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Neutrino currents in the absolute spacetime: Relating the refractive index of the aether to the OPERA excess velocity

R. Tomaschitz[(a)]

Department of Physics, Hiroshima University - 1-3-1 Kagami-yama, Higashi-Hiroshima 739-8526, Japan

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Abstract – The superluminal neutrino velocity measured by the OPERA experiment is explained in a non-relativistic spacetime conception. Spacetime is viewed as a permeable medium of wave propagation. The neutrino wave equation is coupled to a permeability tensor, like electromagnetic fields in dielectric media. The inertial frame in which this tensor is isotropic defines a distinguished frame of reference, the rest frame of the aether. The dispersion relation of the spinorial wave modes gives rise to a superluminal group velocity of the energy flux. The Gordon decomposition of spinor currents in a refractive and dispersive spacetime is performed with finite as well as zero rest mass. The convective and spin components of the superluminal neutrino current are related to the permeability tensor. The refractive index of the aether depends on the neutrino energy, and is inferred in the 10 to 50 GeV range from the measured excess velocity. Implications of the superluminal speed of signal transfer regarding relativity principles and causality are discussed.

Introduction. – The OPERA experiment [1] measured a neutrino excess speed of $v/c-1\approx 2.48\pm 0.6\times 10^{-5}$, at an average neutrino energy of 17 GeV. This was inferred from a time-of-flight measurement, with a muon neutrino beam over a baseline of 730 km between the source at CERN and the OPERA detector at the underground Gran Sasso Laboratory. We explain this excess speed in an absolute spacetime conception, by coupling the neutrino wave equation to the permeability tensor of the aether [2,3].

We discuss the coupling of Dirac spinors to a frequency-dependent permeability tensor, the relation to the Klein-Gordon equation in a dispersive spacetime, as well as the eikonal limit. The permeable spacetime gives rise to a superluminal group velocity. We analyze spinor currents in the aether, and perform the Gordon decomposition into a convection and spin component. The spin current is described in terms of a magnetization and polarization field. We relate the energy flux to the superluminal convection current and show that it propagates with group velocity in the aether.

Finally we identify the superluminal group velocity of a neutrino current with the OPERA excess velocities at three different frequencies, and find, by interpolation, the frequency variation of the refractive index of the aether for neutrinos in the 10–50 GeV interval. In the conclusion, we further elaborate on the absolute spacetime, in particular on causality, inertial frames, and the rest frame of the aether as manifested by the isotropic cosmic microwave background radiation.

Dirac equation coupled to the permeability tensor of the aether. – We start with the Dirac Lagrangian

$$L=\frac{1}{\mathrm{i}}\left(\frac{1}{2}\bar{\psi}\gamma_\mu\eta^{\mu\nu}\psi_{,\nu}-\frac{1}{2}\bar{\psi}_{,\mu}\eta^{\mu\nu}\gamma_\nu\psi+m\bar{\psi}\psi\right),\qquad(1)$$

where we substitute a real symmetric permeability tensor for the Minkowski metric, $\eta^{\mu\nu}\to g^{\mu\nu}$. First, we assume that $g^{\mu\nu}$ is constant. Standard Euler variation gives the Dirac equation and its adjoint,

$$\gamma_\mu g^{\mu\nu}\psi_{,\nu}+m\psi=0,\quad \bar{\psi}_{,\mu}g^{\mu\nu}\gamma_\nu-m\bar{\psi}=0.\qquad(2)$$

$\psi^\dagger$ indicates transposition and complex conjugation, and $\bar{\psi}=\psi^\dagger\gamma^0$ is the adjoint 4-spinor. A subscript comma followed by an index denotes a partial derivative, $\bar{\psi}_{,\mu}=\partial_\mu\bar{\psi}$. Indices are raised and lowered with the Minkowski metric, $\eta_{\mu\nu}=\mathrm{diag}(-1,1,1,1)$, and we have put $\hbar=c=1$.

