (2.6) inserted. We have identified $(\rho_{\mathrm{E}}^{\mathrm{T}}, \mathbf{S}^{\mathrm{T}})$ with ${T_0}^{\mu}$, and $(\rho_{\mathrm{E}}^{\mathrm{L}}, \mathbf{S}^{\mathrm{L}})$ stands for $-{T_0}^{\mu}$, so that the time-averaged densities are positive definite in either case. The averaging is readily carried out by means of the Fourier modes listed in (2.4) and (2.6). We find for the respective products of the transversal modes

$$\frac{1}{T}\int_{-T/2}^{+T/2} (\mathbf{A}^{\mathrm{T}^2}, \mathbf{E}^{\mathrm{T}^2}, \mathbf{B}^{\mathrm{T}^2}, c(\mathbf{E}^{\mathrm{T}} \times \mathbf{B}^{\mathrm{T}}))(\mathbf{x}, t)\, \mathrm{d}t$$
$$= \frac{1}{4(2\pi)^4 c^2 r^2}\frac{2\pi}{T}\int\int_{-\infty}^{+\infty} \delta_{(1)}(\omega - \omega'; T)\hat{\mathbf{J}}^{\mathrm{T}}(\mathbf{x}, \omega)\hat{\mathbf{J}}^{\mathrm{T}*}(\mathbf{x}, \omega')$$
$$\times \left(1, \frac{\omega^2}{c^2}, k^2(\omega), \omega k(\omega)\mathbf{n}\right) \mathrm{d}\omega\, \mathrm{d}\omega' . \tag{2.12}$$

The superscript T always stands for 'transversal' and is not to be confused with the time variable. In the integrand, we have already put $\omega = \omega'$ at several places, to save notation. The integral transform $\hat{\mathbf{J}}^{\mathrm{T}}$ of the current can be singular, cf. (2.7) and Section 3, and therefore, we refrain from this identification in $\hat{\mathbf{J}}^{\mathrm{T}*}$. A limit representation of the Dirac function,

$$\delta_{(1)}(\omega; T) := \frac{1}{2\pi}\int_{-T/2}^{+T/2} \mathrm{e}^{\mathrm{i}\omega t}\, \mathrm{d}t = \frac{1}{\pi}\frac{\sin(T\omega/2)}{\omega} ,$$
$$(2\pi/T)(\delta_{(1)}(\omega; T))^2 =: \delta_{(2)}(\omega; T), \quad \delta_{(1,2)}(\omega; T \to \infty) = \delta(\omega) , \tag{2.13}$$

will be used to avoid ill-defined squares of δ functions. According to (2.10), the time-averaged transversal flux and the energy density can be written as

$$\langle \mathbf{S}^{\mathrm{T}} \rangle \sim c\,\frac{1}{T}\int_{-T/2}^{+T/2} \mathbf{E}^{\mathrm{T}} \times \mathbf{B}^{\mathrm{T}}\, \mathrm{d}t, \quad \langle \rho_{\mathrm{E}}^{\mathrm{T}} \rangle \sim \frac{1}{T}\int_{-T/2}^{+T/2} \mathbf{E}^{\mathrm{T}^2}\, \mathrm{d}t , \tag{2.14}$$

where we insert the Fourier representations (2.12) to obtain

$$\langle \mathbf{S}^{\mathrm{T}} \rangle \sim \frac{\mathbf{n}}{4(2\pi)^4 c^2 r^2}\frac{2\pi}{T}\int\int_{-\infty}^{+\infty} \omega k(\omega)\delta_{(1)}(\omega - \omega'; T)$$
$$\times \hat{\mathbf{J}}^{\mathrm{T}}(\mathbf{x}, \omega)\hat{\mathbf{J}}^{\mathrm{T}*}(\mathbf{x}, \omega')\, \mathrm{d}\omega\, \mathrm{d}\omega' , \tag{2.15}$$

and analogously for $\langle \rho_{\mathrm{E}}^{\mathrm{T}} \rangle$. The longitudinal averages, cf. (2.4) and (2.6),

$$\frac{1}{T}\int_{-T/2}^{+T/2} (\mathbf{A}^{\mathrm{L}^2}, A_0^{\mathrm{L}^2}, \mathbf{E}^{\mathrm{L}^2}, A_0^{\mathrm{L}}\mathbf{A}^{\mathrm{L}})(\mathbf{x}, t)\, \mathrm{d}t$$
$$= \frac{1}{4(2\pi)^4 c^2 r^2}\frac{2\pi}{T}\int\int_{-\infty}^{+\infty} \mathrm{d}\omega\, \mathrm{d}\omega' \delta_{(1)}(\omega - \omega'; T)\hat{\mathbf{J}}^{\mathrm{L}}(\mathbf{x}, \omega)\hat{\mathbf{J}}^{\mathrm{L}*}(\mathbf{x}, \omega')$$

$$\times \left(1, \frac{c^4 k^2(\omega)}{\omega^2}, \frac{m_t^4 c^2}{\omega^2}, -\mathbf{n}\frac{c^2 k(\omega)}{\omega}\right) , \tag{2.16}$$

are substituted into

$$\langle \mathbf{S}^{\mathrm{L}} \rangle \sim -m_t^2 \frac{1}{T} \int_{-T/2}^{+T/2} A_0^{\mathrm{L}} \mathbf{A}^{\mathrm{L}}\, \mathrm{d}t, \quad \langle \rho_{\mathrm{E}}^{\mathrm{L}} \rangle \sim m_t^2 \frac{1}{T} \int_{-T/2}^{+T/2} \mathbf{A}^{\mathrm{L}^2}\, \mathrm{d}t, \tag{2.17}$$

cf. (2.11), and we arrive at

$$\langle \mathbf{S}^{\mathrm{L}} \rangle \sim \frac{m_t^2 \mathbf{n}}{4(2\pi)^4 r^2} \frac{2\pi}{T} \int \int_{-\infty}^{+\infty} \omega^{-1} k(\omega) \delta_{(1)}(\omega - \omega'; T) \hat{\mathbf{J}}^{\mathrm{L}}(\mathbf{x}, \omega) \hat{\mathbf{J}}^{\mathrm{L}*}(\mathbf{x}, \omega')\, \mathrm{d}\omega\, \mathrm{d}\omega' . \tag{2.18}$$

The radiant power is obtained by integrating the flux through a sphere of radius $r \to \infty$,

$$P = P^{\mathrm{T}} + P^{\mathrm{L}}, \quad P^{\mathrm{T,L}} := r^2 \int \mathbf{n} \cdot \langle \mathbf{S}^{\mathrm{T,L}} \rangle\, \mathrm{d}\Omega , \tag{2.19}$$

with the solid angle element $\mathrm{d}\Omega = \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\varphi$. Here, we use the asymptotic Pointing vectors (2.15) and (2.18), with the transforms $\hat{\mathbf{J}}^{\mathrm{T,L}}$ of the current as defined in (2.5) or (2.8). This is applicable to any type of particle motion.

In Section 4, we will replace the classical current in the above formulas by current matrices, appealing to the correspondence principle. To this end, we assume the classical current to consist of a single Fourier mode ω_{mn}:

$$\mathbf{j}_{mn}(\mathbf{x}, t) := \tilde{\mathbf{j}}(\mathbf{x}, \omega_{mn}) \mathrm{e}^{-\mathrm{i}\omega_{mn} t} + \mathrm{c.c.}, \quad \rho_{mn}(\mathbf{x}, t) := \tilde{\rho}(\mathbf{x}, \omega_{mn}) \mathrm{e}^{-\mathrm{i}\omega_{mn} t} + \mathrm{c.c.} , \tag{2.20}$$

so that $\mathrm{i}\omega_{mn} \tilde{\rho}(\mathbf{x}, \omega_{mn}) = \nabla \tilde{\mathbf{j}}(\mathbf{x}, \omega_{mn})$, with an arbitrary $\tilde{\mathbf{j}}$. (The subscript mn is chosen for future reference.) We define the truncated Fourier transform

$$\begin{aligned} \hat{\mathbf{j}}_{mn}(\mathbf{x}, \omega) &:= \int_{-T/2}^{+T/2} \mathbf{j}_{mn}(\mathbf{x}, t) \mathrm{e}^{\mathrm{i}\omega t}\, \mathrm{d}t \\ &= 2\pi \delta_{(1)}(\omega - \omega_{mn}; T) \tilde{\mathbf{j}}(\mathbf{x}, \omega_{mn}) + 2\pi \delta_{(1)}(\omega + \omega_{mn}; T) \tilde{\mathbf{j}}^*(\mathbf{x}, \omega_{mn}) . \end{aligned} \tag{2.21}$$

Such truncations result in smooth limit representations of the δ function, cf. (2.13), which admit unambiguous squares. The $\mathrm{d}\omega$ and $\mathrm{d}\omega'$ integrations in (2.15) and (2.18) get trivial for large T, if we use $\hat{\mathbf{J}}^{\mathrm{T,L}}$ in (2.5) with the current (2.21) inserted. We thus find the radiant powers, cf. (2.19):

$$P^{\mathrm{T}}(\omega_{mn}) = \frac{1}{8\pi^2 c^2} \omega_{mn} k(\omega_{mn}) \int_{r \to \infty} |\tilde{\mathbf{J}}^{\mathrm{T}}(\mathbf{x}, \omega_{mn})|^2\, \mathrm{d}\Omega , \tag{2.22}$$

$$P^{\mathrm{L}}(\omega_{mn}) = \frac{m_t^2}{8\pi^2} \frac{k(\omega_{mn})}{\omega_{mn}} \int_{r \to \infty} |\tilde{\mathbf{J}}^{\mathrm{L}}(\mathbf{x}, \omega_{mn})|^2\, \mathrm{d}\Omega . \tag{2.23}$$

We have defined here, cf. (2.5),

$$\tilde{\mathbf{J}}^{\mathrm{T,L}}(\mathbf{x},\omega_{mn}) := \int \mathrm{d}\mathbf{x}'\,\tilde{\mathbf{j}}^{\mathrm{T,L}}(\mathbf{x}',\mathbf{x},\omega_{mn})\exp(-\mathrm{i}k(\omega_{mn})\mathbf{n}\cdot\mathbf{x}')\ , \tag{2.24}$$

with $\tilde{\mathbf{j}}^{\mathrm{T}} := \tilde{\mathbf{j}}(\mathbf{x}',\omega_{mn}) - \tilde{\mathbf{j}}^{\mathrm{L}}$ and $\tilde{\mathbf{j}}^{\mathrm{L}} := \mathbf{n}(\mathbf{n}\cdot\tilde{\mathbf{j}}(\mathbf{x}',\omega_{mn}))$, where $\mathbf{n} := \mathbf{x}/r$. The longitudinal current transform $\tilde{\mathbf{J}}^{\mathrm{L}}(\mathbf{x},\omega_{mn})$ in (2.24) depends on the tachyonic charge density only. To see this, we use the identity

$$k\mathbf{n}\tilde{\mathbf{j}}^{\mathrm{L}}(\mathbf{x}',\mathbf{x},\omega_{mn})\exp(-\mathrm{i}k\mathbf{n}\cdot\mathbf{x}') = \omega_{mn}\tilde{\rho}(\mathbf{x}',\omega_{mn})\exp(-\mathrm{i}k\mathbf{n}\cdot\mathbf{x}'), \tag{2.25}$$

valid up to a divergence; this is a consequence of current conservation as stated after (2.20). Hence,

$$\tilde{\mathbf{J}}^{\mathrm{L}}(\mathbf{x},\omega_{mn}) = \mathbf{n}\omega_{mn}k^{-1}(\omega_{mn})\int \mathrm{d}\mathbf{x}'\,\tilde{\rho}(\mathbf{x}',\omega_{mn})\exp(-\mathrm{i}k(\omega_{mn})\mathbf{n}\cdot\mathbf{x}')\ . \tag{2.26}$$

Formulas (2.22) and (2.23) for the radiant power are exact; there is no multipole expansion involved. (We will return to them in Sections 4 and 5, when quantizing.) The same holds for the power derived in (2.19) (with the asymptotic flux vectors (2.15) and (2.18) substituted), which is completely general, applying to any conserved current. In the next section we will work out the simplest example, radiation by uniformly moving charges.

