SUPERLUMINAL KINEMATICS IN THE MILNE UNIVERSE: CAUSALITY IN THE COSMIC TIME ORDER

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Abstract. The causality of superluminal signal transfer in the galaxy background is scrutinized. The cosmic time of the comoving galaxy frame determines a distinguished time order for events connected by superluminal signals. Every observer can relate his rest frame to the galaxy frame, and compare so the time order of events in his proper time to the cosmic time order. In this way all observers arrive at identical conclusions on the causality of events connected by superluminal signals. The energy of tachyons (superluminal particles) is defined in the comoving galaxy frame analogous to the energy of subluminal particles. It is positive in the galaxy frame and bounded from below in the rest frames of geodesically moving observers, so that particle–tachyon interactions can be based on energy–momentum conservation. We study tachyons in a Robertson–Walker cosmology with linear expansion factor and open, negatively curved 3-space (Milne universe). This cosmology admits globally geodesic rest frames for uniformly moving observers, synchronized by Lorentz boosts. In this context we show that no signals can be sent into the past of observers. If an observer emits a tachyonic signal, then the response of a second observer can never reach him prior to the emission, i.e., no predetermination can occur. The proof is based on the positivity of tachyonic energy.

Key words: tachyons, superluminal signals, cosmic time, causality, Robertson–Walker cosmology, hyperbolic space

1. Introduction

Superluminal particles (tachyons) are a possibility suggested by a straightforward modification of the formalism of classical relativistic mechanics, they are a natural extension of the classical particle concept (Tanaka, 1961; Terletsky, 1961; Bilaniuk et al., 1962; Feinberg, 1967,1978; Parker, 1969; Davies, 1975, to mention but a few). However, relativistic theories of superluminal motion are marred by causality violation (Feinberg, 1970; Newton, 1970; Pirani, 1970), as Lorentz boosts may change the time order of events connected by superluminal signals. If a uniformly moving observer O_1 sees a tachyon T moving from space point A to space point B, then a second observer O_2 related to the first by a Lorentz boost may well see it heading from B to A. To see a tachyon moving from A to B just means here to observe the change effected by the tachyon at A (emission) prior to the change effected at B (absorption). By definition, emission happens always prior to absorption. Observer O_1 concludes that the change at A (effected by the emission of T) causes the change at B (effected by the absorption of T). Observer O_2, however,

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concludes that the change at *B* (emission) causes the change at A (absorption). Both observers base their conclusion on the assumption that the cause precedes the effect, of course. For observer O_1, the cause is the change that takes place at space point *A* by the emission of the tachyon. The effect is the change that takes place at *B* by its absorption. The same holds for observer O_2, but with *A* and *B* interchanged. What appears as emission to observer O_1 is absorption for observer O_2, and vice versa, as the time order in the two rest frames is different. (In the causality proof given in this paper, there is no need to specify the change effected by the tachyon at *A* and *B* in terms of energy, though we will also address that. In particular, we defined emission and absorption without reference to energy transfer). According to the relativity principle, the conclusions of both observers concerning cause and effect must be regarded as equally real, as physically equivalent. This leads to a violation of the traditional causality principle, which may be stated as follows (Tomaschitz, 1997a): (i) Every effect has a cause. (ii) The cause precedes the effect. (iii) The distinction of cause and effect is unambiguous. The third condition simply means that all observers must come to the same conclusion on what is cause and effect. The conclusions of observers O_1 and O_2 are evidently different.

Remarks: (1) Bilaniuk et al. (1962) and Feinberg (1967) define cause and effect by energy loss and energy gain, respectively, which is a relativistically invariant characterization if properly done, but it conflicts with condition (ii) of the causality principle. (2) As mentioned, we use the terms emission and absorption in a frame-dependent, geometric way. We say that in the rest frame of a given observer the tachyon is emitted at space point *A* and absorbed at *B*, if it appears to this observer as moving from *A* to *B*, i.e., if *A* is the initial and *B* the terminal point of its trajectory, which is parametrized by the proper time of the observer.

In this paper we study tachyons in the Milne universe, a Robertson–Walker (RW) cosmology isometric to the forward light cone (Milne, 1932,1948). The cosmic time of the comoving galaxy frame defines a distinguished time order, to which every observer can relate, and this is the basis of the causality proof for superluminal signals given in this paper. The Milne universe is a RW cosmology with a linear expansion factor and an open, negatively curved 3-space. One can introduce globally geodesic rest frames for uniformly moving observers, and synchronize them by Lorentz transformations (Anderson et al., 1998). In this respect this universe is quite similar to a static Minkowski universe, but it is expanding. Though this cosmology is known for long, it never gained great popularity, presumably due to the fact that it is based on a flat space-time; thus the curvature tensor vanishes and, via the Einstein equations, the energy–momentum tensor. However, the possibilities of evolution of an open universe go far beyond what is predictable by Einstein's equations (Dyson, 1979; Tomaschitz, 1997b), and there is to this day no observational evidence for the validity and applicability of these equations concerning cosmic evolution (Sandage, 1988; Bahcall et al., 1999).

In RW cosmology, there exists a coordinate frame in which all galaxies and galactic observers have constant space coordinates, despite their mutual recession. By this comoving frame, a universal cosmic time is defined, and thus a universal time order of events. Every observer, galactic or not, can compare the time order of events in his proper time to the universal cosmic time order, and all observers can so figure out the cosmic causal connection of events related by superluminal signals, even though the cosmic time order may be inverted in their proper time.

In Minkowski space, there seems to be at first sight a very straightforward generalization of the energy–momentum concept for subluminal particles to tachyons. But it turns out that the sign of the energy of tachyons is not preserved under Lorentz boosts, and because of this ambiguity it cannot be used to define interactions with subluminal particles via energy–momentum conservation. There has been a rescue attempt (Bilaniuk et al. 1962; Feinberg, 1967) to reinterpret tachyons of negative energy as antiparticles with positive energy, similar to the negative energy solutions of the Dirac equation, and to define so a positive energy in an invariant way. However, this does not solve the causality problem (Newton, 1970). In the theory advanced in this paper, the energy of tachyons is defined without using the quantum mechanical antiparticle concept. Tachyonic energy is defined in complete analogy to classical subluminal particles as a positive quantity in the comoving galaxy frame. It is bounded from below in the geodesic rest frames of uniformly moving observers, so that it can be used, via energy–momentum conservation, to define particle–tachyon interactions, without running the risk to create a tachyonic perpetuum mobile. The energy transfer in elastic head-on collisions of tachyons and particles is worked out in Tomaschitz (1998b). A field theory for tachyons (Proca field with negative mass square), a possible cosmic tachyon background radiation, and the interaction of tachyons with matter are studied in Tomaschitz (1999a,b, 2000b,c), where a tachyon mass of $m_{\mathrm{t}} \approx m_{\mathrm{e}}/238 \approx 2.15\,\mathrm{keV}/c^2$ was derived from Lamb shifts in hydrogen-like systems. Here we focus on classical mechanics in the galaxy background and the causality principle.

Remarks: Tachyons are defined as particles with negative mass square, but other mechanisms to generate superluminal motion are quite possible, e. g., light cone fluctuations (Ford, 1995). Quantum mechanical wave packets with superluminal group velocity and their stability are discussed in Aharanov et al. (1998). Observational implications of a speed of light varying in cosmic time (Dyson, 1972; Dirac, 1973) on the luminosity distance, angular diameters, the surface brightness, source counts, and the age of the universe, are discussed in Tomaschitz (1998c, 2000a). An illuminating discussion of the causality principle in the context of a multiply connected spacetime, unrelated to superluminal signal transfer, can be found in Fuller and Wheeler (1962). Hyle and Narlikar (1995) review causality in the cosmic absorber theory (time-symmetric electrodynamics) of Wheeler and Feynman (1945).