(a) E-mail: tom@geminga.org

Greek indices run from 0 to 3, Latin ones from 1 to 3. The inverse of the permeability tensor is denoted by $g^{-1}_{\mu\nu}$, so that $g^{-1}_{\mu\alpha}g^{\alpha\nu}=\delta^{\nu}_{\mu}$, whereas $g_{\mu\nu}=\eta_{\mu\alpha}g^{\alpha\beta}\eta_{\beta\nu}$. The Dirac matrices satisfy $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\eta_{\mu\nu}$, where γ_0 is antihermitian and the γ_i are hermitian [4].

We will consider a homogeneous and isotropic permeability tensor, so that $g^{\mu\nu}$ is diagonal,

$$g^{00}=-\varepsilon, \quad g^{ik}=\delta^{ik}/\mu, \quad g^{0k}=0, \tag{3}$$

where ε and μ are real positive permeabilities, assumed to be constant at this point. The rest frame of the aether defines a distinguished frame of reference, a frame of absolute rest. This frame is physically manifested as the rest frame of the cosmic microwave background radiation, which is isotropic in this frame, with vanishing dipole anisotropy [5,6]. The Solar system barycenter is moving in this frame with a speed of 369 km/s toward $(l,b)\approx(264.0°, 48.26°)$ in Galactic coordinates [7–10]. The angle between the baseline of the neutrino beam and the velocity vector of the barycenter is periodically changing. This affects the superluminal neutrino speed measured in the rest frame of the barycenter, which can substantially differ from the neutrino speed in the aether frame. At superluminal velocities much higher than the speed of light, this angular dependence can even affect the causality interpretation. However, since the neutrino excess speed is very small and the velocity of the barycenter much smaller than the speed of light, we can assume that the measurement of the neutrino velocity is carried out in the rest frame of the microwave background, cf. the conclusion.

We perform a time separation of the wave field, using the factorization $\psi=\hat\psi(\omega)e^{-i\omega t}$, $\bar\psi=\bar{\hat\psi}(\omega)e^{i\omega t}$, where $\hat\psi$ is time independent, and use the permeability tensor (3) to write the Lagrangian (1) as

$$\hat L=\omega\varepsilon\bar{\hat\psi}\gamma_0\hat\psi+\frac{1}{2i\mu}(\bar{\hat\psi}\gamma_i\hat\psi_{,i}-\bar{\hat\psi}_{,i}\gamma_i\hat\psi)+\frac{m}{i}\bar{\hat\psi}\hat\psi. \tag{4}$$

Time-separated quantities will be denoted by a hat.

Applying the Dirac operator $\gamma_\mu g^{\mu\nu}\partial_\nu+m$ onto the first equation in (2), we recover the Klein-Gordon equation coupled to the permeability tensor,

$$(h^{\mu\nu}\partial_\mu\partial_\nu-m^2)\psi=0, \tag{5}$$

where $h^{\mu\nu}=g^{\mu\alpha}\eta_{\alpha\beta}g^{\beta\nu}$. The diagonal $g^{\mu\nu}$ in (3) gives $h^{00}=-\varepsilon^2$, $h^{ik}=\delta^{ik}/\mu^2$, $h^{0k}=0$. When time-separated, this reads

$$(\Delta+k^2)\hat\psi=0, \quad k^2=\mu^2(\varepsilon^2\omega^2-m^2), \tag{6}$$

where Δ is the Euclidean Laplace operator, and the time-separated Dirac equations (2) are

$$\gamma_i\hat\psi_{,i}+(i\gamma_0\mu\varepsilon\omega+\mu m)\hat\psi=0, \quad \bar{\hat\psi}_{,i}\gamma_i-\bar{\hat\psi}(i\gamma_0\mu\varepsilon\omega+\mu m)=0. \tag{7}$$