3. Does a uniformly moving charge radiate?

We turn to the conceptually most interesting case, superluminal radiation emitted by uniformly moving charges. We derive here the classical theory, the first and second quantization will be carried out in the subsequent sections. We consider a tachyonic charge q, moving along the z-axis, $z = vt$, $0 \leqslant v < c$, so that $\mathbf{n}\mathbf{e}_3 = \cos\theta$, $\mathbf{n} = \mathbf{x}/r$. The integral transform (2.8) of the transversal and longitudinal current projections is easily calculated:

$$\hat{\mathbf{J}}^{\mathrm{T}}(\mathbf{x},\omega) = qv(\mathbf{e}_3 - \cos\theta\,\mathbf{n})\tilde{J}(\theta,\omega), \qquad \hat{\mathbf{J}}^{\mathrm{L}}(\mathbf{x},\omega) = qv\cos\theta\mathbf{n}\tilde{J}(\theta,\omega)\ ,$$

$$\tilde{J}(\theta,\omega) := \int_{-T/2}^{+T/2}\mathrm{d}t\exp[\mathrm{i}t(\omega - k(\omega)v\cos\theta)] = 2\pi\delta_{(1)}(\omega - k(\omega)v\cos\theta; T)\ , \tag{3.1}$$

where $k(\omega)$ is negative for negative ω, cf. (2.4), and $\delta_{(1)}(\omega;T)$ is defined in (2.13). We have restricted the trajectory to a finite time interval $[-T/2, T/2]$, so that the asymptotic formulas (2.4) and (2.6) apply, also compare (2.21). The time-averaged transversal Pointing vector is readily assembled, cf. (2.15):

$$\langle\mathbf{S}^{\mathrm{T}}\rangle \sim \frac{\mathbf{n}q^2\sin^2\theta}{4(2\pi)^4r^2}\frac{v^2}{c^2}\frac{2\pi}{T}\int\int_{-\infty}^{+\infty}\omega k(\omega)\delta_{(1)}(\omega-\omega';T)\tilde{J}^2(\theta,\omega)\,\mathrm{d}\omega\,\mathrm{d}\omega'\ . \tag{3.2}$$

By making use of (2.13) and

$$\delta(\omega - k(\omega)v\cos\theta) = \frac{\Theta(\cos\theta)}{1-(v^2/c^2)\cos^2\theta}(\delta(\omega-\omega^+)+\delta(\omega-\omega^-)) ,$$

$$\omega^\pm := \pm\frac{m_t v\cos\theta}{\sqrt{1-(v^2/c^2)\cos\theta}}, \quad \omega^\pm = k(\omega^\pm)v\cos\theta , \tag{3.3}$$

we may write this as

$$\langle \mathbf{S}^T\rangle \sim \frac{\mathbf{n}q^2}{2(2\pi)^2 r^2}\frac{v^2}{c^2}\frac{\omega^+ k(\omega^+)\sin^2\theta}{1-(v^2/c^2)\cos^2\theta}\Theta(\cos\theta) . \tag{3.4}$$

The argument of the δ function in (3.3) can only get zero for $\cos\theta > 0$, therefore the Heaviside function $\Theta(\cos\theta)$. In (3.2), the limit $T \to \infty$ can be performed without compromising the asymptotics in (2.4). In this limit, the singular accelerations inflicted by the artificial, but technically convenient discontinuous truncation in (3.1) do not show in the time averages. We thus find the transversally radiated power, cf. (2.19) and (3.4):

$$\begin{aligned} P^T &= \frac{q^2}{4\pi}\frac{m_t^2 v^3}{c^2}\int_0^{\pi/2}\frac{\cos\theta\sin^3\theta}{(1-(v^2/c^2)\cos^2\theta)^2}\,\mathrm{d}\theta \\ &= \frac{1}{2}\frac{q^2}{4\pi}\frac{m_t^2 c^2}{v}\left(\log\frac{1}{1-v^2/c^2} - \frac{v^2}{c^2}\right) . \end{aligned} \tag{3.5}$$

The spectral energy density is identified by a variable change according to (3.3):

$$p^T(\omega) := \frac{q^2}{4\pi} m_t^2 v\,\frac{\omega(1-\omega^2/\omega_{\max}^2)}{\omega^2 + m_t^2 c^2}, \quad P^T = \int_0^{\omega_{\max}} p^T(\omega)\,\mathrm{d}\omega , \tag{3.6}$$

with $\omega_{\max} := m_t v\gamma$ as the highest frequency radiated. The tachyon mass m_t is a shortcut for $m_t c/\hbar$ and γ is the subluminal Lorentz factor $(1-v^2/c^2)^{-1/2}$, so that $\omega_{\max}$ is just an m_t/m fraction of the electron energy. Another way to obtain the spectral density is to insert (3.2) and (3.1) into (2.19), and to perform the $\mathrm{d}\omega'$ integration as above, followed by the angular integration:

$$\begin{aligned} P^T &= \frac{1}{2}\frac{q^2}{4\pi}\frac{v^2}{c^2}\int_{-\infty}^{+\infty}\int_{-1}^{+1} d\cos\theta\sin^2\theta\,\delta(\omega - k(\omega)v\cos\theta)\omega k(\omega)\,\mathrm{d}\omega \\ &= \frac{q^2}{4\pi}\frac{v}{c^2}\int_0^{\omega_{\max}}(1-\omega^2 k^{-2}v^{-2})\omega\,\mathrm{d}\omega, \quad k(\omega_{\max})v = \omega_{\max} . \end{aligned} \tag{3.7}$$

This derivation is simpler, but conceals the angular dependence, explicit in (3.4).

The longitudinal flux is calculated via (2.18)

$$\langle \mathbf{S}^L\rangle \sim \frac{\mathbf{n}m_t^2 q^2 v^2\cos^2\theta}{4(2\pi)^4 r^2}\frac{2\pi}{T}\int\int_{-\infty}^{+\infty}\omega^{-1}k(\omega)\delta_{(1)}(\omega-\omega';T)\tilde{J}^2(\theta,\omega)\,\mathrm{d}\omega\,\mathrm{d}\omega' , \tag{3.8}$$

which can be evaluated in the same way as (3.2), resulting in

$$\langle \mathbf{S}^{\mathrm{L}} \rangle \sim \frac{\mathbf{n} q^2 m_{\mathrm{t}}^2}{2(2\pi)^2 r^2} \frac{v \cos\theta}{1-(v^2/c^2)\cos^2\theta} \Theta(\cos\theta) \, . \tag{3.9}$$

We thus find the longitudinal power

$$P^{\mathrm{L}} = \frac{q^2}{4\pi} m_{\mathrm{t}}^2 v \int_0^{\pi/2} \frac{\cos\theta \sin\theta}{1-(v^2/c^2)\cos^2\theta} \, \mathrm{d}\theta = \frac{1}{2} \frac{q^2}{4\pi} \frac{m_{\mathrm{t}}^2 c^2}{v} \log \frac{1}{1-v^2/c^2} \, , \tag{3.10}$$

which in turn leads to the spectral density

$$p^{\mathrm{L}}(\omega) := \frac{q^2}{4\pi} \frac{1}{v} m_{\mathrm{t}}^2 c^2 \frac{\omega}{\omega^2 + m_{\mathrm{t}}^2 c^2}, \quad P^{\mathrm{L}} = \int_0^{\omega_{\max}} p^{\mathrm{L}}(\omega) \, \mathrm{d}\omega \, , \tag{3.11}$$

with $\omega_{\max}$ defined after (3.6). Alternatively, we may interchange the $\mathrm{d}\omega$ and the angular integrations as done in (3.7).

$$\begin{aligned} P^{\mathrm{L}} &= \frac{1}{2} \frac{q^2}{4\pi} m_{\mathrm{t}}^2 v^2 \int_{-\infty}^{+\infty} \int_{-1}^{+1} d\cos\theta \cos^2\theta \, \delta(\omega - k(\omega) v \cos\theta) k(\omega) \omega^{-1} \, \mathrm{d}\omega \\ &= \frac{q^2}{4\pi} \frac{m_{\mathrm{t}}^2}{v} \int_0^{\omega_{\max}} \omega k^{-2} \, \mathrm{d}\omega \, , \end{aligned} \tag{3.12}$$

which coincides with (3.11). Flux vector and energy density relate in the usual way, $\langle \mathbf{S}^{\mathrm{T,L}} \rangle = \left\langle \rho_{\mathrm{E}}^{\mathrm{T,L}} \right\rangle v_{\mathrm{gr}} \mathbf{n}$, with $v_{\mathrm{gr}} = c^2/v_{\mathrm{ph}}$, and $v_{\mathrm{ph}} = v\cos\theta$. There is no backward radiation, that is, for $\cos\theta \leqslant 0$. In the limit $\theta \to \pi/2$, the emitted tachyons approach infinite speed and zero energy. Radiation angle and frequency relate via $\omega = k(\omega) v \cos\theta$. To restore the units, we have to substitute $m_{\mathrm{t}} \to m_{\mathrm{t}} c/\hbar$ in the above formulas. A detailed discussion of the spectral densities and powers will be given in the next section, after quantization. The classical formulas derived here are only valid if $v/c \gg m_{\mathrm{t}}/m$. The Planck constant does not show in this constraint; however, the tachyon mass already enters in the classical field equations by the combination $m_{\mathrm{t}} c/\hbar$, cf. the beginning of Section 2.

4. Quantization of the superluminal spectral densities and the radiant power

We will investigate how far quantization modifies the classical picture given in Section 3, tachyon radiation by a structureless particle in uniform motion. To derive the quantized version of the spectral densities (3.6) and (3.11), we replace the classical current by the current matrix of a subluminal quantum particle carrying tachyonic charge as outlined at the end of Section 2. In doing so, we assume the correspondence principle; in Section 5, we will demonstrate that the spectral densities and powers calculated in this way can be recovered from the spontaneous emission rates in second quantization. We will not consider spin or antiparticles, and content ourselves with positive frequency solutions of the Klein–Gordon equation. The inclusion of spin is interesting if the electron orbits in a magnetic field, resulting in tachyonic cyclotron and synchrotron radiation, but there are otherwise no conceptual changes, the current being replaced by the matrix elements of the spinor current followed by polarization averages.

We start with the Klein–Gordon equation of a subluminal particle, $c^{-2}\psi_{,tt} - \Delta\psi + m^2\psi = 0$, where m is a shortcut for $mc/\hbar$. We define the 4-current functionals

$$\rho(\psi,\varphi) := \mathrm{i}q(\varphi^*\psi_{,t} - \psi\,\varphi^*_{,t}), \quad \mathbf{j}(\psi,\varphi) := -\mathrm{i}qc^2(\varphi^*\nabla\psi - \psi\nabla\varphi^*)\,, \tag{4.1}$$

and note the continuity equation $\rho_{,t} + \mathrm{div}\,\mathbf{j} = 0$, where ψ and φ are arbitrary solutions of the wave equation. We use the separation ansatz $\psi_i = u_i\exp(-\mathrm{i}\omega_i t)$, $\omega_i > 0$, and define the shortcuts $\rho_{mn} := \rho(\psi_m,\psi_n)$ and $\mathbf{j}_{mn} := \mathbf{j}(\psi_m,\psi_n)$, as well as $\rho_{mn} =: \tilde{\rho}_{mn}\exp(-\mathrm{i}\omega_{mn}t)$ and $\mathbf{j}_{mn} =: \tilde{\mathbf{j}}_{mn}\exp(-\mathrm{i}\omega_{mn}t)$, with $\omega_{mn} := \omega_m - \omega_n$. We hence find the time-separated wave equation $\Delta u_i = (m^2 - c^{-2}\omega_i^2)u_i$, as well as the Hermitian current matrices

$$\tilde{\rho}_{mn} = q(\omega_m + \omega_n)u_m u_n^*, \quad \tilde{\mathbf{j}}_{mn} = -\mathrm{i}qc^2(u_n^*\nabla u_m - u_m\nabla u_n^*)\,. \tag{4.2}$$

We consider periodic boundary conditions on a box of size L and conveniently normalized eigenfunctions:

$$u_i = (2\omega_i)^{-1/2}L^{-3/2}\exp(\mathrm{i}\mathbf{k}_i\mathbf{x}), \quad \int_{L^3}\tilde{\rho}_{mn}\,\mathrm{d}^3x = q\delta_{mn}\,, \tag{4.3}$$

with $\mathbf{k}_i = 2\pi\mathbf{n}_i/L$ and $\mathbf{n}_i \in Z^3$. The frequencies depend on the wave vectors via the subluminal dispersion relation $k_i^2 = \omega_i^2/c^2 - m^2$. The current matrices $\tilde{\rho}_{mn}$ and $\tilde{\mathbf{j}}_{mn}$ in (4.2) are composed with the u_i in (4.3), and we substitute them into (2.24) and (2.26) (where all spatial integrations extend over the box size):

$$\tilde{\mathbf{J}}^{\mathrm{T}}(\mathbf{x},\omega_{mn}) = \frac{qc^2}{\sqrt{\omega_m\omega_n}}\frac{(2\pi)^3}{L^3}(\mathbf{k}_m - \mathbf{n}(\mathbf{n}\mathbf{k}_m))\delta_{(1)}(\mathbf{K}_{mn};L)\,, \tag{4.4}$$

$$\tilde{\mathbf{J}}^{\mathrm{L}}(\mathbf{x},\omega_{mn}) = \frac{q\omega_{mn}(\omega_m+\omega_n)}{2k(\omega_{mn})\sqrt{\omega_m\omega_n}}\frac{(2\pi)^3}{L^3}\mathbf{n}\delta_{(1)}(\mathbf{K}_{mn};L)\,, \tag{4.5}$$

$$\mathbf{K}_{mn} := \mathbf{k}_m - \mathbf{k}_n - k(\omega_{mn})\mathbf{n}, \quad \mathbf{n} := \mathbf{x}/r.$$