As mentioned, we do not make use of tachyonic antiparticles, nor do we consider the related concept of superluminal motion backwards in time, which can be characterized by negative energies and which leads to a causality that allows for effects to precede their causes (Feinberg, 1967; Bilaniuk et al., 1962). As pointed out by Pirani (1970), superluminal signal exchange becomes rather confusing on the basis of a generalized causality, in which the meaning of cause and effect gets blurred by dropping the second or third condition of the causality principle. The purpose of this paper is to demonstrate that the frame of reference defined by the cosmic microwave background offers a very conventional causality interpretation of superluminal signal transfer.

Though tachyons are treated in this paper as classical point particles, this theory is to be understood as the geometric optics limit of a real Proca field coupled to subluminal matter in a similar way as the electromagnetic field (Tomaschitz, 1999a, b). Tachyons are viewed as an extension of the photon concept, a sort of photons with negative mass square, they do not carry any kind of charge, tachyonic charge is a property of subluminal particles, as is electric charge. Accordingly, we do not adopt an antiparticle concept for tachyons.

Cosmic space is generated by the galaxy grid, which provides a natural reference frame, practically realized by the Planckian microwave background. The consequences of this frame have never been seriously faced, though it is widely acknowledged that comoving coordinates define a distinguished cosmic time. The state of absolute rest can be defined with respect to the galaxy background, and uniform motion and rest become easily distinguishable states. Whether an observer is at rest or in uniform motion with respect to the microwave background, this can really be unambiguously decided, quantitatively, by measuring the dipole antisotropy of the background temperature, caused by a Doppler shift. If tachyons are defined with respect to this universal rest frame, as we are going to do, a causality problem does not even arise, since the cosmic time order of events is unambiguously defined by this frame. All uniformly moving observers, irrespective of their location in the universe, can relate their proper time to cosmic time, by determining their motion relative to the background radiation, and arrive in this way at the same conclusion on causal connections. To figure the causality of an experiment involving tachyons, one has to connect the lab to the rest of the universe and to determine its motion relative to the galaxy background. This is quite possible today, thanks to the microwave radiation; the solar barycenter is moving at some 370 km/s (Smoot and Scott, 1998), fast enough to even neglect the relative motions of the Earth in a first approximation.

The background radiation is the practical tool to determine the observer's velocity in the galaxy grid, a photon gas pervading space. However, if there is an absolute cosmic space as defined by the galaxy grid, we are again permitted to contemplate on the substance of space itself, the ether, the medium that makes wave propagation possible (Whittaker, 1951; Tomaschitz, 1998a,c,d). The galaxy grid is anchored in the ether, the local manifestation of the absolute cosmic space, and a quantum particle or tachyon propagating in the ether 'knows' its state of motion quite without reference to the background radiation. Superluminal wave propagation in the ether, the Proca equation coupled to the permeability tensor, the refractive index of the ether (Tomaschitz, 2000a) with regard to tachyons, and the Wheeler-Feynman absorber theory in the context of an absolute cosmic space-time will be discussed elsewhere.

In Section 2, we determine the world-lines of particles (observers) and tachyons in the comoving galaxy frame as well as in the globally geodesic rest frames of uniformly moving observers, and we define the energy concept for tachyons. In Section 3, we study superluminal signal exchange between two galactic observers, in Section 4 between a galactic and a non-galactic (but uniformly moving) observer, and in Section 5 between two non-galactic observers. The causality proof is given as follows. An observer emits a tachyon which is absorbed by a second observer. As soon as the absorption takes place, the second observer emits as his response a tachyon which is in turn absorbed by the first. In Sections 3–5 it is demonstrated that in the rest frame of the first observer the response of the second does not arrive prior to the emission of the first tachyon. We show that in the rest frame of the first observer the emission of the tachyon is not predetermined by the response to it. The proof is based on the positivity of the energy of the tachyons in the rest frames in which they are emitted. In Section 6 we present our conclusions.

2. Tachyonic World-Line in Cosmic and Local Time

The Milne universe is a RW cosmology with negatively curved 3-space and linear expansion factor, defined by the line element

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \tau^2 u^{-2}(\mathrm{d}u^2 + |\mathrm{d}\xi|^2), \tag{2.1}$$

(u, ξ) denote Cartesian coordinates in the Poincaré half-space H^3, $u > 0, \xi = x_1 + ix_2$ (complex notation), cf. Fenchel (1983) and Balazs and Voros (1986). Cosmic time τ ranges in $0 < \tau < \infty$. This 4-manifold can be isometrically mapped onto the interior of the forward light cone, $t^2 - \mathbf{x}^2 > 0$, $t > 0$, $\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2$ (globally geodesic coordinates),

$$t = \frac{1}{2}\frac{\tau}{u}(|\xi|^2 + u^2 + 1), \qquad \omega = \frac{\tau}{u}\xi, \qquad x = \frac{1}{2}\frac{\tau}{u}(|\xi|^2 + u^2 - 1), \tag{2.2}$$

$(\omega := y + iz)$, cf. Infeld and Schild (1945). In the following we consider geodesic motion along the u-semiaxis of H^3. This is without loss of generality, as H^3 is homogeneous. So we put $\mathrm{d}\xi = 0$ in (2.1) and $\omega = \xi = 0$ in (2.2). Then the inverse of (2.2) reads

$$\tau = \sqrt{t^2 - x^2}, \qquad u = \sqrt{\frac{t + x}{t - x}}. \tag{2.3}$$

The orientation preserving symmetry group of the forward light cone is the proper orthochronous Lorentz group $SO^+(3,1)$, which also happens to be the orientation preserving symmetry group of the spacelike slices $\tau = \text{const}$ of the RW cosmology, by virtue of the isometry (2.2); its explicit action on H^3 can be found in Tomaschitz (1999b). The boost

$$t' = (1-\alpha^2)^{-1/2}(t-\alpha x), \qquad x' = (1-\alpha^2)^{-1/2}(x-\alpha t) \tag{2.4}$$

corresponds to the transformation

$$\tau' = \tau, \qquad u' = \eta^{-1}u, \tag{2.5}$$

with $\eta = (1+\alpha)^{1/2}(1-\alpha)^{-1/2}$ or $\alpha = (\eta^2-1)(\eta^2+1)^{-1}$. This easily follows from (2.2) and (2.3). We have $|\alpha| < 1$ and $\eta > 0$, of course.

Because of the homogeneity of H^3, it is sufficient to focus on geodesic motion along the u-semiaxis of H^3. All other geodesics are generated by applying the symmetry group $SO^+(3,1)$. We obtain from (2.1) a first integral of motion,

$$\dot{\tau}^2(s) - \tau^2 u^{-2}\dot{u}^2(s) = \varepsilon, \tag{2.6}$$

with $\varepsilon = 1$ for particles, $\varepsilon = -1$ for tachyons, and $\varepsilon = 0$ for rays. A second integral follows from the cyclicity of log u, namely

$$\tau^2 u^{-1}\dot{u} = v, \tag{2.7}$$

with a real integration constant v. Combining (2.6) and (2.7), we obtain for the velocity along the u-semiaxis

$$\mathbf{v} = \tau u^{-1}\mathrm{d}u/\mathrm{d}\tau = \mathrm{sign}(v)(1+\varepsilon v^{-2}\tau^2)^{-1/2}. \tag{2.8}$$

Integrating (2.6) and (2.7), we have for particles ($\varepsilon = 1$)

$$\tau(s) = \sqrt{s^2 - v^2}, \qquad u(s) = \kappa\sqrt{\frac{s-v}{s+v}}, \tag{2.9}$$

$$u(\tau) = \kappa\left[\frac{-v+\sqrt{\tau^2+v^2}}{v+\sqrt{\tau^2+v^2}}\right]^{1/2}. \tag{2.10}$$

Here κ is a positive integration constant, and s ranges in $|v| < s < \infty$. All galaxies and galactic observers ($v = 0$) have constant space coordinates in this (τ, u)-frame, the comoving frame discussed in the Introduction. For light rays we obtain instead of (2.10) $u = \kappa\tau$, or $u = \kappa\tau^{-1}$, depending on whether the photon moves up or down the u-semiaxis. For tachyons ($\varepsilon = -1$), we find from (2.6) and (2.7)

$$\tau(s) = \sqrt{v^2 - s^2}, \qquad u(s) = \kappa\sqrt{\frac{v+s}{v-s}}, \tag{2.11}$$

$$u(\tau) = \kappa\left(\frac{v-\sqrt{v^2-\tau^2}}{v+\sqrt{v^2-\tau^2}}\right)^{1/2}. \tag{2.12}$$

κ is again positive, v is real, and s now ranges in $-|v| < s < 0$. Also note that the lifetime of a tachyon is restricted to $\tau < |v|$ in this frame. Equations (2.10) and (2.12) comprises all particle and tachyon trajectories along the u-semiaxis. They are determined by two integration constants, $\kappa > 0$ and v.