Here, $\delta_{(1)}(\mathbf{k};L)$ is the three-dimensional analog to the truncated integral representation of the δ function in (2.13); the limit procedure outlined there has likewise an obvious 3-d generalization by factorization, which we use when squaring these $\tilde{\mathbf{J}}^{\mathrm{T,L}}$ in the integrands of the classical powers (2.22) and (2.23):

$$P^{\mathrm{T}} = \frac{q^2c^2}{8\pi^2}\frac{(2\pi)^3}{L^3}\frac{\omega_{mn}k(\omega_{mn})}{\omega_m\omega_n}\int\mathrm{d}\Omega(k_m^2 - (\mathbf{n}\mathbf{k}_m)^2)\delta_{(1)}(\mathbf{K}_{mn};L)\,, \tag{4.6}$$

$$P^{\mathrm{L}} = \frac{q^2m_{\mathrm{t}}^2}{32\pi^2}\frac{(2\pi)^3}{L^3}\frac{\omega_{mn}(\omega_m+\omega_n)^2}{k(\omega_{mn})\omega_m\omega_n}\int\mathrm{d}\Omega\delta_{(1)}(\mathbf{K}_{mn};L)\,. \tag{4.7}$$

The solid angle integration refers to the unit vector $\mathbf{n}$ and is easily done by means of the substitution $\int_{|\mathbf{n}|=1} \mathrm{d}\Omega \to 2\int_{R^3} \mathrm{d}^3\mathbf{n}\,\delta(\mathbf{n}^2-1)$. Hence,

$$P^{\mathrm{T}} = \frac{q^2c^2}{4\pi^2}\,\frac{(2\pi)^3}{L^3}\,\frac{\omega_{mn}(k_m^2k_n^2-(\mathbf{k}_m\mathbf{k}_n)^2)}{\omega_m\omega_n k^2(\omega_{mn})}\,\delta((\mathbf{k}_m-\mathbf{k}_n)^2-k^2(\omega_{mn}))\,, \tag{4.8}$$

$$P^{\mathrm{L}} = \frac{q^2m_{\mathrm{t}}^2c^4}{16\pi^2}\,\frac{(2\pi)^3}{L^3}\,\frac{(k_m^2-k_n^2)^2}{\omega_{mn}\omega_m\omega_n k^2(\omega_{mn})}\,\delta((\mathbf{k}_m-\mathbf{k}_n)^2-k^2(\omega_{mn}))\,. \tag{4.9}$$

The total power radiated is obtained by summing over the final states and performing the continuum limit:

$$P_{\mathrm{tot}}^{\mathrm{T,L}}(\mathbf{k}_m) = \sum_{k_n} P^{\mathrm{T,L}}(\mathbf{k}_m,\mathbf{k}_n), \qquad \mathrm{d}P_{\mathrm{tot}}^{\mathrm{T,L}} = L^3(2\pi)^{-3}P^{\mathrm{T,L}}\,\mathrm{d}^3\mathbf{k}_n\,. \tag{4.10}$$

We introduce polar coordinates for $\mathbf{k}_n$, with $\mathbf{k}_m$ as polar axis, and integrate $\mathrm{d}P_{\mathrm{tot}}^{\mathrm{T,L}}$ over the angular variables. This is easily done by means of the δ functions in (4.8) and (4.9), if we replace $\mathrm{d}^3\mathbf{k}_n$ with $2\pi k_n^2\mathrm{d}k_n\int_{-1}^{1}\mathrm{d}\cos\theta$. We thus obtain

$$\mathrm{d}P_{\mathrm{tot}}^{\mathrm{T}} = \frac{q^2c^2}{16\pi}\,\frac{\omega_{mn}D_{mn}}{\omega_m\omega_n k_m k^2(\omega_{mn})}\,\Theta(D_{mn})k_n\,\mathrm{d}k_n\,, \tag{4.11}$$

$$\mathrm{d}P_{\mathrm{tot}}^{\mathrm{L}} = \frac{q^2m_{\mathrm{t}}^2c^4}{16\pi}\,\frac{(k_m^2-k_n^2)^2}{\omega_{mn}\omega_m\omega_n k_m k^2(\omega_{mn})}\,\Theta(D_{mn})k_n\,\mathrm{d}k_n\,, \tag{4.12}$$

$$D_{mn} := 4k_m^2k_n^2 - (k_m^2+k_n^2-k^2(\omega_{mn}))^2\,, \tag{4.13}$$

where Θ is the Heaviside function. The tachyonic wave vector relates to the subluminal frequencies by $k(\omega_{mn}) = \sqrt{\omega_{mn}^2/c^2+m_{\mathrm{t}}^2}$, with $\omega_{mn}=\omega_m-\omega_n$. The dispersion relation for the subluminal charge is $k_n=\sqrt{\omega_n^2/c^2-m^2}$, and the same for k_m and ω_m. The initial state is denoted by a subscript m, the final state by n, so that for emission $\omega_{mn}>0$. This designation of 'initial' and 'final' is arbitrary, just for the purpose of defining the radiation modes. By making use of the dispersion relations, we write D_{mn} as a function of ω_{mn}:

$$D_{mn} = 4\,\frac{m^2}{c^2}\left(\frac{m_{\mathrm{t}}^2}{m^2}(\omega_m^2-\omega_0^2) - \frac{m_{\mathrm{t}}^2}{m^2}\,\omega_m\omega_{mn} - \omega_{mn}^2\right)\,, \tag{4.14}$$

with $\omega_0 := mc\sqrt{1+\frac{1}{4}m_{\mathrm{t}}^2/m^2}$. There are two zeros, $D_{mn}(\omega_{mn}^{\pm})=0$, where

$$\omega_{mn}^{\pm} = \pm\frac{k_m\omega_0}{m}\,\frac{m_{\mathrm{t}}}{m} - \frac{\omega_m}{2}\,\frac{m_{\mathrm{t}}^2}{m^2}. \tag{4.15}$$

Emission means $\omega_{mn}>0$, thus we can ignore the negative root and we will write $\omega_{\max}(\omega_m)$ for ω_{mn}^{+}. It is easy to see that $\omega_{\max}$ is positive only if $\omega_m>\omega_0$, cf. (4.14), and $\omega_{\max}(\omega_0)=0$. Clearly, $\omega_m \geqslant mc$ from the outset. It is likewise evident that

$D_{mn}(\omega_{mn}) > 0$ for $0 < \omega_{mn} < \omega_{\max}$ and negative for larger frequencies. If $\omega_m < \omega_0$, then $\Theta(D_{mn}(\omega_{mn}))$ in (4.11) and (4.12) vanishes for all $\omega_{mn} > 0$, and hence $\omega_m > \omega_0$ is a necessary condition for the emission of superluminal quanta. The spectral range is $0 < \omega_{mn} < \omega_{\max}$, defined by $\Theta(D_{mn}) = 1$.

The total power radiated is $P_{\rm tot}^{\rm T,L} = \int_0^{k_m} {\rm d}P_{\rm tot}^{\rm T,L}(k_n)$, cf. (4.11) and (4.12). To obtain the frequency distributions, we introduce ω_{mn} as integration variable. Using the dispersion relation for k_n, we find $\omega_n\, {\rm d}\omega_{mn} = -c^2 k_n\, {\rm d}k_n$ and $P_{\rm tot}^{\rm T,L} = -\int_0^{\omega_m - mc} {\rm d}P_{\rm tot}^{\rm T,L}(\omega_{mn})$. Finally, $\omega_{\max}(\omega_m) \leqslant \omega_m - mc$ for $\omega_m > \omega_0$, which is easily seen from (4.15). (There is a double zero at $\omega_m = mc(1 + \frac{1}{2}m_{\rm t}^2/m^2)$.) Thus we can replace the upper integration boundary by $\omega_{\max}$ and drop $\Theta(D_{mn})$ in (4.11) and (4.12). We write in the following ω for ω_{mn}, and define the densities $p^{\rm T,L}(\omega)\,{\rm d}\omega := -{\rm d}P_{\rm tot}^{\rm T,L}(\omega_{mn})$. We thus find, via the sub- and superluminal dispersion relations as stated after (4.13), the transversally and longitudinally radiated powers, the number counts, and the respective spectral functions:

$$P_{\rm tot}^{\rm T,L} = \int_0^{\omega_{\max}} p^{\rm T,L}(\omega)\,{\rm d}\omega, \qquad N_{\rm tot}^{\rm T,L} = \hbar^{-1}\int_0^{\omega_{\max}} p^{\rm T,L}(\omega)\omega^{-1}\,{\rm d}\omega\,, \tag{4.16}$$

$$\omega_{\max} := \sqrt{\omega_m^2 - m^2c^2}\,\frac{\omega_0}{mc}\,\frac{m_{\rm t}}{m} - \frac{1}{2}\,\omega_m\,\frac{m_{\rm t}^2}{m^2}, \qquad \frac{\omega_0}{mc} := \sqrt{1 + \frac{1}{4}\,\frac{m_{\rm t}^2}{m^2}}\,, \tag{4.17}$$

$$p^{\rm T}(\omega) = \frac{q^2}{4\pi}\,\frac{m_{\rm t}^2}{\omega_m k_m}\,\frac{\omega}{\omega^2 + m_{\rm t}^2c^2}\,(\omega_m^2 - \omega_0^2 - \omega_m\omega - (m/m_{\rm t})^2\omega^2)\,, \tag{4.18}$$

$$p^{\rm L}(\omega) = \frac{q^2}{4\pi}\,\frac{m_{\rm t}^2}{\omega_m k_m}\,\frac{\omega}{\omega^2 + m_{\rm t}^2c^2}\,(\omega_m^2 - \omega_m\omega + \tfrac{1}{4}\omega^2)\,, \tag{4.19}$$

where $k_m = c^{-1}\sqrt{\omega_m^2 - m^2c^2}$. The upper edge $\omega_{\max}$ of the spectral range is positive only if $\omega_m > \omega_0$. Spontaneous emission can only occur if the subluminal source surpasses a finite threshold energy ω_0. This is unparalleled in the classical radiation theory, cf. Section 3.

At the upper edge of the spectrum, we have $p^{\rm T}(\omega_{\max}) = 0$, cf. (4.14) and (4.15), but the longitudinal density $p^{\rm L}(\omega)$ is still positive at $\omega_{\max}$. It may even happen that the integration in (4.16) is cut off before the maximum of $p^{\rm L}(\omega)$ is reached, so that $p^{\rm L}(\omega)$ is increasing throughout the spectral range, cf. the discussion following (4.26). The tachyonic mean energy is $\hbar\omega_{\rm av}^{\rm T,L} := P_{\rm tot}^{\rm T,L}/N_{\rm tot}^{\rm T,L}$, the emission rates $N_{\rm tot}^{\rm T,L}$ (tachyons per unit time) are defined in (4.16). To get the dimensions right in (4.17)–(4.19), we still have to rescale the masses, $m_{\rm (t)} \to m_{\rm (t)}c/\hbar$. The integrals in (4.16) are elementary, and we find the total transversally emitted power and the transversal count rate as

$$P_{\rm tot}^{\rm T} = \frac{\varepsilon_{\rm q}}{2}\left[\left(\frac{\omega_m^2}{m^2c^2} - \frac{1}{4}\,\frac{m_{\rm t}^2}{m^2}\right)\log\left(1 + \frac{\omega_{\max}^2}{m_{\rm t}^2c^2}\right) - \frac{\omega_{\max}^2}{m_{\rm t}^2c^2} + 2\frac{\omega_m}{mc}\,\frac{m_{\rm t}}{m}\left(\arctan\frac{\omega_{\max}}{m_{\rm t}c} - \frac{\omega_{\max}}{m_{\rm t}c}\right)\right]\,,$$

$$N^{\mathrm{T}}_{\mathrm{tot}} = \frac{\varepsilon_{\mathrm{q}}}{m_{\mathrm{t}} c\hbar} \left[\left(\frac{\omega_m^2}{m^2 c^2} - \frac{1}{4}\frac{m_{\mathrm{t}}^2}{m^2} \right) \arctan \frac{\omega_{\max}}{m_{\mathrm{t}} c} - \frac{\omega_{\max}}{m_{\mathrm{t}} c} - \frac{1}{2}\frac{\omega_m}{mc}\frac{m_{\mathrm{t}}}{m} \log\left(1 + \frac{\omega_{\max}^2}{m_{\mathrm{t}}^2 c^2}\right)\right] . \tag{4.20}$$

Here, $\omega_{\max}$ is the break frequency defined in (4.17), the power scale is set by

$$\varepsilon_{\mathrm{q}} := \frac{q^2}{4\pi c}\frac{mc}{\omega_m}\frac{m_{\mathrm{t}}^2 c^2}{\sqrt{\omega_m^2/(mc)^2 - 1}} , \tag{4.21}$$

and $0 < \arctan < \pi/2$. The longitudinal power and count rate are

$$P^{\mathrm{L}}_{\mathrm{tot}} = P^{\mathrm{T}}_{\mathrm{tot}} + \frac{\varepsilon_{\mathrm{q}}}{2}\frac{\omega_0^2}{m^2c^2}\frac{\omega_{\max}^2}{m_{\mathrm{t}}^2 c^2}, \quad N^{\mathrm{L}}_{\mathrm{tot}} = N^{\mathrm{T}}_{\mathrm{tot}} + \frac{\varepsilon_{\mathrm{q}}}{m_{\mathrm{t}} c\hbar}\frac{\omega_0^2}{m^2c^2}\frac{\omega_{\max}}{m_{\mathrm{t}} c} , \tag{4.22}$$

with ω_0 defined in (4.17).

For the rest of this section, we will study asymptotic limits of the spectral densities, powers and number counts derived above. There are three asymptotic regimes giving a comprehensive picture of the radiation. To see this, we introduce the shortcut $\alpha := \sqrt{1 - \omega_0^2/\omega_m^2}$ and write $\omega_m = mc\gamma$, with the subluminal $\gamma = (1 - v^2/c^2)^{-1/2}$. Since $\omega_m > \omega_0$, we have apparently $\alpha < 1$ or

$$v > v_{\min} := \frac{1}{2}\frac{m_{\mathrm{t}} c^2}{\omega_0}, \quad \gamma_{\min} := \gamma(v_{\min}) = \frac{\omega_0}{mc} , \tag{4.23}$$

which is equivalent to $\omega_{\max} > 0$. The velocity v refers to the subluminal particle. Condition (4.23) defines the threshold velocity $v_{\min}$ for tachyon radiation. To figure out the asymptotic regimes of the spectral functions with regard to v, we parametrize $\omega_{\max}$ and ω_m with α:

$$\omega_{\max} = \frac{m_{\mathrm{t}}}{m}\frac{\omega_0}{\sqrt{1-\alpha^2}}\left(\sqrt{\alpha^2 + \frac{1}{4}\frac{m_{\mathrm{t}}^2}{m^2}} - \frac{1}{2}\frac{m_{\mathrm{t}}}{m}\right), \quad \omega_m = \frac{\omega_0}{\sqrt{1-\alpha^2}} . \tag{4.24}$$

It is evident that $\omega_{\max} < \omega_m m_{\mathrm{t}}/m$. If $\alpha^2 \ll m_{\mathrm{t}}^2/m^2 \ll 1$, which defines the extreme non-relativistic regime, we find $\omega_{\max} \approx mc\alpha^2$. In the non-relativistic limit, $m_{\mathrm{t}}^2/m^2 \ll \alpha^2 \ll 1$, we find $\omega_{\max} \approx m_{\mathrm{t}} c\alpha$. In the ultra-relativistic regime, with $\alpha^2 \approx 1$ (and $m_{\mathrm{t}}^2/m^2 \ll 1$), we find $\omega_{\max} \approx m_{\mathrm{t}} c(1-\alpha^2)^{-1/2}$. This is to be compared to $\omega_m \approx mc$ in the two non-relativistic regimes, and to $\omega_m \approx mc(1-\alpha^2)^{-1/2}$ in the ultra-relativistic limit. (All these estimates are meant as leading orders in asymptotic double series expansions.) The extreme non-relativistic limit only applies to a very narrow velocity range, to velocities close to the threshold $v_{\min}$, which is evident from $\gamma = (1-\alpha^2)^{-1/2}\gamma_{\min}$.

We can now compare the foregoing to the classical radiation theory of Section 3. The upper edge of the spectrum, $\omega_{\max}$ in (4.24), coincides with its classical counterpart defined after (3.6) in the limit $m_{\mathrm{t}}^2/m^2 \ll \alpha^2$. This is the condition for the classical theory to apply. In this limit we can apparently identify $\alpha \approx v/c$, and it is also evident that

$\omega_m \gg \omega_{\max}$ (where $\omega_{\max}$ is the highest frequency radiated, and ω_m is the energy of the source). In the transversal spectral density (4.18), we may therefore replace ω_0 by mc and drop the subsequent term $\omega_m \omega$, in this way recovering the classical density (3.6). The same reasoning applies to the longitudinal density $p^{\mathrm{L}}(\omega)$, which coincides with the classical formula (3.11) if we drop the $\omega_m\omega$ and $\omega^2/4$ terms in (4.19). The powers (4.20) and (4.22) are turned into the classical ones in (3.5) and (3.10), by discarding all terms explicitly depending on the m_{t}/m ratio.

The peak frequency of the transversal spectral density $p^{\mathrm{T}}(\omega)$, cf. (4.18), is a zero of

$$y_{\mathrm{T}}^4 + \left(\alpha^2 + 3(1-\alpha^2)\frac{m^2c^2}{\omega_0^2}\right)\frac{m_{\mathrm{t}}^2}{m^2}\, y_{\mathrm{T}}^2 + 2(1-\alpha^2)\,\frac{m^2c^2}{\omega_0^2}\,\frac{m_{\mathrm{t}}^4}{m^4}\, y_{\mathrm{T}} - \alpha^2(1-\alpha^2)\,\frac{m^2c^2}{\omega_0^2}\,\frac{m_{\mathrm{t}}^4}{m^4} = 0\,, \tag{4.25}$$

where $y_{\mathrm{T}} := \omega^{\mathrm{T}}_{\mathrm{peak}}/\omega_m$. We calculate this maximum in the three asymptotic regimes enumerated above. If $\alpha^2 \ll m_{\mathrm{t}}^2/m^2 \ll 1$, we can ignore the fourth order in (4.25) as well as the second, and find $\omega^{\mathrm{T}}_{\mathrm{peak}} \approx mc\alpha^2/2$, just in the middle of the spectral range. If $m_{\mathrm{t}}^2/m^2 \ll \alpha^2 \ll 1$, we find, by dropping the fourth and first-order terms, $\omega^{\mathrm{T}}_{\mathrm{peak}} \approx m_{\mathrm{t}}c\alpha/\sqrt{3}$, which is likewise located almost in the center of the frequency range. Finally, in the ultra-relativistic regime, $\alpha^2 \approx 1$, we may again drop the first- and fourth-order terms in (4.25), so that the density is peaked at $\omega^{\mathrm{T}}_{\mathrm{peak}} \approx m_{\mathrm{t}}c$.

The maximum of the longitudinal spectral function (4.19) is found by solving

$$y_{\mathrm{L}}^4 - \left(4 - 3\,\frac{m_{\mathrm{t}}^2c^2}{\omega_m^2}\right) y_{\mathrm{L}}^2 - 8\,\frac{m_{\mathrm{t}}^2c^2}{\omega_m^2}\, y_{\mathrm{L}} + 4\,\frac{m_{\mathrm{t}}^2c^2}{\omega_m^2} = 0\,, \tag{4.26}$$

with $y_{\mathrm{L}} := \omega^{\mathrm{L}}_{\mathrm{peak}}/\omega_m$, and we may always assume $m_{\mathrm{t}}c/\omega_m \ll 1$. There are two positive solutions; the first, $y_{\mathrm{L}} \approx 2$, lies outside the integration range in (4.16), the relevant one is $\omega^{\mathrm{L}}_{\mathrm{peak}} \approx m_{\mathrm{t}}c$, stemming from the second and zeroth orders of (4.26). Also this peak lies beyond $\omega_{\max}$ if $\alpha^2 \leqslant 1/2$, since $\omega_{\max} \approx m_{\mathrm{t}}c$ for $\alpha^2 \approx 1/2$. Hence, if $\alpha^2 \leqslant 1/2$, then $p^{\mathrm{L}}(\omega)$ is increasing throughout the spectral range; if $\alpha^2 \geqslant 1/2$, it admits a maximum at $\omega^{\mathrm{L}}_{\mathrm{peak}} \approx m_{\mathrm{t}}c$ like the transversal density.

We turn to the asymptotic limits of the radiant powers and the count rates in (4.20) and (4.22). In the extreme non-relativistic regime, $\alpha^2 \ll m_{\mathrm{t}}^2/m^2 \ll 1$,

$$P^{\mathrm{T}}_{\mathrm{tot}} \sim \frac{1}{3}\,\frac{q^2}{4\pi c}\,\frac{m}{m_{\mathrm{t}}}\, m^2c^2\alpha^6, \quad N^{\mathrm{T}}_{\mathrm{tot}} \sim \frac{q^2}{4\pi c\hbar}\,\frac{m}{m_{\mathrm{t}}} mc\alpha^4, \quad \omega^{\mathrm{T}}_{\mathrm{av}} \sim \frac{1}{3}\, mc\alpha^2\,,$$

$$P^{\mathrm{L}}_{\mathrm{tot}} \sim \frac{q^2}{4\pi c}\,\frac{m}{m_{\mathrm{t}}}\, m^2c^2\alpha^4, \quad N^{\mathrm{L}}_{\mathrm{tot}} \sim 2\,\frac{q^2}{4\pi c\hbar}\,\frac{m}{m_{\mathrm{t}}} mc\alpha^2, \quad \omega^{\mathrm{L}}_{\mathrm{av}} \sim \frac{1}{2}\, mc\alpha^2\,. \tag{4.27}$$

In the non-relativistic limit, $m_{\mathrm{t}}^2/m^2 \ll \alpha^2 \ll 1$,

$$P^{\mathrm{T}}_{\mathrm{tot}} \sim \frac{1}{2}\,\frac{q^2}{4\pi c}\, m_{\mathrm{t}}^2c^2\alpha^3, \quad N^{\mathrm{T}}_{\mathrm{tot}} \sim \frac{q^2}{4\pi c\hbar}\, m_{\mathrm{t}}c\alpha^2, \quad \omega^{\mathrm{T}}_{\mathrm{av}} \sim \frac{1}{2}\, m_{\mathrm{t}}c\alpha\,,$$

$$P_{\rm tot}^{\rm L} \sim \frac{1}{2}\frac{q^2}{4\pi c} m_{\rm t}^2 c^2 \alpha, \quad N_{\rm tot}^{\rm L} \sim \frac{q^2}{4\pi c\hbar} m_{\rm t} c, \quad \omega_{\rm av}^{\rm L} \sim \omega_{\rm av}^{\rm T} . \tag{4.28}$$

The non-relativistic mean frequencies $\omega_{\rm av}^{\rm T,L}$ are close to the transversal spectral peak $\omega_{\rm peak}^{\rm T}$. In the extreme relativistic regime, $\alpha^2 \approx 1$, we find

$$P_{\rm tot}^{\rm T} \sim \frac{1}{2}\frac{q^2}{4\pi c} m_{\rm t}^2 c^2 \left(\log\frac{1}{1-\alpha^2} - 1\right), \quad N_{\rm tot}^{\rm T} \sim \frac{\pi}{2}\frac{q^2}{4\pi c\hbar} m_{\rm t} c ,$$

$$\omega_{\rm av}^{\rm T} \sim \frac{1}{\pi} m_{\rm t} c \left(\log\frac{1}{1-\alpha^2} - 1\right) , \tag{4.29}$$

and the same formulas for the longitudinal radiation with the -1 after the log-terms dropped. The parameter α defining the three asymptotic regimes has been introduced after (4.22); it relates to the subluminal velocity of the source and the tachyon mass by

$$\frac{v^2}{c^2} = \frac{\alpha^2 + \frac{1}{4} m_{\rm t}^2/m^2}{1 + \frac{1}{4} m_{\rm t}^2/m^2} . \tag{4.30}$$

We still have to rescale the masses, $m_{(\rm t)} \to m_{(\rm t)} c/\hbar$, in all formulas of this section, and we define the tachyonic fine structure constant as $\alpha_{\rm q} := q^2/(4\pi\hbar c)$, which is not to be confused with the expansion parameter α. We illustrate the quantities listed in (4.27)–(4.29) with a freely moving electron as source. The electron–tachyon mass ratio is $m_{\rm t}/m \approx 1/238$, resulting in a tachyonic Compton wavelength of $\hbar/(m_{\rm t} c) \approx 0.92$ Å, and the quotient of tachyonic and electric fine structure constants reads $\alpha_{\rm q}/\alpha_{\rm e} \approx 1.4 \times 10^{-11}$, inferred from Lamb shifts in hydrogenic systems [18]. We will use $\alpha_{\rm q} \approx 1.0 \times 10^{-13}$ and $m_{\rm t} \approx 2.15$ keV$/c^2$. The quantities in (4.27)–(4.29) can easily be assembled with these ratios and $m_{\rm t} c^2/\hbar \approx 3.27 \times 10^{18}$ s^{-1}.

As an example for the extreme non-relativistic limit (4.27), we assume the electron at $v_{\rm LG}/c \approx 2.10 \times 10^{-3}$, which is the velocity of the Galaxy in the microwave background, inferred from the dipole anisotropy of the background temperature, cf. the review article of Smoot and Scott in Ref. [23]. This speed coincides with the threshold velocity $v_{\rm min}$ in (4.23), recovered by putting $\alpha = 0$ in (4.30). This suggests that the velocity of the Local Group in the ether is linked to the tachyon mass as stated in (1.1). I do not have a real explanation for that, perhaps it is just a coincidence, but it is most intriguing indeed that $v_{\rm LG}$ is the speed at which free electrons cease to emit tachyons, cease to drain energy from the ether.