The trajectories (2.10) and (2.12) are mapped into the forward light cone (x-axis) by (2.2). In this geodesic frame (t, x), we find for particles the world-lines

$$x = \frac{\kappa^2 - 1}{\kappa^2 + 1} t - \frac{2\kappa v}{\kappa^2 + 1}, \tag{2.13}$$

for tachyons

$$x = \frac{\kappa^2 + 1}{\kappa^2 - 1} t - \frac{2\kappa |v|}{\kappa^2 - 1}, \tag{2.14}$$

and for photons $x = t - \kappa^{-1}$ if $u = \kappa\tau$, or $x = -t + \kappa$, if $u = \kappa\tau^{-1}$.

Remark. In this paper we assume a constant tachyon mass. Conformally coupled tachyons are studied in Tomaschitz (1999b, 2000b); the tachyon mass then scales inversely proportional to the expansion factor, depending in a non-covariant way on the cosmic time parameter (Dyson, 1972; Dirac, 1973). The world-lines of conformal tachyons in the forward light cone are not any more straight-lines, and double images of tachyons can emerge in geodesic rest frames (Tomaschitz, 1998a,b).

In Section 3 we will attach globally geodesic rest frames (truncated copies of the forward light cone) to uniformly moving observers. We denote by (t', x') such a geodesic frame, connected to (t, x) by the Lorentz boost (2.4). In this frame the world-lines (2.13) and (2.14) keep their shape, but with κ replaced by κ/η, cf. (2.5). A world-line specified by (κ, v) is mapped by (2.4) into a world-line $(\kappa\eta^{-1}, v)$.

It is important to know the range of the time parameter in (2.13) and (2.14). In the case of particle trajectories, t ranges in the interval $[|v|\kappa^{-\mathrm{sign}(v)}, \infty]$, which corresponds via (2.2) and (2.3) to the τ-interval $[0, \infty]$.

As for the tachyon trajectory (2.14), t ranges in $[v\kappa^{-1}, \frac{1}{2}v\kappa^{-1}(1+\kappa^2)]$ if $v > 0$. Here, the interval boundary $t = v\kappa^{-1}$ corresponds to $\tau = 0$, and $\frac{1}{2}v\kappa^{-1}(1 + \kappa^2)$ to $\tau = v$. If $\kappa < 1$, then the time order of events labeled by the proper time t is inverted (as compared to the cosmic time order); if τ increases, then t decreases. Finally, if $v < 0$, the t-range of the tachyon trajectory is $[-v\kappa, -\frac{1}{2}v\kappa^{-1}(1 + \kappa^2)]$. In this case $\tau = 0$ corresponds to $t = -v\kappa$, and $\tau = -v$ to $t = -\frac{1}{2}v\kappa^{-1}(1 + \kappa^2)$, which means there is a time inversion if $\kappa > 1$. Note in particular that two trajectories as in (2.12) which differ only by the sign of v are mapped onto the same straight-line (2.14), but in disjoint t-ranges. The particle trajectories (2.13) extend through the whole forward light cone, from boundary to boundary. Tachyonic trajectories have one end point at the boundary and one inside the cone, which reflects their finite lifetime.

We define energy and momentum in the comoving frame by

$$E = m\dot{\tau}(s), \qquad \mathbf{p} = m\dot{u}(s), \tag{2.15}$$

for particles and tachyons alike, with $m > 0$, cf. (2.9) and (2.11). Introducing cosmic time as curve parameter, we obtain

$$E = m\tau^{-1}\sqrt{v^2 + \varepsilon\tau^2}, \qquad \mathbf{p} = mvu\tau^{-2}, \tag{2.16}$$

and $E^2 - |\mathbf{p}|^2 = m^2\varepsilon$ (with $|\mathbf{p}|^2 := \mathbf{p}^2 u^{-2}\tau^2$ of course). In the geodesic (t, x)-frame, we define energy by transforming $(E, \mathbf{p})$ in (2.15) like a contravariant 2-vector via (2.2). We obtain so for particles

$$E^{\mathrm{P}} = m\frac{\kappa^2 + 1}{2\kappa} = \frac{m}{\sqrt{1 - \mathbf{v}^2}}, \qquad \mathbf{p}^{\mathrm{P}} = m\frac{\kappa^2 - 1}{2\kappa} = \frac{m\mathbf{v}}{\sqrt{1 - \mathbf{v}^2}}. \tag{2.17}$$

The particle trajectories in the (t, x)-frame are given in (2.13), and their velocity is of course $\mathbf{v} = (\kappa^2 - 1)(\kappa^2 + 1)^{-1}$. Energy and momentum of tachyons read

$$E^{\mathrm{T}} = m\,\mathrm{sign}(v)\frac{\kappa^2 - 1}{2\kappa} = \frac{m\,\mathrm{sign}[(\kappa - 1)v]}{\sqrt{\mathbf{v}^2 - 1}},$$

$$\mathbf{p}^{\mathrm{T}} = m\,\mathrm{sign}(v)\frac{\kappa^2 + 1}{2\kappa} = \frac{m\mathbf{v}\,\mathrm{sign}[(\kappa - 1)v]}{\sqrt{\mathbf{v}^2 - 1}}. \tag{2.18}$$

Here κ and v define the trajectory (2.12), and $\mathbf{v} = (\kappa^2 + 1)(\kappa^2 - 1)^{-1}$, cf. (2.14). In the limit of infinite speed, the energy of tachyons is zero, but their momenta stay finite. Equations (2.18) are not a covariant definition of energy and momentum, as they are based on the comoving reference frame and the cosmic time parameter. (The sign of E^{T} is not preserved under Lorentz boosts, unlike the sign of E^{P}.) The time inversion as discussed after (2.14) happens only if E^{T} is negative. In any other geodesic frame (t', x') related to the (t, x)-frame via (2.4), energy and momentum are defined by transforming $(E^{\mathrm{P}}, \mathbf{p}^{\mathrm{P}})$ and $(E^{\mathrm{T}}, \mathbf{p}^{\mathrm{T}})$ like contravariant 2-vectors. As pointed out after (2.14), this just amounts to replace in (2.17) and (2.18) κ by κ/η. The comoving reference frame is necessary to unambiguously define energy and momentum of tachyons in all geodesic frames, unless one is prepared to introduce an acausal antiparticle concept for classical tachyons (Feinberg, 1967, 1970). The finite lifetime of tachyons as a result of the space expansion was interpreted by Chaliasos (1987) in terms of tachyon-antitachyon pairs, that move with opposite velocity and join at zero energy.

Finally we derive a bound for the energy transfer in a head-on collision (not necessarily elastic) of a tachyon with a particle. The collision takes place, say, at $(\tau_{\mathrm{c}}, u_{\mathrm{c}})$. We may write, by means of (2.12),

$$\kappa = u_{\mathrm{c}}\left(\frac{v + \sqrt{v^2 - \tau_{\mathrm{c}}^2}}{v - \sqrt{v^2 - \tau_{\mathrm{c}}^2}}\right)^{1/2}. \tag{2.19}$$

Inserting this into (2.18), we obtain

$$E^{\mathrm{T}} = \frac{1}{2}\frac{m}{\tau_{\mathrm{c}}}[v(u_{\mathrm{c}}^2 - 1) + \sqrt{v^2 - \tau_{\mathrm{c}}^2}(u_{\mathrm{c}}^2 + 1)]. \tag{2.20}$$

This formula holds both for incoming and outgoing tachyon (with different v-values, of course). If the outgoing tachyon has zero energy in the comoving galaxy frame, this means $v_{\mathrm{out}} = \pm\tau_{\mathrm{c}}$, cf. (2.16). The minimum energy of the outgoing tachyon in the geodesic rest frame cannot be smaller than $-\frac{1}{2}m|u_{\mathrm{c}}^2 - 1|$, because v in (2.20) lies outside the range $[-\tau_{\mathrm{c}}, \tau_{\mathrm{c}}]$, and $E^{\mathrm{T}}(v)$ takes its minimum value either at $v = \tau_{\mathrm{c}}$ or $v = -\tau_{\mathrm{c}}$. Accordingly, only a finite amount of energy can be extracted from a tachyon.

3. Tachyonic Communication Between Galactic Observers

Galactic observers are characterized by constant space coordinates in the galaxy frame, as pointed out in the Introduction. In (2.14), this simply means $v = 0$ and $u = \kappa$. Their energy (2.16) is therefore constant. In globally geodesic coordinates, the world-line of a galactic observer κ reads $x = (\kappa^2 - 1)(\kappa^2 + 1)^{-1}t$, cf. (2.13); his time coordinate ranges in $[0, \infty]$, see the discussion after (2.14). As in Section 2, we focus on geodesic motion along the x-axis (u-semiaxis). Galactic observers are related by Lorentz boosts (2.4). We can introduce for every galactic observer κ the forward light cone as geodesic rest frame by applying the Lorentz boost (2.4) with $\alpha = (\kappa^2 - 1)(\kappa^2 + 1)^{-1}$, so that his world-line is just $x' = 0$ in his rest frame $t'^2 - x'^2 > 0, t' > 0$. In the comoving frame, this boost corresponds to a simple rescaling of the space coordinate, $u' = \kappa^{-1}u$, leaving cosmic time unchanged, cf. (2.5).

A non-galactic, geodesically moving observer is characterized by a world-line (2.10) with $v \neq 0$. In globally geodesic coordinates his world-line is given by (2.13), with time ranging in $[|v|\kappa^{-\mathrm{sign}(v)}, \infty]$. The rest frame of this observer (κ, v) is obtained, like for the galaxy $(\kappa, v = 0)$, by a Lorentz boost (2.4) with α as above. His world-line reads there $x' = -v$, with t' ranging in $[|v|, \infty]$. The rest frame of observer (κ, v) is therefore $t'^2 - x'^2 > 0, t' > |v|$. The galaxies radially emanate from $x' = 0$, and because he is sitting at $x' = -v$, the galactic recession appears anisotropic. From this anisotropy, he can determine his movement in the comoving galaxy frame, and relate so his proper time to cosmic time. (In practice this is done by measuring the angular anisotropy of the temperature of the microwave background.)

At first we study the superluminal communication process in the comoving galaxy frame. A tachyon T_A is emitted at $(\tau_A, u_A = 1)$ and absorbed at (τ_B, u_B), $u_B > 1, \tau_B > \tau_A$, by two galactic observers sitting at $u_A = 1$ and u_B, respectively. The trajectory of the tachyon is given in (2.12). Its initial velocity at $(\tau_A, u_A = 1)$ is determined by the integration constant v_A, cf. (2.8), which must be chosen in a way that the tachyon can reach u_B, see (3.3). The integration constant κ in (2.12) is therefore

$$\kappa_A = \left(\frac{v_A + \sqrt{v_A^2 - \tau_A^2}}{v_A - \sqrt{v_A^2 - \tau_A^2}} \right)^{1/2}, \tag{3.1}$$

and the time at which tachyon T_A reaches u_B is

$$\tau_B = v_A \frac{2u_B \kappa_A}{u_B^2 + \kappa_A^2} = \frac{2v_A u_B \tau_A}{v_A(1+u_B^2) + \sqrt{v_A^2 - \tau_A^2}(1-u_B^2)}. \tag{3.2}$$

In calculating τ_B, we assumed that $u_B < \kappa_A$ or, equivalently,

$$v_A > \tau_A \frac{1+u_B^2}{2u_B}, \tag{3.3}$$

only then T_A can reach u_B. If equality holds in (3.3), i. e., if $u_B = \kappa_A$, then the tachyon still reaches u_B at $\tau_B = v_A$, but with infinite speed and zero energy, cf. (2.16); we will not deal with such limit cases in the following, since no energy is transferred.

The geodesic rest frame (t, x) of the galactic observer $u_A = 1$, who emits T_A, is linked to the galaxy frame via (2.2). In the (t, x)-frame, the trajectory of T_A is given by (2.14) with integration parameters (κ_A, v_A). Its initial and terminal points are

$$\begin{pmatrix} t_A \\ x_A \end{pmatrix} = \begin{pmatrix} \tau_A \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} t_B \\ x_B \end{pmatrix} = \frac{1}{2} \frac{\tau_B}{u_B} (u_B^2 \pm 1). \tag{3.4}$$

Substituting (3.2) into (3.4), we easily see that $t_B > t_A$. The energy of tachyon T_A is positive in this frame, because $v_A > 0$ and $\kappa_A > 1$, cf. (2.18). The world-line of observer $u_A = 1$ is of course $x = 0$, and the world-line of observer u_B reads as $x = \tilde{\alpha} t$ with $\tilde{\alpha} = (u_B^2 - 1)(u_B^2 + 1)^{-1}$. To obtain the geodesic rest frame (t', x') of observer u_B, we apply a Lorentz boost (2.4) with this $\tilde{\alpha}$ (or, equivalently, a coordinate change $u' = u/u_B$ in the galaxy frame, followed by the transformation (2.2)). In this frame the world-line of observer u_B is $x' = 0$, and the world-line of observer $u_A = 1$ reads $x' = -\tilde{\alpha} t'$. The world-line of the tachyon is as in (2.14) with $(\kappa = \kappa_A/u_B, v_A)$, and its initial and terminal points are

$$\begin{pmatrix} t'_A \\ x'_A \end{pmatrix} = \frac{1}{2} \frac{\tau_A}{u_B} (1 \pm u_B^2), \qquad \begin{pmatrix} t'_B \\ x'_B \end{pmatrix} = \begin{pmatrix} \tau_B \\ 0 \end{pmatrix}. \tag{3.5}$$

It is easy to see that also here $t'_A < t'_B$. Using (3.2), we may write this inequality as

$$v_A(1 - u_B^2) + \sqrt{v_A^2 - \tau_A^2}(1 + u_B^2) > 0, \tag{3.6}$$

which is satisfied for υ_A in the range (3.3). (If $u_B = \kappa_A$, then $t'_A = t'_B$, and the tachyon has in this limit infinite speed and zero energy). Since $\upsilon_A > 0$, and $u_B < \kappa_A$, the energy of the tachyon is positive also in this frame, cf. (2.18). As $t_A < t_B$ as well as $t'_A < t'_B$, it follows that whenever two galactic observers are connected by a tachyonic signal, they will observe the same time order in their respective geodesic rest frames. For a third galactic observer this need not be the case, as we will demonstrate now.