In the extreme non-relativistic regime, $\alpha^2 \ll m_{\rm t}^2/m^2$ (that is $0 < \alpha \ll 10^{-3}$), the electronic speed (parametrized as in (4.30)) is virtually independent of α, and very nearly coincides with the threshold velocity. We find with the above constants, cf. (4.27):

$$P_{\rm tot}^{\rm T}\ ({\rm eV\ s^{-1}}) \approx 3.1 \times 10^{15}\alpha^6, \quad N_{\rm tot}^{\rm T}\ ({\rm s^{-1}}) \approx 1.8 \times 10^{10}\alpha^4,$$

$$\hbar\omega_{\rm av}^{\rm T}\ ({\rm eV}) \approx 1.7 \times 10^5 \alpha^2 ,$$

$$P_{\rm tot}^{\rm L}\ ({\rm eV\ s^{-1}}) \approx 9.4 \times 10^{15}\alpha^4, \quad N_{\rm tot}^{\rm L}\ ({\rm s^{-1}}) \approx 3.7 \times 10^{10}\alpha^2,$$

$$\hbar\omega_{\rm av}^{\rm L}\ ({\rm eV}) \approx 2.55 \times 10^5 \alpha^2 ,$$

$$\hbar\omega_{\rm max}\ ({\rm eV}) \approx 5.1\times 10^5\alpha^2, \quad \hbar\omega^{\rm T}_{\rm peak}\ ({\rm eV}) \approx 2.55\times 10^5\alpha^2, \tag{4.31}$$

and $\omega^{\rm L}_{\rm peak} \approx \omega_{\rm max}$. The speed of the radiated quanta at the peak frequencies is readily found by the Einstein relation, $\hbar\omega^{\rm T,L}_{\rm peak} = m_{\rm t}c^2(v^2_{\rm tach}/c^2 - 1)^{-1/2}$. For $\alpha \to 0$, the peak frequencies converge to zero, so that $v_{\rm tach}/c \approx m_{\rm t}c^2/(\hbar\omega^{\rm T,L}_{\rm peak})$ can attain virtually any value, with modest energy transfer, however. This applies to electrons freely propagating very close to the speed of the Local Group, $v_{\rm LG} \approx 627$ km/s, below this threshold no tachyons can be emitted in uniform motion.

The normal non-relativistic regime as defined by (4.28) is covered by $10^{-3} \ll \alpha \ll 1$; in this case we may identify $\alpha \approx v/c$, cf. (4.30), and find

$$\begin{aligned}
&P^{\rm T}_{\rm tot}\ ({\rm keV\ s^{-1}}) \approx 3.5\times 10^5\alpha^3, \quad N^{\rm T}_{\rm tot}\ ({\rm s^{-1}}) \approx 3.3\times 10^5\alpha^2,\\
&\quad \hbar\omega^{\rm T,L}_{\rm av}\ ({\rm keV}) \approx 1.1\alpha\ ,\\
&P^{\rm L}_{\rm tot}\ ({\rm keV\ s^{-1}}) \approx 3.5\times 10^5\alpha, \quad N^{\rm L}_{\rm tot}\ ({\rm s^{-1}}) \approx 3.3\times 10^5\ ,\\
&\hbar\omega_{\rm max}\ ({\rm keV}) \approx 2.15\alpha, \quad \hbar\omega^{\rm T}_{\rm peak}\ ({\rm keV}) \approx 1.2\alpha, \ \ \omega^{\rm L}_{\rm peak} \approx \omega_{\rm max}\ .
\end{aligned} \tag{4.32}$$

In contrast to the extreme non-relativistic limit, these quantities scale with the particle speed, and the energy scale of this radiation is, by at least a factor of 10^3, larger.

Next-generation linear colliders will yield electrons with $E \approx 0.5$ TeV or $\gamma \approx 9.785\times 10^5$ and our first ultra-relativistic example. In (4.29) we put $1-\alpha^2 \approx \gamma^{-2}$, and find

$$\begin{aligned}
&P^{\rm T}_{\rm tot}\ ({\rm GeV\ s^{-1}}) \approx 9.35, \quad N^{\rm T,L}_{\rm tot}\ ({\rm s^{-1}}) \approx 5.1\times 10^5, \quad \hbar\omega^{\rm T}_{\rm av}\ ({\rm keV}) \approx 18.2\ ,\\
&P^{\rm L}_{\rm tot}\ ({\rm GeV\ s^{-1}}) \approx 9.7, \quad \hbar\omega^{\rm L}_{\rm av}\ ({\rm keV}) \approx 18.9\ ,\\
&\hbar\omega_{\rm max}\ ({\rm GeV}) \approx 2.1, \quad \hbar\omega^{\rm T,L}_{\rm peak}\ ({\rm keV}) \approx 2.15\ .
\end{aligned} \tag{4.33}$$

The spectral range is much larger than in the previous examples, and the longitudinal spectral density has a genuine maximum coinciding with the transversal peak frequency; in contrast to the non-relativistic limits, where the longitudinal density is truncated at the break frequency $\omega_{\rm max}$ before the peak is reached. The spectral peaks are not very pronounced, they deviate from the mean frequencies by one order of magnitude. A further increase of the Lorentz factor does not substantially change the radiant powers and the number counts in (4.29), although it strongly affects the shape of the spectral densities, since $\omega_{\rm max}/\omega^{\rm T,L}_{\rm peak} \sim \gamma$. For instance, electrons shock-accelerated to $E \approx 200$ TeV ($\gamma \approx 3.91\times 10^8$) in supernova remnants [25–27] radiate

$$\begin{aligned}
&P^{\rm T}_{\rm tot}\ ({\rm GeV\ s^{-1}}) \approx 13.6, \quad N^{\rm T,L}_{\rm tot}\ ({\rm s^{-1}}) \approx 5.1\times 10^5, \quad \hbar\omega^{\rm T}_{\rm av}\ ({\rm keV}) \approx 26.4\ ,\\
&P^{\rm L}_{\rm tot}\ ({\rm GeV\ s^{-1}}) \approx 13.9, \quad \hbar\omega^{\rm L}_{\rm av}\ ({\rm keV}) \approx 27.1\ , \quad \hbar\omega_{\rm max}\ ({\rm TeV}) \approx 0.84,
\end{aligned} \tag{4.34}$$

with $\hbar\omega^{\rm T,L}_{\rm peak}$ as in (4.33). At this point, one could be tempted to define a radiation lifetime, something like E/P. However, tachyon radiation is generated by a time-symmetric

Green function and an absorber field. Contrary to electromagnetic radiation, there is no deceleration due to radiation loss, the energy spontaneously radiated is contained in the absorber field, supplied by the oscillators of the absorber. In the next section, we will demonstrate that the classical time symmetry (discussed after (2.5)) has its quantal counterpart in the symmetry of the Einstein A-coefficients, the spontaneous emission being balanced by spontaneous absorption.

5. Spontaneous emission and absorption outside the lightcone: Einstein coefficients for free charges

We will study induced and spontaneous radiation in second quantization. A non-relativistic example to that effect, tachyonic transitions between bound states in a Coulomb potential, has already been given in Ref. [24]. Here, we consider tachyon radiation by freely propagating electrons. In this case, the Einstein coefficients can be calculated without multipole approximations. The B-coefficients reflect the symmetry of the induced radiation, however, the A-coefficients are symmetric as well. In electrodynamics, there is no time-symmetric counterpart to spontaneous emission, but outside the lightcone there is spontaneous absorption, the radiated energy being recovered from the absorber medium. The Green function is time symmetric, and so is spontaneous radiation. The spontaneous absorption corresponds to the advanced component of the classical radiation field, cf. Section 2. The quantum statistics of the free tachyon field was studied in Ref. [20], we repeat some formulas needed to compile the matrix elements of the Hamiltonian. Then we calculate and balance the emission rates for uniformly moving charges. Finally we show that the spectral densities (4.18) and (4.19) derived by means of the correspondence principle survive the second quantization. In this paper, the Fourier transforms of the dielectric and magnetic permeabilities of the ether are put equal to one, $\hat{\varepsilon}(\omega)=\hat{\mu}(\omega)=1$, that is, we assume a negligible refractivity and absorptivity, cf. Ref. [20]. Otherwise we would have to specify more parameters, apart from the tachyon mass and the tachyonic fine structure constant.

We start with the plane wave decomposition of the spatial component of the vector potential

$$\mathbf{A}(\mathbf{x},t) = L^{-3/2}\sum_{\mathbf{k}}(\hat{\mathbf{A}}(\mathbf{k})\exp(\mathrm{i}(\mathbf{kx}-\omega t)) + \text{c.c.}), \quad \hat{\mathbf{A}}(\mathbf{k}) := \sum_{\lambda=1}^{3}\boldsymbol{\varepsilon}_{\mathbf{k},\lambda}\hat{a}(\mathbf{k},\lambda) \tag{5.1}$$

with $\mathbf{k} := 2\pi\mathbf{n}/L$. The summation is over integer lattice points $\mathbf{n}$ in R^3, corresponding to periodic boundary conditions in (2.3), so that the $L^{-3/2}\exp(\mathrm{i}\mathbf{kx})$ are orthonormal and complete in a box of size L. The $\boldsymbol{\varepsilon}_{\mathbf{k},1}$ and $\boldsymbol{\varepsilon}_{\mathbf{k},2}$ are arbitrary real unit vectors (linear polarization vectors) orthogonal to $\boldsymbol{\varepsilon}_{\mathbf{k},3} := \mathbf{k}_0 = \mathbf{k}/|\mathbf{k}|$, so that the $\boldsymbol{\varepsilon}_{\mathbf{k},\lambda}$ constitute an orthonormal triad for every $\mathbf{n}$. The amplitudes $\hat{a}(\mathbf{k},\lambda)$ are arbitrary complex numbers.

The Fourier coefficients $\hat{A}_0(\mathbf{k})$ of the time component $A_0(\mathbf{x},t)$ of the 4-potential are defined as in (5.1), and the same holds for the field strengths,

$$\hat{\mathbf{E}}(\mathbf{k}) = \mathrm{i}c^{-1}(\mathbf{k}\hat{A}_0(\mathbf{k}) + \omega\hat{\mathbf{A}}(\mathbf{k})), \quad \hat{\mathbf{B}}(\mathbf{k}) = \mathrm{i}\mathbf{k}\times\hat{\mathbf{A}}(\mathbf{k})\,. \tag{5.2}$$

For $\mathbf{A}$ to be a solution of (2.3) (with $\rho = 0$, $\mathbf{j} = 0$), the dispersion relation,

$$k^2 = \omega^2/c^2 + m_t^2 , \tag{5.3}$$

has to be satisfied, which we henceforth assume; ω and $k := |\mathbf{k}|$ are positive.

We split the potential and the field strengths into transversal and longitudinal components:

$$\begin{aligned}
&\hat{\mathbf{A}}^{\mathrm{T}}(\mathbf{k}) := \sum_{\lambda=1,2} \boldsymbol{\varepsilon}_{\mathbf{k},\lambda}\hat{a}(\mathbf{k},\lambda), \quad \hat{A}_0^{\mathrm{T}} = 0 , \\
&\hat{\mathbf{A}}^{\mathrm{L}}(\mathbf{k}) := \mathbf{k}_0\hat{a}(\mathbf{k},3), \quad \hat{A}_0^{\mathrm{L}}(\mathbf{k}) = -c^2k\omega^{-1}\hat{a}(\mathbf{k},3) , \\
&\hat{\mathbf{E}}^{\mathrm{T}}(\mathbf{k}) = \mathrm{i}c^{-1}\omega\hat{\mathbf{A}}^{\mathrm{T}}(\mathbf{k}), \quad \hat{\mathbf{B}}^{\mathrm{T}} = \mathrm{i}\mathbf{k} \times \hat{\mathbf{A}}^{\mathrm{T}} , \\
&\hat{\mathbf{E}}^{\mathrm{L}} = -\mathrm{i}m_t^2c\omega^{-1}\mathbf{k}_0\hat{a}(\mathbf{k},3), \quad \hat{\mathbf{B}}^{\mathrm{L}} = 0 .
\end{aligned} \tag{5.4}$$

The amplitudes $\hat{a}(\mathbf{k},\lambda)$, $\lambda = 1,2,3$, can be arbitrarily prescribed. Time averages of products such as $\int_{L^3} \mathbf{E}^2(\mathbf{x},t)\,\mathrm{d}\mathbf{x}$, over a period of $2\pi/\omega$, are readily calculated according to $\langle \int \Psi\Phi \rangle = \sum_{\mathbf{k}} \hat{\Psi}(\mathbf{k})\hat{\Phi}^*(\mathbf{k}) + \mathrm{c.c.}$ We find the spatially integrated and time-averaged energy ${T_0}^0$ and the flux vector ${T_0}^m$ in (2.9) as, cf. (5.4),

$$\begin{aligned}
&\left\langle \int \rho_{\mathrm{E}} \right\rangle = \left\langle \int \rho_{\mathrm{E}}^{\mathrm{T}} \right\rangle + \left\langle \int \rho_{\mathrm{E}}^{\mathrm{L}} \right\rangle, \quad \left\langle \int \mathbf{S} \right\rangle = \left\langle \int \mathbf{S}^{\mathrm{T}} \right\rangle + \left\langle \int \mathbf{S}^{\mathrm{L}} \right\rangle , \\
&\left\langle \int \rho_{\mathrm{E}}^{\mathrm{T}} \right\rangle := 2c^{-2} \sum_{\mathbf{k};\lambda=1,2} \omega^2\hat{a}\hat{a}^*, \quad \left\langle \int \mathbf{S}^{\mathrm{T}} \right\rangle := 2 \sum_{\mathbf{k};\lambda=1,2} \mathbf{k}\omega\hat{a}\hat{a}^* , \\
&\left\langle \int \rho_{\mathrm{E}}^{\mathrm{L}} \right\rangle := -2m_t^2 \sum_{\mathbf{k}} \hat{a}(3)\hat{a}^*(3), \quad \left\langle \int \mathbf{S}^{\mathrm{L}} \right\rangle := -2m_t^2c^2 \sum_{\mathbf{k}} \mathbf{k}\omega^{-1}\hat{a}(3)\hat{a}^*(3) .
\end{aligned} \tag{5.5}$$