Let us consider a galactic observer sitting at $u = u_C$ in the galaxy frame. In the geodesic frame (t, x) of observer $u_A = 1$, this observer has the world-line $x = \hat{\alpha}t$, $\hat{\alpha} = (u_C^2 - 1)(u_C^2 + 1)^{-1}$. By applying a Lorentz boost (2.4) with this $\hat{\alpha}$, we obtain the rest frame (t'', x'') of observer u_C. It is easy to show that $t''_A < t''_B$ only holds if

$$u_C^2 < u_B \frac{u_B\tau_B - \tau_A}{u_B\tau_A - \tau_B} = \kappa_A^2. \tag{3.7}$$

If $u_C > \kappa_A$, then the time order in the rest frame of observer u_C is inverted, $t''_B < t''_A$, he sees the signal emitted at x''_B and absorbed at x''_A. The trajectory of tachyon T_A in the rest frame of observer u_C is defined by $(\kappa = \kappa_A/u_C, \upsilon_A)$ in (2.14). The energy (2.18) of the tachyon is positive in this frame only if $u_C < \kappa_A$. Negative energy indicates to the observer that the cosmic time order is inverted in his proper time; what he perceives as emission is actually absorption in the galaxy frame, and vice versa.

4. Tachyonic Communication Between a Galactic and a Uniformly Moving Observer

As in the preceding section, we consider a galactic observer sitting at $u_A = 1$ and emitting at τ_A a tachyon T_A specified by some $\upsilon_A > 0$. This tachyon reaches at (τ_B, u_B), $u_B > 1$, a uniformly moving observer specified by some $\upsilon_B \neq 0$. (The case $\upsilon_B = 0$ was dealt with in Section 3). The trajectory of this observer is defined by (2.10) with

$$\kappa_B = u_B \left(\frac{\upsilon_B + \sqrt{\upsilon_B^2 + \tau_B^2}}{-\upsilon_B + \sqrt{\upsilon_B^2 + \tau_B^2}} \right)^{1/2}. \tag{4.1}$$

In the geodesic rest frame (t, x) of the galactic observer $u_A = 1$, this trajectory (κ_B, υ_B) reads as in (2.13). The coordinates for emission and absorption events are given in (3.4); in particular, $t_A < t_B$. The geodesic rest frame $(\hat{t}, \hat{x})$ of observer (κ_B, υ_B) is obtained by applying a Lorentz boost (2.4) with $\alpha = (\kappa_B^2 - 1)(\kappa_B^2 + 1)^{-1}$ to the (t, x)-frame, corresponding to the coordinate change $\hat{u} = u/\kappa_B$ in the comoving galaxy frame, cf. (2.5). The world-line of this observer reads $\hat{x} = -\upsilon_B$ in his rest frame. The world-line of the galactic observer $u_A = 1$ reads there $\hat{x} = -\alpha\hat{t}$, which means $(\kappa = \kappa_B^{-1}, v = 0)$ in (2.13). Finally, the trajectory of tachyon T_A reads as in (2.14), with $(\kappa = \kappa_A/\kappa_B, v_A)$. The emission of T_A at $(\tau_A, u_A = 1)$, and the absorption of T_A at (τ_B, u_B) take place in the rest frame of observer (κ_B, v_B) at

$$\begin{pmatrix} \hat{t}_A \\ \hat{x}_A \end{pmatrix} = \frac{1}{2}\frac{\tau_A}{\kappa_B}(1 \pm \kappa_B^2) = \frac{1}{2}\frac{\tau_A}{\tau_B u_B}[-v_B(1 \mp u_B^2) + \\ + \sqrt{v_B^2 + \tau_B^2}(1 \pm u_B^2)], \tag{4.2}$$

$$\begin{pmatrix} \hat{t}_B \\ \hat{x}_B \end{pmatrix} = \tau_B \frac{u_B^2 \pm \kappa_B^2}{2u_B\kappa_B} = \begin{pmatrix} \sqrt{\tau_B^2 + v_B^2} \\ -v_B \end{pmatrix}, \tag{4.3}$$

respectively. By means of (4.2) and (4.3), we write the inequality $\hat{t}_A < \hat{t}_B$ equivalently as

$$\sqrt{\tau_B^2 + v_B^2}[v_A(1 - u_B^2) + \sqrt{v_A^2 - \tau_A^2}(1 + u_B^2)] > v_B[v_A(1 + u_B^2) + \\ + \sqrt{v_A^2 - \tau_A^2}(1 - u_B^2)]. \tag{4.4}$$

From (3.6) we know that the right side of (4.4) is positive. Therefore, if $v_B < 0$, inequality (4.4) is satisfied, and so is $\hat{t}_A < \hat{t}_B$. If $v_B > 0$, we square both sides of (4.4) and obtain

$$\tau_B[v_A(1 - u_B^2) + \sqrt{v_A^2 - \tau_A^2}(1 + u_B^2)] > 2v_B\tau_A u_B. \tag{4.5}$$

Inserting here τ_B as calculated in (3.2), we arrive at

$$v_B < v_A \frac{v_A(1 - u_B^2) + \sqrt{v_A^2 - \tau_A^2}(1 + u_B^2)}{v_A(1 + u_B^2) + \sqrt{v_A^2 - \tau_A^2}(1 - u_B^2)}, \tag{4.6}$$

which is equivalent to $\hat{t}_A < \hat{t}_B$. (Note that v_A as well as nominator and denominator of the ratio in (4.6) are positive.) The integration constants v_A and v_B define the velocities of tachyon and observer, respectively, in the galaxy frame, cf. (2.8); no time inversion can occur in the observer's rest frame if the velocities point in opposite directions. For the energy of tachyon T_A to be positive in the frame $(\hat{t}, \hat{x})$, we need $\kappa_A > \kappa_B$ (because $\kappa = \kappa_A/\kappa_B$ and $v = v_A > 0$ in (2.18)), which is easily seen to be equivalent to (4.4), and thus to condition (4.6).

Let us consider the response of observer (κ_B, v_B), who emits at (τ_B, u_B) a tachyon T_R, which reaches at a later instant (cosmic time) the galactic observer at $u_A = 1$. The trajectory of the tachyon reads as in (2.10) with $v_R < 0$ (because it moves the u-semiaxis downwards) and with κ replaced by

$$\kappa_R = u_B \left(\frac{|v_R| - \sqrt{v_R^2 - \tau_B^2}}{|v_R| + \sqrt{v_R^2 - \tau_B^2}} \right)^{1/2}. \tag{4.7}$$

Tachyon $T_R(\kappa_R, v_R)$ can reach $u_A = 1$ only if $\kappa_R < 1$, which means

$$|v_R| > \tau_B \frac{1+u_B^2}{2u_B} \tag{4.8}$$

as condition on the integration constant v_R, compare (3.3). T_R reaches $u_A = 1$ at

$$\tau_R = \frac{2\kappa_R|v_R|}{1+\kappa_R^2} = \frac{2u_B\tau_B|v_R|}{|v_R|(1+u_B^2)+\sqrt{v_R^2-\tau_B^2}(1-u_B^2)}. \tag{4.9}$$

In the geodesic rest frame (t, x) of the galactic observer $u_A = 1$, we have for the event (τ_B, u_B) (emission of T_R in the galaxy frame) the coordinates (t_B, x_B) as given in (3.4), and for event $(\tau_R, u_A = 1)$ (absorption of T_R in the galaxy frame) the coordinates

$$x_R = 0, \qquad t_R = \tau_R. \tag{4.10}$$

We see by means of (4.9) and (3.4) that $t_B < t_R$ is equivalent to

$$|v_R|(1-u_B^2)+\sqrt{v_R^2-\tau_B^2}(1+u_B^2) > 0, \tag{4.11}$$

which is satisfied for $|v_R|$ in the range (4.8), also compare (3.6). Moreover, since $v_R < 0$ and $\kappa_R < 1$, the energy (2.18) of T_R is positive in this frame.

Thus we have proven that $t_A < t_B < t_R$ holds in the geodesic rest frame of observer $u_A = 1$. ($t_A < t_B$ was already pointed out after (3.4)). Emission and absorption of the tachyons appear to observer u_A as they actually happen in the galaxy frame. The cosmic time order $\tau_A < \tau_B < \tau_R$ is preserved in the proper time t of this galactic observer. To recapitulate, t_A is the time at which T_A is emitted by observer u_A; at time t_B tachyon T_A is absorbed and T_R is emitted by observer (κ_B, v_B), and at time t_R tachyon T_R is absorbed by observer u_A. In the geodesic rest frame (t, x) of observer u_A, the emission of T_A takes place prior to the absorption of the response T_R, and thus no predetermination can arise.

In the geodesic rest frame $(\hat{t}, \hat{x})$ of observer (κ_B, v_B) introduced after (4.1), we have for (τ_B, u_B) the coordinates $(\hat{t}_B, \hat{x}_B)$ as calculated in (4.3), and for $(\tau_R, u_A = 1)$

$$\begin{pmatrix} \hat{t}_R \\ \hat{x}_R \end{pmatrix} = \frac{1}{2}\frac{\tau_R}{\kappa_B}(1 \pm \kappa_B^2) = |v_R|\frac{-v_B(1 \mp u_B^2)+\sqrt{v_B^2+\tau_B^2}(1 \pm u_B^2)}{|v_R|(1+u_B^2)+\sqrt{v_R^2-\tau_B^2}(1-u_B^2)}. \tag{4.12}$$

Thus we may write $\hat{t}_B < \hat{t}_R$ equivalently as

$$|v_R| > -\text{sign}\,(v_B)\sqrt{v_B^2+\tau_B^2}. \tag{4.13}$$

This is the condition on v_R for the cosmic time order to be preserved in the geodesic rest frame of observer (κ_B, v_B), so that $(\hat{t}_B, \hat{x}_B)$ is the emission and $(\hat{t}_R, \hat{x}_R)$

the absorption event in his frame. The integration parameter v_R determines the speed of tachyon T_R, cf. (2.8), and we assumed from the outset $v_R < 0$, cf. (4.7). Inequality (4.13) is easily seen to be equivalent to $\kappa_R < \kappa_B$. In the $(\hat{t}, \hat{x})$-frame the tachyon is defined by $(\kappa = \kappa_R/\kappa_B, v_R)$, cf. (2.14). Its energy (2.18) is positive in this frame only if emission and absorption events are the same as in the galaxy frame.

Remark. If observer (κ_B, v_B) emits a tachyon $(\kappa_T, v_T), v_T > 0$, at (τ_B, u_B), then $\kappa_T > \kappa_B$ or, equivalently, the restriction

$$v_T > \operatorname{sign}(v_B)\sqrt{v_B^2 + \tau_B^2} \tag{4.14}$$

on the integration parameters is required for the energy of the tachyon to be positive in his geodesic rest frame. (If the energy of a tachyon were negative in the rest frame in which it is emitted, then it would appear there before its emission). Observer (κ_B, v_B) can only emit a tachyon at (τ_B, u_B) whose integration parameter v satisfies either condition (4.13), if $v(= v_R) < 0$, or (4.14), if $v(= v_T) > 0$. The positivity of the tachyonic energy imposes a constraint on the velocity of tachyons emitted by uniformly moving observers; only galactic observers $(v_B = 0)$ can emit tachyons of any velocity. This is further discussed in the next section, cf. (5.10).

5. Tachyonic Communication Between Two Non-Galactic, Uniformly Moving Observers

We generalize the communication process considered in Section 4 by assuming that the observer starting the signal exchange by emitting tachyon T_A is himself moving in the galaxy background. We will demonstrate that no signals can be sent into the past of this observer; the response T_R of a second observer to tachyon T_A does not appear prior to the emission of T_A in the rest frame of the first observer.

The observer emitting tachyon T_A is defined in the galaxy frame by the trajectory (2.10) and integration parameters (κ_C, v_C). He may move up or down the u-axis, i.e., we do not specify the sign of v_C. We assume, without loss of generality, that at time τ_A at which the observer emits tachyon T_A his space coordinate is $u_A = 1$, so that

$$\kappa_C = \left(\frac{v_C + \sqrt{\tau_A^2 + v_C^2}}{-v_C + \sqrt{\tau_A^2 + v_C^2}} \right)^{1/2} . \tag{5.1}$$

Tachyon T_A is defined as in Section 3, by integration parameters (κ_A, v_A) and trajectory (2.12). κ_A is given by (3.1), since T_A is emitted at $(\tau_A, u_A = 1)$. We assume again, without loss of generality, that T_A moves the u-axis upwards, i.e., $v_A > 0$. The second observer $(\kappa_B . v_B)$, who receives T_A at $(\tau_B, u_B), u_B > 1$, is defined in Section 4; in particular (4.1) holds, and there are no restrictions on the sign of v_B. Finally, tachyon T_R, which observer (κ_B, v_B) emits as his response as soon as he absorbs T_A at (τ_B, u_B), is likewise defined in Section 4, namely by integration parameters (κ_R, v_R), $v_R < 0$, and (4.7). Tachyon T_A can reach observer (κ_B, v_B) only if the condition $u_B < \kappa_A$ is satisfied, cf. (3.3), and τ_B reads as in (3.2). It follows from (5.1) that $\kappa_C > 1$ if $v_C > 0$, and $\kappa_C < 1$ if $v_C < 0$. We have $\kappa_R < u_B$, cf. (4.7), and $u_B > 1$.

The coordinates at which the collision of tachyon T_R with observer (κ_C, v_C) takes place are readily calculated in the geodesic rest frame $(\tilde{t}, \tilde{x})$ of this observer. This frame is the same as for the galactic observer κ_C defined in Section 3, and it is linked to the galaxy frame via

$$\begin{pmatrix} \tilde{t} \\ \tilde{x} \end{pmatrix} = \frac{1}{2}\frac{\tau}{u}\frac{1}{\kappa_C}(u^2 \pm \kappa_C^2), \tag{5.2}$$

obtained by replacing in (2.2) u by u/κ_C. Observer (κ_C, v_C) is located at $\tilde{x} = -v_C$, and the trajectory of T_R reads as in (2.14), with $(\kappa = \kappa_R/\kappa_C, v_R)$. The collision happens at

$$\tilde{t}_C = (\kappa_R^2 + \kappa_C^2)^{-1}[v_C(\kappa_C^2 - \kappa_R^2) + 2\kappa_R\kappa_C|v_R|], \tag{5.3}$$

$$\tilde{x}_C = -v_C. \tag{5.4}$$

In the comoving frame, we find for this event (by means of (2.3), with u/κ_C substituted for u) the coordinates

$$\tau_C = 2\sqrt{\kappa_R\kappa_C}(\kappa_R^2 + \kappa_C^2)^{-1}[(\kappa_C|v_R| - v_C\kappa_R)(\kappa_R|v_R| + v_C\kappa_C)]^{1/2}, \tag{5.5}$$

$$u_C = \sqrt{\kappa_R\kappa_C}[(\kappa_C|v_R| - v_C\kappa_R)(\kappa_R|v_R| + v_C\kappa_C)^{-1}]^{1/2}. \tag{5.6}$$

(Clearly, we recover (5.3) and (5.4), if we insert (5.5) and (5.6) into (5.2). Moreover, u_C, τ_C and κ_C are connected by (2.10), and by substituting (2.10) into (5.2), we readily find $\tilde{t}_C = \sqrt{\tau_C^2 + v_C^2}$.) As in (3.3), we have to figure out under which conditions the collision can occur at all. Firstly, $(\tilde{t}_C, \tilde{x}_C)$ lies in the forward light cone, which means that $\tilde{t}_C > |v_C|$ must hold. Secondly, as was pointed out after (2.14), the $\tilde{t}$-range of the trajectory of T_R is the interval $[|v_R|\kappa_R\kappa_C^{-1}, \frac{1}{2}|v_R|\kappa_R^{-1}\kappa_C^{-1}(\kappa_R^2 + \kappa_C^2)]$, and therefore $\tilde{t}_C$ must lie in this range. Under these two restrictions on the integration constants a collision takes place as indicated in (5.3)–(5.6). Moreover, this collision happens after the emission of T_R at (τ_B, u_B), and because T_R moves the u-axis downwards, this means a third condition on the integration constants, namely $u_C < u_B$, with u_C as in (5.6).