The interference term of the longitudinal and transversal modes vanishes in the averaging procedure. The sign change of the longitudinal components of energy and flux, anticipated in (2.11), will be effected by Fermi statistics. By comparing the individual modes in these series, we find

$$\left\langle \int \mathbf{S}^{\mathrm{T,L}} \right\rangle_{\mathbf{k},\lambda} = \left\langle \int \rho_{\mathrm{E}}^{\mathrm{T,L}} \right\rangle_{\mathbf{k},\lambda} \mathbf{v}_{\mathrm{gr}}, \quad \mathbf{v}_{\mathrm{gr}} := \mathbf{k}_0\,\mathrm{d}\omega/\mathrm{d}k . \tag{5.6}$$

The group velocity $\mathbf{v}_{\mathrm{gr}}$ follows from the dispersion relation (5.3), $\mathrm{d}\omega/\mathrm{d}k = c^2k/\omega$, cf. the discussion after (3.12). We introduce rescaled Fourier coefficients $a_{\mathbf{k},\lambda}$ in the preceding time averages,

$$\hat{a}(\mathbf{k},\lambda) =: 2^{-1/2}c\hbar^{1/2}\omega^{-1/2}a_{\mathbf{k},\lambda}, \quad \hat{a}(\mathbf{k},3) =: 2^{-1/2}\hbar^{1/2}\omega^{1/2}m_t^{-1}a_{\mathbf{k},3} , \tag{5.7}$$

where $\lambda = 1, 2$, so that the field energy and the flux get amendable to statistical interpretation

$$\left\langle \int \rho_{\mathrm{E}}^{\mathrm{T}} \right\rangle = \sum_{\mathbf{k};\lambda=1,2} \hbar\omega_k a_{\mathbf{k},\lambda} a^*_{\mathbf{k},\lambda}, \quad \left\langle \int \mathbf{S}^{\mathrm{T}} \right\rangle = \sum_{\mathbf{k};\lambda=1,2} \mathbf{v}_{\mathrm{gr}} \hbar\omega_k a_{\mathbf{k},\lambda} a^*_{\mathbf{k},\lambda} , \tag{5.8}$$

$$\left\langle \int \rho_{\mathrm{E}}^{\mathrm{L}} \right\rangle = -\sum_{\mathbf{k}} \hbar\omega_k a_{\mathbf{k},3} a^*_{\mathbf{k},3}, \quad \left\langle \int \mathbf{S}^{\mathrm{L}} \right\rangle = -\sum_{\mathbf{k}} \mathbf{v}_{\mathrm{gr}} \hbar\omega_k a_{\mathbf{k},3} a^*_{\mathbf{k},3} . \tag{5.9}$$

These time averages are the starting point for quantization. We sketch here only very shortly the overall reasoning, for details see Ref. [20]. The Fourier coefficients $a_{\mathbf{k},\lambda}$ are interpreted as operators, and the complex conjugates $a^*_{\mathbf{k},\lambda}$ as their adjoints $a^+_{\mathbf{k},\lambda}$. We use commutation relations, $[a_{\mathbf{k},\lambda}, a^+_{\mathbf{k}',\lambda'}] = \delta_{\mathbf{k}\mathbf{k}'}\delta_{\lambda\lambda'}$, for the transversal modes $\lambda = 1, 2$, which admit the occupation number representation

$$a_i|n_1,\ldots,n_i,\ldots,n_\infty\rangle = \sqrt{n_i}|n_1,\ldots,n_i-1,\ldots,n_\infty\rangle ,$$
$$a_i^+|n_1,\ldots,n_i,\ldots,n_\infty\rangle = \sqrt{n_i+1}|n_1,\ldots,n_i+1,\ldots,n_\infty\rangle . \tag{5.10}$$

Anticommutators, $[a_{\mathbf{k},3}, a^+_{\mathbf{k}',3}]_+ = \delta_{\mathbf{k}\mathbf{k}'}$, are employed for the longitudinal modes, to turn the longitudinal energy (5.9) into a positive definite operator. These Fermi operators admit the representation

$$a_i|n_1,\ldots,n_i,\ldots,n_\infty\rangle = (-)^{n_{<i}} n_i |n_1,\ldots,1-n_i,\ldots,n_\infty\rangle, \quad n_{<i} := \sum_{k=1}^{i-1} n_k ,$$
$$a_i^+|n_1,\ldots,n_i,\ldots,n_\infty\rangle = (-)^{n_{<i}}(1-n_i)|n_1,\ldots,1-n_i,\ldots,n_\infty\rangle , \tag{5.11}$$

where the occupation numbers are now restricted to zero and one. The time-averaged transversal Hamilton operator for the free tachyon field and the transversal flux operator are thus given in (5.8), with the Fourier amplitudes $a_{\mathbf{k},\lambda}a^*_{\mathbf{k},\lambda}$ replaced by the operator product $a^+_{\mathbf{k},\lambda}a_{\mathbf{k},\lambda}$. The energy and flux operators of the longitudinal radiation are obtained by the substitution $a_{\mathbf{k},3}a^*_{\mathbf{k},3} \to -a^+_{\mathbf{k},3}a_{\mathbf{k},3}$ in (5.9). The partition function is easily assembled, the lattice sums being replaced by the continuum limit [28,29], and we find the spectral densities of the transversal and longitudinal radiations as

$$\rho_{\mathrm{T}}(\omega) = \frac{\hbar}{\pi^2 c^3} \frac{\omega^2\sqrt{\omega^2 + m_{\mathrm{t}}^2 c^2}}{\exp(\beta\hbar\omega) - 1}, \quad \rho_{\mathrm{L}}(\omega) = \frac{\hbar}{2\pi^2 c^3} \frac{\omega^2\sqrt{\omega^2 + m_{\mathrm{t}}^2 c^2}}{\exp(\beta\hbar\omega) + 1} . \tag{5.12}$$

We turn to the interaction with subluminal matter. As in Section 4, we consider a spinless quantum particle, a Klein–Gordon field coupled to the tachyonic vector potential by minimal substitution. We write the Lagrangian of the coupled system as $L = L_{\mathrm{P}} + L_\psi$, with the Lagrangian L_{P} of the free Proca field as in (2.1), and

$$L_\psi := c^{-2}\partial_t^A\psi\partial_t^{A*}\psi^* - \nabla^A\psi\,\nabla^{A*}\psi^* - (mc/\hbar)^2\psi\psi^* ,$$
$$\partial_t^A := \partial_t - \mathrm{i}(q/\hbar c)A_0, \quad \nabla^A := \nabla - \mathrm{i}(q/\hbar c)\mathbf{A} . \tag{5.13}$$

The energy density of the free matter field reads

$$H_\psi^{\rm free} := c^{-2}\psi_{,t}\psi_{,t}^* + \nabla\psi\nabla\psi^* + (mc/\hbar)^2\psi\psi^* = c^{-2}(\psi_{,t}\psi_{,t}^* - \psi^*\psi_{,tt}) , \quad (5.14)$$

the second equality is valid up to a divergence, and we used the free field equation as stated before (4.1). The 4-current, the time separation, and the spectral resolution are given in (4.1)–(4.3). We expand the free Klein–Gordon field, $\psi = \sqrt{\hbar}c\sum_n b_n u_n {\rm e}^{-{\rm i}\omega_n t}$, with arbitrary complex amplitudes b_n, normalized eigenfunctions u_n, cf. (4.3), and positive frequencies ω_n. We thus find the energy of the free field, $E = \int H_\psi^{\rm free}{\rm d}^3x = \sum_n \hbar\omega_n b_n b_n^*$, via the orthonormality (4.3). In the 4-current (4.1), we at first put $\varphi = \psi$ and then expand the wave field, so that

$$\rho(\psi) = \hbar c^2 \sum_{m,n} b_m b_n^* \tilde{\rho}_{mn} {\rm e}^{-{\rm i}\omega_{mn}t}, \quad \mathbf{j}(\psi) = \hbar c^2 \sum_{m,n} b_m b_n^* \tilde{\mathbf{j}}_{mn} {\rm e}^{-{\rm i}\omega_{mn}t} , \quad (5.15)$$

with $\tilde{\rho}_{mn}$ and $\tilde{\mathbf{j}}_{mn}$ defined in (4.2) and (4.3). The interaction Hamiltonian can be read off from the Lagrangian (5.1),

$$H_\psi^{\rm int} = \frac{{\rm i}q}{\hbar c}(-c^{-2}A_0\psi^*\psi_{,t} + c^{-2}A_0\psi\psi_{,t}^* + \mathbf{A}\psi^*\nabla\psi - \mathbf{A}\psi\nabla\psi^*) , \quad (5.16)$$

up to terms of $O(q^2)$. Hence, by means of (4.1),

$$H_\psi^{\rm int} = -\frac{1}{\hbar c^3}(A_0\rho(\psi) + \mathbf{A}\mathbf{j}(\psi)) . \quad (5.17)$$

Here we substitute the Fourier expansions (5.15) as well as those of the tachyon field defined by (5.1), (5.4) and (5.7). Finally, we replace the $b_m b_n^*$ in (5.15) by operator products $b_n^+ b_m$, and the tachyonic field amplitudes $a_{\mathbf{k},\lambda}^{(*)}$ by operators $a_{\mathbf{k},\lambda}^{(+)}$ as done after (5.11) for the free field. The subluminal spinless Klein–Gordon field is quantized in Bose statistics, $[b_m, b_n^+] = \delta_{mn}$, so that the representation (5.10) is applicable, and the (anti)commutator brackets and representations for the tachyonic operators $a_{\mathbf{k},\lambda}^{(+)}$ are stated in (5.10) and (5.11).

First we study interaction with transversal tachyons. We consider a fixed linear polarization λ (that is, no summation over λ in the Fourier series). The transversal component of the interaction Hamiltonian (5.17) reads $H_{\rm int}^{\rm T} := -\hbar^{-1}c^{-3}\mathbf{A}^{\rm T}\mathbf{j}(\psi)$, where we substitute the Fourier decompositions (5.1), (5.4), (5.7), and (5.15),

$$\int H_{\rm int}^{\rm T}\,{\rm d}^3x = -\frac{\hbar^{1/2}}{\sqrt{2}L^{3/2}} \sum_{m,n,\mathbf{k}} b_n^+ b_m \omega_k^{-1/2} \left(a_{\mathbf{k},\lambda} \int \boldsymbol{\varepsilon}_{\mathbf{k},\lambda}\tilde{\mathbf{j}}_{mn} {\rm e}^{{\rm i}\mathbf{kx}}\,{\rm d}^3x {\rm e}^{-{\rm i}(\omega_{mn}+\omega_k)t} \right.$$
$$\left. + a_{\mathbf{k},\lambda}^+ \int \boldsymbol{\varepsilon}_{\mathbf{k},\lambda}\tilde{\mathbf{j}}_{mn} {\rm e}^{-{\rm i}\mathbf{kx}}\,{\rm d}^3x {\rm e}^{-{\rm i}(\omega_{mn}-\omega_k)t} \right) \quad (5.18)$$

The amplitudes have been replaced by operators $b_i^{(+)}$ and $a_{\mathbf{k},\lambda}^{(+)}$ as indicated after (5.17). The transversal $a_{\mathbf{k},\lambda=1,2}^{(+)}$ satisfy Bose statistics. We compile the matrix elements of (5.18) with an initial state m and a final state n representing a single subluminal particle and the absorption or emission of a tachyon of polarization λ,

$$\left\langle n \left| \int H_{\rm int}^{\rm T} \right| m \right\rangle_{\rm abs} = \sqrt{n_{\mathbf{k}}} \langle T_{\rm abs}^{\rm T} \rangle {\rm e}^{-{\rm i}(\omega_{mn}+\omega_k)t} , \quad \omega_{mn} < 0 ,$$

$$\left\langle n \left| \int H_{\rm int}^{\rm T} \right| m \right\rangle_{\rm em} = \sqrt{n_{\mathbf{k}}+1} \langle T_{\rm em}^{\rm T} \rangle {\rm e}^{-{\rm i}(\omega_{mn}-\omega_k)t} , \quad \omega_{mn} > 0 ,$$

$$\left\langle T_{\rm abs/em}^{\rm T} \right\rangle := -\frac{\hbar^{1/2}}{\sqrt{2}\omega_k^{1/2} L^{3/2}} \int \boldsymbol{\varepsilon}_{\mathbf{k},\lambda} \tilde{\mathbf{j}}_{mn} {\rm e}^{\pm {\rm i}\mathbf{kx}}\, {\rm d}^3 x . \tag{5.19}$$

The $n_{\mathbf{k}}$ are tachyonic occupation numbers for a state of polarization λ. At this point, $\mathbf{k}$ is a discrete lattice vector, cf. (5.1). The $\langle T_{\rm abs,em}^{\rm T} \rangle$ just differ by a sign change of the wave vector in the exponential. (The upper sign always refers to absorption.) The preceding formulas are standard time-dependent perturbation theory with a periodic potential [30]; the $n_{\mathbf{k}}$-dependent factors stem from the bosonic representation (5.10). The tachyonic wave vector $\mathbf{k}$ relates to the tachyonic frequency ω_k by the dispersion relation (5.3); k and ω_k are positive, and the $\omega_{mn} := \omega_m - \omega_n$ refer to energy levels of the free wave equation, cf. (4.3). The initial state will be denoted by a subscript m and the final state by n, so that a positive ω_{mn} stands for emission.