Observer (κ_C, v_C) can receive the response $T_R(\kappa_R, v_R)$ only if the foregoing three conditions on the integration constants are met. They can be combined and made more explicit as follows:

(i) If $\kappa_R < \kappa_C$ and $v_C < 0$, then

$$v_C > -|v_R| \frac{\kappa_R}{\kappa_C} \frac{u_B^2 - \kappa_C^2}{u_B^2 + \kappa_R^2} \tag{5.7}$$

must hold. ((5.7) is in fact equivalent to $u_C < u_B$).

(ii) If $\kappa_R < \kappa_C$ and $v_C > 0$, or if $\kappa_R > \kappa_C$ and $v_C < 0$, then

$$-|v_R| \frac{\kappa_R^2 - \kappa_C^2}{2\kappa_R \kappa_C} > v_C > -|v_R| \frac{\kappa_R}{\kappa_C} \frac{u_B^2 - \kappa_C^2}{u_B^2 + \kappa_R^2} \tag{5.8}$$

must hold. (As mentioned at the beginning of this section, we have $u_B > \kappa_R$. Therefore, the left term in (5.8) is always larger than the right).

Either criterion (i) or (ii) must apply for the signal T_R to be received by observer (κ_C, v_C). Conditions (i) or (ii) imply that $\tau_C > \tau_B$, with τ_C as in (5.5) and

$$\tau_B = 2(\kappa_R^2 + u_B^2)^{-1} |v_R| \kappa_R u_B. \tag{5.9}$$

(τ_B is here calculated from (4.7); it is of course the same τ_B as in (3.2)). Also $\tau_A < \tau_B$ is satisfied, which follows from (3.2) (second equation) and condition (3.3).

The energy of tachyon T_A is positive in the rest frame $(\tilde{t}, \tilde{x})$ of observer (κ_C, v_C), because it is emitted there. In $(\tilde{t}, \tilde{x})$-coordinates, T_A moves along trajectory (2.14) with integration parameters $(\kappa = \kappa_A/\kappa_C, v_A)$. Since $v_A > 0, \kappa_A > \kappa_C$ must hold for its energy (2.18) to be positive in this frame. We may write this condition simply as $\mathbf{v}_A \mathbf{v}_C < 1$, with $\mathbf{v}_{A,C} = v_{A,C}(v_{A,C}^2 + \varepsilon \tau_{A,C}^2)^{-1/2}$, cf. (2.8). Likewise, the energy of tachyon T_R is positive in the rest frame $(\hat{t}, \hat{x})$ of observer (κ_B, v_B). This requires $\kappa_R < \kappa_B$ (with $v_R < 0$), as pointed out after (4.13), or, equivalently, $\mathbf{v}_R \mathbf{v}_B < 1$. In fact, if an observer moves with speed $\mathbf{v}_{\text{obs}}$ in the comoving reference frame, then he can only emit tachyons whose speed satisfy

$$\mathbf{v}_{\text{tach}}(\tau_{\text{em}}) \cdot \mathbf{v}_{\text{obs}}(\tau_{\text{em}}) < 1 \tag{5.10}$$

at emission time. (The product is taken with respect to the 3-space metric, of course.) This reference to the observer's speed in the comoving galaxy frame underscores once more the non-relativistic nature of superluminal signals. They are defined with respect to the galaxy background, the universal frame of reference. Only if the observer's state relative to the galaxy background is known, is a proper evaluation of tachyonic signals in his geodesic rest frame possible, but all observers can arrive at unambiguous conclusions. Clearly, inequality (5.10) does not impose a bound on $|\mathbf{v}_{\text{tach}}(\tau_{\text{em}})|$ if tachyon and observer head in opposite directions.

As the energy of T_A is positive in the geodesic rest frame of observer (κ_C, v_C), he will perceive emission and absorption as they occur in the galaxy frame. This is easy to check. In the galaxy frame, T_A is emitted at $(\tau_A, u_A = 1)$, which corresponds in the geodesic rest frame $(\tilde{t}, \tilde{x})$ of observer (κ_C, v_C) to the time coordinate

$$\tilde{t}_A = \tfrac{1}{2}\tau_A \kappa_C^{-1}(1 + \kappa_C^2), \tag{5.11}$$

cf. (5.2). Tachyon T_A is absorbed at (τ_B, u_B), which corresponds to

$$\tilde{t}_B = \tfrac{1}{2}\tau_B u_B^{-1} \kappa_C^{-1}(u_B^2 + \kappa_C^2). \tag{5.12}$$

We obtain from (3.1)

$$\tau_A = 2v_A \kappa_A (1 + \kappa_A^2)^{-1}, \tag{5.13}$$

and we see that $\tilde{t}_B > \tilde{t}_A$ holds if we insert (5.13) and (3.2) (first equation) into (5.11) and (5.12), respectively, and make use of $v_A > 0$, $u_B > 1$, and $\kappa_A > \kappa_C$.

In the galaxy frame, the emission of tachyon T_R takes place at (τ_B, u_B). In the rest frame $(\hat{t}, \hat{x})$ of observer (κ_B, v_B), this event has the coordinates $(\hat{t}_B, \hat{x}_B)$ as calculated in (4.3). The absorption takes place at (τ_C, u_C), cf. (5.5) and (5.6), which corresponds to the time coordinate

$$\hat{t}_C = \tfrac{1}{2}\tau_C u_C^{-1} \kappa_B^{-1}(u_C^2 + \kappa_B^2) \tag{5.14}$$

in the $(\hat{t}, \hat{x})$-frame. Emission and absorption events appear as in the galaxy frame, there is no time inversion provided the energy of T_R is positive: if $\kappa_R < \kappa_B$, then $\hat{t}_B < \hat{t}_C$ follows, which is again easy to show. We obtain from (5.5) and (5.6)

$$\tau_C u_C^{-1} = 2(\kappa_R^2 + \kappa_C^2)^{-1}(\kappa_R |v_R| + v_C \kappa_C). \tag{5.15}$$

($\kappa_R |v_R| + v_C \kappa_C$ is always positive; if $v_C < 0$ then $\kappa_C < 1$, and the positivity of this factor follows from conditions (i) or (ii), cf. (5.7) and (5.8)). If we substitute (5.9) into (4.3) (first equation), and insert (5.15) and (5.6) into (5.14), we easily see that $\hat{t}_B < \hat{t}_C$ reduces to (5.7), provided $\kappa_R < \kappa_B$. Inequality (5.7) is also contained in (5.8). Thus $\hat{t}_B < \hat{t}_C$ is satisfied, because conditions (i) or (ii) are met.

In the rest frame $(\tilde{t}, \tilde{x})$ of observer (κ_C, v_C), the cosmic time order may well be inverted, $\tilde{t}_B > \tilde{t}_C$, so that emission and absorption of tachyon T_R are interchanged compared to the galaxy frame. Likewise, in the rest frame $(\hat{t}, \hat{x})$ of observer (κ_B, v_B), the cosmic time order of emission and absorption of tachyon T_A may be inverted, $\hat{t}_B < \hat{t}_A$, so that emission in the galaxy frame appears as absorption in the geodesic rest frame, and vice versa. But these time inversions cannot cause predetermination, since observer (κ_C, v_C) does not emit T_R, nor does observer (κ_B, v_B) emit tachyon T_A. However, in the rest frame $(\tilde{t}, \tilde{x})$ of observer (κ_C, v_C), the response T_R of observer (κ_B, v_B) must arrive after the emission of tachyon $T_A(\kappa_A, v_A)$, otherwise this emission would indeed be predetermined and causality violating: Tachyon T_A is emitted at time $\tilde{t}_A = \sqrt{\tau_A^2 + v_C^2}$, cf. (5.11) and (5.1), and tachyon T_R is absorbed at $\tilde{t}_C = \sqrt{\tau_C^2 + v_C^2}$, cf. (5.3) and (5.5). Accordingly, $\tilde{t}_C > \tilde{t}_A$ holds; the cosmic time order $\tau_C > \tau_A$ is preserved in the proper time of observer (κ_C, v_C), and thus predetermination is excluded.

6. Conclusion

The cosmic time of the comoving galaxy frame determines a distinguished time order to which every observer can relate; every observer can connect his rest frame to the galaxy frame, and compare so the time order of events in his proper time to the cosmic time order. The time order in the proper time of galactic or uniformly moving observers may be inverted as compared to the cosmic time order, but all observers arrive at the same conclusion on the causality of the observed process as defined by the cosmic time order. In this paper it was demonstrated that the causality of superluminal signal transfer can be unambiguously defined by virtue of the cosmic time of the comoving reference frame, so that the causality principle as stated in the Introduction is adhered to. The high isotropy of the microwave background makes it in practice possible for every observer to determine his movement relative to the galaxy background, and in this way to infer the cosmic time order of events connected by tachyons.

We defined the kinematics of superluminal signal transfer in the comoving galaxy frame, discussed how tachyons appear in the geodesic rest frames of uniformly moving observers, and demonstrated that no signals can be sent into the past of an observer. The impossibility of predetermination was pointed out in Sections 3 and 4 at hand of two special cases, and proven in Section 5 in complete generality. If a geodesically moving observer emits a superluminal signal, then the response of a second observer cannot reach him before the emission of this signal. The proof makes use of the energy concept for tachyons developed in Section 2, which is based on the comoving galaxy frame as the universal reference frame. Tachyonic energy is defined in the galaxy frame analogous to the energy of subluminal particles, it is positive and varying in cosmic time. In the geodesic rest frames it is bounded from below, and thus particle–tachyon interactions can be based on energy–momentum conservation. Whenever the energy of a tachyon is negative in a geodesic rest frame, this indicates to the observer time inversion; the cosmic time order of events connected by the tachyon is interchanged in his proper time. Hence, an observer can infer the cosmic time order either from the energy of the tachyon relating the respective events, or from his own movement relative to the background radiation, as pointed out above.

In the Milne universe, one can attach to every uniformly moving observer a globally geodesic rest frame, a copy of the forward light cone. These rest frames are related by Lorentz transformations, and there are global isometries which synchronize the proper time of the geodesic rest frames with cosmic time. The distinctive feature of the Milne universe is, that the synchronization of clocks can be carried out in a straightforward way, like in Minkowski space, without resorting to chains of infinitesimal, locally geodesic neighborhoods, but otherwise the overall reasoning in this paper is not bound to a specific expansion factor. In such infinitesimal neighborhoods, causality can be proven along the same lines, with linearized equations of motion for observers and tachyons. The causality proof given here holds globally, for events separated by finite space and time intervals.

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References

Aharanov, Y., Reznik, B. and Stern, A.: 1998, *Phys. Rev. Lett.* **81**, 2190.
Anderson, R., Vetharaniam, I. and Stedman, G. E.: 1998, *Phys. Rep.* **295**, 93.
Bahcall, N. A. et al.: 1999, *Science* **284**, 1481.
Balazs, N. L. and Voros, A.: 1986, *Phys. Rep.* **143**, 109.
Bilaniuk, O. M. P., Deshpande, V. P. and Sudarshan, E. C. G.: 1962, *Am. J. Phys.* **30**, 718.
Chaliasos, E.: 1987, *Physica A* **144**, 390.
Davies, P. C. W.: 1975, *Nuovo Cimento B* **25**, 571.
Dirac, P. A. M.: 1973, in: J. Mehra (ed.), *The Physicist's Conception of Nature*, Reidel, Dordrecht, p. 45.
Dyson, F. J.: 1972, in: A. Salam and E. P. Wigner (eds), *Aspects of Quantum Theory*, Cambridge Univ. Press, p. 213.
Dyson, F. J.: 1979, *Rev. Mod. Phys.* **51**, 447.
Feinberg, G.: 1967, *Phys. Rev.* **159**, 1089.
Feinberg, G.: 1970, *Sci. Am.* **222**(2), 69.
Feinberg, G.: 1978, *Phys. Rev. D* **17**, 1651.
Fenchel, W.: 1989, *Elementary Geometry in Hyperbolic Space*, de Gruyter, Berlin.
Ford, L. H.: 1995, *Phys. Rev. D* **51**, 1692.
Fuller, R. W. and Wheeler, J. A.: 1962, *Phys. Rev.* **128**, 919.
Hoyle, F. and Narlikar, J. V.: 1995, *Rev. Mod. Phys.* **67**, 113.
Infeld, L. and Schild, A.: 1945, *Phys. Rev.* **68**, 250.
Milne, E. A.: 1932, *Nature* **130**, 9.
Milne, E. A.: 1948, *Kinematic Relativity*, Clarendon, Oxford.
Newton, R.: 1970, *Science* **167**, 1569.
Parker, L.: 1969, *Phys. Rev.* **188**, 2287.
Pirani, F. A. E.: 1970, *Phys. Rev. D* **1**, 3224.
Sandage, A.: 1988, *Annu. Rev. Astron. Astrophys.* **26**, 561.
Smoot, G. F. and Scott, D.: 1998, in C. Caso *et al.*, *Eur. Phys. J. C* **3**, 1.
Tanaka, S.: 1961, *Progr. Theoret. Phys.* **24**, 171.
Terletsky, Ya. P.: 1961, *Sov. Phys. Dokl.* **5**, 782.

Tomaschitz, R.: 1997a, *Chaos, Solitons & Fractals* **8**, 761.
Tomaschitz, R.: 1997b, *Int. J. Bifurcation & Chaos* **7**, 1847.
Tomaschitz, R.: 1998a, *Int. J. Theoret. Phys.* **37**, 1121.
Tomaschitz, R.: 1998b, *Int. J. Mod. Phys. D* **7**, 279.
Tomaschitz, R.: 1998c, *Astrophys. & Space Sci.* **259**, 255.
Tomaschitz, R.: 1998d, *Chaos, Solitons & Fractals* **9**, 1199.
Tomaschitz, R.: 1999a, *Int. J. Mod. Phys. A* **14**, 4275, *ibid. A* **14**, 5137.
Tomaschitz, R.: 1999b, *Class. Quant. Grav.* **16**, 3349.
Tomaschitz, R.: 2000a, *Astrophys. & Space Sci.* **271**, 181.
Tomaschitz, R.: 2000b, *Int. J. Mod. Phys. A* **15**, 3019.
Tomaschitz, R.: 2000c, *Eur. Phys. J. B* **17**, 523.
Wheeler, J. A. and Feynman, R. P.: 1945, *Rev. Mod. Phys.* **17**, 157.
Whittaker, E.: 1951, *A History of the Theories of Aether and Electricity*, Thomas Nelson & Sons, London.