We turn to the longitudinal component of the interaction (5.17), $H_{\rm int}^{\rm L} = H_{\rm int}^{\rm L(1)} + H_{\rm int}^{\rm L(2)}$, where $H_{\rm int}^{\rm L(1)} = -\hbar^{-1} c^{-3} \mathbf{A}^{\rm L} \mathbf{j}(\psi)$ and $H_{\rm int}^{\rm L(2)} = -\hbar^{-1} c^{-3} A_0 \rho(\psi)$, with the Fourier series for $\mathbf{A}^{\rm L}$ and A_0 defined in (5.1), (5.4) and (5.7). We find, analogously to (5.18),

$$\int H_{\rm int}^{\rm L(1)} {\rm d}^3 x = -\frac{\hbar^{3/2}}{\sqrt{2} m_{\rm t} c^2 L^{3/2}} \sum_{m,n,\mathbf{k}} b_n^+ b_m \omega_k^{1/2}$$
$$\times \left(a_{\mathbf{k},3} \int \mathbf{k}_0 \tilde{\mathbf{j}}_{mn} {\rm e}^{{\rm i}\mathbf{kx}} {\rm d}^3 x {\rm e}^{-{\rm i}(\omega_{mn}+\omega_k)t} + a_{\mathbf{k},3}^+ \int \mathbf{k}_0 \tilde{\mathbf{j}}_{mn} {\rm e}^{-{\rm i}\mathbf{kx}} {\rm d}^3 x {\rm e}^{-{\rm i}(\omega_{mn}-\omega_k)t} \right) ,$$

$$\int H_{\rm int}^{\rm L(2)} {\rm d}^3 x = \frac{\hbar^{3/2}}{\sqrt{2} m_{\rm t} L^{3/2}} \sum_{m,n,\mathbf{k}} b_n^+ b_m k \omega_k^{-1/2}$$
$$\times \left(a_{\mathbf{k},3} \int \tilde{\rho}_{mn} {\rm e}^{{\rm i}\mathbf{kx}} {\rm d}^3 x {\rm e}^{-{\rm i}(\omega_{mn}+\omega_k)t} + a_{\mathbf{k},3}^+ \int \tilde{\rho}_{mn} {\rm e}^{-{\rm i}\mathbf{kx}} {\rm d}^3 x {\rm e}^{-{\rm i}(\omega_{mn}-\omega_k)t} \right) . \tag{5.20}$$

We have here restored the units, $m_{\rm t} \to m_{\rm t} c/\hbar$. The longitudinal operators $a_{\mathbf{k},3}^{(+)}$ anticommute, the representation (5.11) applies, and we assemble the matrix elements of the longitudinal interaction operator as

$$\left\langle n \left| \int H_{\rm int}^{\rm L} \right| m \right\rangle_{\rm abs} = (-)^{n<m} n_{\mathbf{k}} \langle T_{\rm abs}^{\rm L} \rangle {\rm e}^{-{\rm i}(\omega_{mn}+\omega_k)t} ,$$

$$\left\langle n \left| \int H_{\rm int}^{\rm L} \right| m \right\rangle_{\rm em} = (-)^{n<m} (1-n_{\mathbf{k}}) \langle T_{\rm em}^{\rm L} \rangle {\rm e}^{-{\rm i}(\omega_{mn}-\omega_k)t} ,$$

$$\left\langle T^{\rm L}_{\rm abs/em}\right\rangle := \langle T^{\rm L(1)}_{\rm abs/em}\rangle + \langle T^{\rm L(2)}_{\rm abs/em}\rangle, \quad \langle T^{\rm L(1)}_{\rm abs/em}\rangle := \frac{-\hbar^{3/2}\omega_k^{1/2}}{\sqrt{2}m_{\rm t}c^2L^{3/2}}\int \mathbf{k}_0\tilde{\mathbf{j}}_{mn}{\rm e}^{\pm{\rm i}\mathbf{kx}}\,{\rm d}^3x\,,$$

$$\left\langle T^{\rm L(2)}_{\rm abs/em}\right\rangle := \frac{\hbar^{3/2}k}{\sqrt{2}m_{\rm t}\omega_k^{1/2}L^{3/2}}\int \tilde{\rho}_{mn}{\rm e}^{\pm{\rm i}\mathbf{kx}}\,{\rm d}^3x\,. \tag{5.21}$$

Here, $n_{\mathbf{k}}$ is an occupation number in Fermi statistics, zero or one, and $(-)^{n<m}$ denotes the sign factor occurring in the fermionic representation (5.11); $\mathbf{k}_0 = \mathbf{k}/k$ is the tachyonic unit wave vector. The generalization of the matrix elements (5.19) and (5.21) to a refractive and absorptive spacetime can be found in Ref. [20]. Finally we return to (4.1)–(4.3), and inspect the integral $\int(u_m\Delta u_n^* - u_n^*\Delta u_m){\rm e}^{\pm{\rm i}\mathbf{kx}}\,{\rm d}^3x$, once by applying the Gauss theorem, and once by using the Klein–Gordon equation. In this way we derive $\mathbf{k}_0\tilde{\mathbf{j}}_{mn}{\rm e}^{\pm{\rm i}\mathbf{kx}} = \mp k^{-1}\omega_{mn}\tilde{\rho}_{mn}{\rm e}^{\pm{\rm i}\mathbf{kx}}$, valid under the integral sign, cf. (2.25). Thus, we can express the longitudinal T-matrix by the charge density alone:

$$\left\langle T^{\rm L}_{\rm abs/em}\right\rangle = \frac{m_{\rm t}c^2}{\sqrt{2}\hbar^{1/2}\omega_k^{1/2}kL^{3/2}}\int \tilde{\rho}_{mn}{\rm e}^{\pm{\rm i}\mathbf{kx}}\,{\rm d}^3x\,, \tag{5.22}$$

where we used energy conservation, $\omega_k = \mp\omega_{mn}$ in (5.21), as well as the tachyonic dispersion relation (5.3) (with $m_{\rm t} \to m_{\rm t}c/\hbar$.)

Once the matrix elements are known, the transition rate for transversally induced absorption and emission in a given polarization λ is obtained by a standard procedure [30],

$$w^{\rm T,ind}_{\rm abs/em} \sim \frac{1}{t\hbar^2}\sum_{\mathbf{k}} n_{\mathbf{k}}\left|\left\langle T^{\rm T}_{\rm abs/em}\right\rangle\right|^2\left|\int_{-t/2}^{t/2}{\rm e}^{-{\rm i}(\omega_{mn}\pm\omega_k)t}\,{\rm d}t\right|^2$$

$$\sim \frac{2\pi}{\hbar^2c^2}\frac{L^3}{(2\pi)^3}\int\left|\left\langle T^{\rm T}_{\rm abs/em}\right\rangle\right|^2\delta_{(1)}(\omega_{mn}\pm\omega;t)\,\frac{k(\omega)\omega}{{\rm e}^{\beta\hbar\omega}-1}\,{\rm d}\omega\,{\rm d}\Omega\,, \tag{5.23}$$

valid for large times t, with the smooth Dirac limit function $\delta_{(1)}$ as defined in (2.13). We have here replaced the box-summation by the continuum limit, $L^3(2\pi)^{-3}\int{\rm d}\mathbf{k}$, and the occupation numbers by their averages $\langle n_{\mathbf{k}}\rangle = ({\rm e}^{\beta\hbar\omega_k}-1)^{-1}$, cf. (5.12). ${\rm d}\Omega = \sin\theta\,{\rm d}\theta\,{\rm d}\varphi$, the solid angle element of the tachyonic wave vector, and $k(\omega)$ is given in (5.3). The same formula also applies to spontaneous radiation, $w^{\rm T,sp}_{\rm em}$, but with the $\langle n_{\mathbf{k}}\rangle$-factor dropped, since $w^{\rm T,sp}_{\rm em}$ stems from the $+1$ under the root in (5.19). In the limit $t\to\infty$, the ${\rm d}\omega$-integration in (5.23) gets trivial, and by substituting (5.19) we find

$$\begin{aligned}{\rm d}w^{\rm T,ind}_{\rm abs/em} &\sim \frac{1}{8\pi^2}\frac{k}{\hbar c^2}\frac{1}{{\rm e}^{\beta\hbar\omega}-1}\left|\int \boldsymbol{\varepsilon}_{\mathbf{k},\lambda}\tilde{\mathbf{j}}_{mn}{\rm e}^{\pm{\rm i}\mathbf{kx}}\,{\rm d}^3x\right|^2 d\Omega \\ &=: \tfrac{1}{2}B^{\rm T}_{mn}(\mp\mathbf{k},\lambda)\rho_{\rm T}(\omega)\,{\rm d}\Omega\,,\end{aligned} \tag{5.24}$$

$${\rm d}w^{\rm T,sp}_{\rm em} \sim ({\rm e}^{\beta\hbar\omega}-1){\rm d}w^{\rm T,ind}_{\rm em} =: A^{\rm T}_{mn}(\mathbf{k},\lambda)\,{\rm d}\Omega\,, \tag{5.25}$$

where ω (and $k(\omega)$) is taken at $|\omega_{mn}|$. The upper sign refers to absorption, and m to the initial state. The transversal tachyonic spectral density $\rho_{\rm T}$ is defined in (5.12). (The spectral densities in (5.12) refer to the tachyonic heat bath triggering the induced radiation.) The total emission rate is $\mathrm{d}w_{\mathrm{em}}^{\mathrm{T}} = \mathrm{d}w_{\mathrm{em}}^{\mathrm{T,ind}} + \mathrm{d}w_{\mathrm{em}}^{\mathrm{T,sp}}$. In equilibrium, induced emission and absorption compensate each other, due to the detailed balancing symmetry $B_{mn}^{\mathrm{T}}(\mathbf{k},\lambda) = B_{nm}^{\mathrm{T}}(-\mathbf{k},\lambda)$, which follows from the hermiticity of the current matrices (4.2). The spontaneous emission of transversal tachyons is temperature independent, unaffected by the tachyonic heat bath, in contrast to the longitudinal emission discussed below. The unpolarized transversal radiation rates are obtained by replacing $\varepsilon_{\mathbf{k},\lambda}\tilde{\mathbf{j}}_{mn}$ in (5.24) by the transversal current, $\tilde{\mathbf{j}}_{mn}^{\mathrm{T}} := \tilde{\mathbf{j}}_{mn} - \mathbf{k}_0(\mathbf{k}_0\tilde{\mathbf{j}}_{mn})$, where $\mathbf{k}_0 := \mathbf{k}/k$ and

$$\tilde{\mathbf{j}}_{mn} = c^2 \frac{\mathbf{k}_m + \mathbf{k}_n}{\omega_m + \omega_n} \tilde{\rho}_{mn}, \qquad \tilde{\rho}_{mn} = \frac{q(\omega_m + \omega_n)}{2L^3\sqrt{\omega_m\omega_n}} \exp(\mathrm{i}(\mathbf{k}_m - \mathbf{k}_n)\mathbf{x}) , \tag{5.26}$$

cf. (4.2) and (4.3).

The spontaneous emission rate (5.25) is symmetric, $A_{mn}^{\mathrm{T}}(\mathbf{k},\lambda) = A_{nm}^{\mathrm{T}}(-\mathbf{k},\lambda)$, reflecting the time symmetry of the classical radiation field, cf. Section 2. (The radiation discussed in the previous sections is all spontaneous.) The retarded field, which we have quantized, results from the absorber field complementing the time-symmetric field of the particle, as pointed out after (2.5). The net energy balance of the time-symmetric field is zero, as the spontaneous emission of a tachyon is accompanied by the absorption of an absorber quantum. This restores the initial state of the source in the reverse transition. Spontaneous absorption stands as the quantal analog to the advanced modes of the time-symmetric classical wave field. Induced transitions are not affected by the absorber field, and in equilibrium induced emission and absorption cancel each other, due to the mentioned symmetry of the B-coefficients. In the energy balance for the equilibrium distribution $\rho_{\mathrm{T}}(\omega)$ in (5.12), the different Boltzmann weights are accounted for by the A-coefficients,

$$N_m(\tfrac{1}{2}B_{mn}^{\mathrm{T}}(\mathbf{k},\lambda)\rho_{\mathrm{T}}(\omega) + A_{mn}^{\mathrm{T}}(\mathbf{k},\lambda)) = \tfrac{1}{2}N_n B_{nm}^{\mathrm{T}}(-\mathbf{k},\lambda)\rho_{\mathrm{T}}(\omega) , \tag{5.27}$$

and the occupation numbers relate by $N_m/N_n = \exp(-\beta\hbar\omega_{mn})$, quite independent of the statistics.

We turn to longitudinal radiation. The induced absorption/emission rate for longitudinal tachyons is composed analogously to the transversal rates (5.23),

$$w_{\mathrm{abs/em}}^{\mathrm{L,ind}} \sim \frac{1}{t\hbar^2} \sum_{\mathbf{k}} n_{\mathbf{k}} |\langle T_{\mathrm{abs/em}}^{\mathrm{L}}\rangle|^2 \left| \int_{-t/2}^{t/2} \mathrm{e}^{-\mathrm{i}(\omega_{mn} \pm \omega_k)t}\, \mathrm{d}t \right|^2 , \tag{5.28}$$

with $\langle T_{\mathrm{abs/em}}^{\mathrm{L}}\rangle$ in (5.22). The fermionic occupation numbers are replaced in the continuum limit by the averages $\langle n_{\mathbf{k}}\rangle = (\mathrm{e}^{\beta\hbar\omega_k} + 1)^{-1}$, cf. (5.12). Hence,

$$\mathrm{d}w_{\mathrm{abs/em}}^{\mathrm{L,ind}} \sim \frac{1}{8\pi^2} \frac{m_{\mathrm{t}}^2 c^2}{\hbar^3 k} \frac{1}{\mathrm{e}^{\beta\hbar\omega} + 1} \left| \int \tilde{\rho}_{mn} \mathrm{e}^{\pm\mathrm{i}\mathbf{k}\mathbf{x}}\, \mathrm{d}^3x \right|^2 \mathrm{d}\Omega =: B_{mn}^{\mathrm{L}}(\mp\mathbf{k})\rho_{\mathrm{L}}(\omega)\, \mathrm{d}\Omega , \tag{5.29}$$

where m denotes the initial state, both for absorption and emission, and $\omega = |\omega_{mn}|$. The longitudinal spontaneous emission is identified as follows. The $n_{\mathbf{k}}$ in (5.21) can only take the values zero and one, so that the factor $1 - n_{\mathbf{k}}$ does not change if squared.

Thus the total emission rate is $dw^{L}_{em} = dw^{L,sp}_{em,T=0} - dw^{L,ind}_{em}$, with $dw^{L,ind}_{em}$ as defined by (5.29) and

$$dw^{L,sp}_{em,T=0} := (e^{\beta\hbar\omega} + 1)\, dw^{L,ind}_{em} . \tag{5.30}$$

This is the spontaneous transition rate in the zero temperature limit, obtained from (5.28) with the $n_{\mathbf{k}}$-factors dropped. At finite temperature, the spontaneous emission is $dw^{L,sp}_{em} = dw^{L,sp}_{em,T=0} - 2dw^{L,ind}_{em}$, so that the total emission $dw^{L}_{em} = dw^{L,ind}_{em} + dw^{L,sp}_{em}$. Hence,

$$dw^{L,sp}_{em} \sim (e^{\beta\hbar\omega} - 1)\, dw^{L,ind}_{em} = \tanh(\beta\hbar\omega/2)\, dw^{L,sp}_{em,T=0} =: A^{L}_{mn}(\mathbf{k})\, d\Omega , \tag{5.31}$$

which reduces in the absence of a tachyonic heat bath to $dw^{L,sp}_{em,T=0}$ The basic symmetries $B^{L}_{mn}(\mathbf{k}) = B^{L}_{nm}(-\mathbf{k})$ and $A^{L}_{mn}(\mathbf{k}) = A^{L}_{nm}(-\mathbf{k})$ also extend to longitudinal radiation, so that the induced transitions cancel each other, and a spontaneous transition is instantaneously restored by an absorber quantum. The longitudinal spontaneous emission (5.31) is temperature dependent and vanishes in the high temperature limit. At finite temperature, the equilibrium condition, cf. (5.27),

$$B^{L}_{mn}(\mathbf{k})\rho_{L}(\omega) + A^{L}_{mn}(\mathbf{k}) = B^{L}_{nm}(-\mathbf{k})\rho_{L}(\omega)\exp(\beta\hbar\omega_{mn}) , \tag{5.32}$$

requires the longitudinal density $\rho_{L}(\omega)$ in (5.12).

I take this opportunity to correct a mistake in the dipole approximation of the longitudinal transition probability calculated in Ref. [20]. The squared ratio $\hbar\omega_{ji}/(m_t c^2)$ in Eqs. (5.17), (5.23) and (5.27) of Ref. [20] should be inverted. At 2.2 MeV, the ratio of the longitudinal and transversal dipole transition rates reads $\omega^{L}/\omega^{T} \approx 3.8 \times 10^{-7}$, from which we conclude that the longitudinal background radiation has reached equilibrium within 10^{18} s. This is to be compared with a cosmic age of $H_0^{-1} \approx 14$ Gyr $\approx 4.4 \times 10^{20}$ s. The reasoning behind this is explained in Ref. [20].

At zero temperature, the power spontaneously radiated by a freely propagating charge was calculated in Section 4 by means of the correspondence principle, which amounts to identify in (2.20)–(2.26) $\tilde{\rho}(\mathbf{x}, \omega_{mn})$ and $\tilde{\mathbf{j}}(\mathbf{x}, \omega_{mn})$ with the hermitian current matrices $\tilde{\rho}_{mn}$ and $\tilde{\mathbf{j}}_{mn}$ in (5.26). The powers (4.20) and (4.22) can be recovered from the emission rates $dw^{T,sp}_{em}$ and $dw^{L,sp}_{em,T=0}$ in (5.24), (5.25), (5.29), and (5.30). The angular-integrated power radiated at $\omega = \omega_{mn}$ is apparently

$$P^{T}(\omega_{mn}) = \hbar\omega_{mn} \int_{\Omega} dw^{T,sp}_{em}, \quad P^{L}(\omega_{mn}) = \hbar\omega_{mn} \int_{\Omega} dw^{L,sp}_{em,T=0} . \tag{5.33}$$

We consider unpolarized transversal radiation, which means to replace $\boldsymbol{\varepsilon}_{\mathbf{k},\lambda}\tilde{\mathbf{j}}_{mn}$ in $dw^{T,sp}_{em}$ by the transversal current $\tilde{\mathbf{j}}^{T}_{mn}$. If we substitute the current (5.26) into the powers (2.22) and (2.23), we obtain (5.33); thus the spectral densities (4.18) and (4.19) also hold in second quantization.

6. Conclusion

The absorber theory [12] was motivated by Dirac's covariant version of radiation damping [15], where the absorber field, half-retarded minus half-advanced, enters as Lorentz force. In the non-relativistic derivation of Abraham and Lorentz [14] it does so as well, of course, but in a less explicit way. In any case, this field is not perceived as stemming from an absorber medium, but rather as generated by the charge itself. In Dirac's theory, the absorber field does not show as radiation field in the equations of motion, but is exclusively applied along the trajectory of the charge, defining the damping force. Here we have elaborated on superluminal radiation fields at large distance from the source, the opposite limit. The asymptotic fields are quite sufficient to calculate the spectral densities and the radiant power, classically as well as in second quantization. It is not advisable to rely on the short distance behavior of Green functions; the self-energy problem indicates that the Maxwell theory may just be the asymptotic limit of a non-linear Born–Infeld type of electrodynamics [31]. If so, one cannot use the linearized theory in the vicinity of the radiating sources. The same holds for the Proca field.

Wheeler and Feynman designed the absorber theory for electrodynamics, and they interpreted the half-retarded minus half-advanced Liénard–Wiechert potential, cf. Section 2, as generated by an absorber medium, which they proposed to be the collection of electric charges in the universe [12]. They used this potential in an action-at-a-distance electrodynamics [10,11,13], in an attempt to solve the radiation damping problem. In the Maxwell theory, we do not consider an absorber medium because there is a retarded Green function. Outside the lightcone, however, retardation can only be achieved by an absorber field, as the Green function supported there is time symmetric. A causal theory of superluminal signals needs an absolute spacetime, since Lorentz boosts do not preserve the time order in spacelike connections. Once the absolute nature of space is acknowledged, it is only a small step to identify space itself as the absorber medium, the ether, whose microscopic oscillators generate the absorber field [16,20].

I conclude by comparing the absolute spacetime underlying superluminal radiation to the relativistic spacetime view. Radiation by inertial charges may be unimaginable in relativity theory, but in the absolute cosmic spacetime this is easy to comprehend, since accelerated and inertial frames are treated on the same basis. There is a universal reference frame, the rest frame of the ether, generated by the comoving galaxy grid and manifested by the microwave background and other background radiations. The spectral density of the radiation is determined by the velocity of the uniformly moving charge. This is not a relative velocity, it stands for the absolute motion of the charge in the ether. Relative velocities only affect the appearance of the radiation in moving frames. In the rest frames of inertial observers, the radiation field may appear advanced, the transversal and longitudinal modes may appear tangled, or they may not appear at all, as it happens in the rest frame of the radiating charge [24], but all this is a consequence of the observer's individual motion. Whatever the appearance of the superluminal radiation field in a moving frame, the observer can infer the radiation in the rest frame of the ether (such as the power, the spectral densities and the frequencies radiated) by measuring the absolute velocity of the charge in the microwave background.

More generally, the relativity principle asserts that the laws of nature are the same in all inertial frames, in particular, uniform motion and rest are not distinguishable in this respect. In the absolute cosmic spacetime, the laws of nature are inherent in the rest frame of the ether, and their appearance in inertial frames is determined by the observer's state of motion. This is in sharp contrast to relativity theory, where the laws of nature are thought of as attached to individual and equivalent inertial frames. The absolute spacetime concept is centered at the state of rest, tantamount to the universal reference frame generated by the galaxy grid. Particles move in the ether, subjected to the flow of cosmic time as defined by the galactic recession, without resort to the inertial frames and proper times of individual observers. This is again in strong contrast to relativity theory, where inertial frames are the substitute for the universal rest frame. In the absolute cosmic spacetime, the crucial distinction is not between inertial and accelerated frames, but simply between motion and rest, and therefore it is not surprising that uniformly moving charges radiate.

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References

[1] G. Feinberg, Phys. Rev. 159 (1967) 1089.
[2] G. Feinberg, Sci. Am. 222 (2) (1970) 69.
[3] R. Newton, Science 167 (1970) 1569.
[4] R. Tomaschitz, Chaos Solitons Fractals 7 (1996) 753.
[5] R. Tomaschitz, Chaos Solitons Fractals 8 (1997) 761.
[6] Th. Des Coudres, Arch. Néerland. Sci. (II) 5 (1900) 652.
[7] S. Tanaka, Prog. Theor. Phys. 24 (1960) 171.
[8] Ya.P. Terletsky, Sov. Phys. Dokl. 5 (1961) 782.
[9] G. Feinberg, Phys. Rev. D 17 (1978) 1651.
[10] A.D. Fokker, Physica 9 (1929) 33.
[11] A.D. Fokker, Physica 12 (1932) 145.
[12] J.A. Wheeler, R.P. Feynman, Rev. Mod. Phys. 17 (1945) 157.
[13] J.A. Wheeler, R.P. Feynman, Rev. Mod. Phys. 21 (1949) 425.
[14] H.A. Lorentz, Theory of Electrons, Dover, New York, 1952.
[15] P.A.M. Dirac, Proc. Roy. Soc. A 167 (1937) 148.
[16] R. Tomaschitz, Class. Quant. Grav. 18 (2001) 4395.
[17] A. Proca, J. Phys. 7 (1936) 347.
[18] R. Tomaschitz, Eur. Phys. J. B 17 (2000) 523.
[19] R. Tomaschitz, Celest. Mech. Dyn. Astron. 77 (2000) 107.
[20] R. Tomaschitz, Physica A 307 (2002) 375.
[21] E. Wiechert, Arch. Néerland. Sci. (II) 5 (1900) 549.
[22] A. Sommerfeld, Proc. K. Ned. Akad. Wet. (Sec. Sci.) 7 (1904) 346.
[23] K. Hagiwara, et al., Phys. Rev. D 66 (2002) 010001.
[24] R. Tomaschitz, Physica A 293 (2001) 247.
[25] K. Koyama, et al., Nature 378 (1995) 255.

[26] O.C. de Jager, et al., Astrophys. J. 457 (1996) 253.
[27] S.P. Reynolds, J.W. Keohane, Astrophys. J. 525 (1999) 368.
[28] B. Mashhoon, Phys. Rev. D 8 (1973) 4297.
[29] S. Singh, R.K. Pathria, Phys. Rev. A 30 (1984) 442.
[30] L.D. Landau, E.M. Lifshitz, Quantum Mechanics: Non-relativistic Theory, Pergamon, London, 1991.
[31] M. Born, L. Infeld, Proc. Roy. Soc. A 144 (1934) 425.