These equations can directly be derived by Euler variation of the Lagrangian (4). From now on, we take $\hat L$ in (4) and the Dirac equations (7) as starting point (instead of (1) and (2)), and substitute frequency-dependent permeabilities $\varepsilon(\omega)$ and $\mu(\omega)$. Moreover, applying the operator $\gamma_i\partial_i+(i\gamma_0\omega\mu\varepsilon+\mu m)$ onto the first equation in (7), one can derive the time-separated Klein-Gordon equation (6) with frequency-dependent permeabilities. If the spinor is a plane-wave solution $\hat\psi=\tilde\psi e^{i\mathbf{kx}}$ of (7), where $\tilde\psi$ is a constant 4-spinor, it also satisfies (6), which requires the dispersion relation

$$k^2=\mu^2(\omega)(\varepsilon^2(\omega)\omega^2-m^2), \tag{8}$$

connecting the wave number $k(\omega)=|\mathbf{k}|$ with the permeabilities. A positive squared wave number implies the frequency condition $\varepsilon(\omega)>m/|\omega|$. Dispersion is modeled by frequency-dependent permeabilities in time-separated quantities [11,12]. The dispersion relation (8) also follows from the Hamilton-Jacobi equation, $h^{\mu\nu}S_{,\mu}S_{,\nu}+m^2=0$, since $\psi\propto e^{iS}$, with the action $S=\mathbf{k}(\omega)\mathbf{x}-\omega t$, so that the semiclassical approximation of (5) is exact: the phase of the quantum-mechanical elementary waves is the classical action or eikonal.

The permeability tensor $g^{\mu\nu}$ in the Dirac equation (2) is unrelated to electromagnetic permeabilities, even though the formalism resembles that of electromagnetic fields in dielectrics. Electromagnetic radiation is not affected by the permeability tensor of neutrinos. In fact, this would even contradict the Michelson-Morley experiment. Different particles couple with different permeability tensors to the aether.

Convection and spin currents in the dispersive spacetime. – We start with a constant permeability tensor $g^{\mu\nu}$, and consider two solutions ψ and φ of the Dirac equation (2), so that the current $j^\mu=-q\bar\varphi\gamma_\nu g^{\nu\mu}\psi$ is conserved, $\partial_\mu j^\mu=0$. q is some charge carried by the current. The charge density is defined by $\rho=j^0$. Using the diagonal permeability tensor (3) (frequency independent), we find $\rho=q\varepsilon\bar\varphi\gamma_0\psi$, $\mathbf{j}=-(q/\mu)\bar\varphi\gamma_n\psi$, and the continuity equation $\partial\rho/\partial t+\operatorname{div}\mathbf{j}=0$.

To derive the continuity equation in the case of frequency-dependent permeabilities, we employ the time-separated Dirac equations,

$$\gamma_i\hat\psi_{,i}(\omega)+[i\gamma_0\mu(\omega)\varepsilon(\omega)\omega+\mu(\omega)m]\hat\psi(\omega)=0, \tag{9}$$

$$\bar{\hat\varphi}_{,i}(\omega')\gamma_i-\bar{\hat\varphi}(\omega')[i\gamma_0\mu(\omega')\varepsilon(\omega')\omega'+\mu(\omega')m]=0, \tag{10}$$

so that the divergence $(\bar{\hat\varphi}\gamma_i\hat\psi)_{,i}$ vanishes at $\omega'=\omega$. The spinors $\hat\psi(\omega)$ and $\bar{\hat\varphi}(\omega')$ also satisfy the Klein-Gordon equation (6) with dispersion relation (8). The time-dependent Fourier modes are $\psi=\hat\psi(\omega)e^{-i\omega t}$ and $\bar\varphi=\bar{\hat\varphi}(\omega')e^{i\omega' t}$. If not indicated otherwise, $\hat\psi$ means $\hat\psi(\omega)$ and $\bar{\hat\varphi}$ stands for $\bar{\hat\varphi}(\omega')$.

We use the time separation $\rho=\hat\rho e^{i(\omega'-\omega)t}$ and $\mathbf{j}=\hat{\mathbf{j}}e^{i(\omega'-\omega)t}$, and define the current as

$$\hat{\mathbf{j}}=-\frac{q}{2}\left(\frac{\bar{\hat\varphi}(\omega')\gamma_k\hat\psi(\omega)}{\mu(\omega')}+\frac{\bar{\hat\varphi}(\omega')\gamma_k\hat\psi(\omega)}{\mu(\omega)}\right), \tag{11}$$

which coincides with $\hat{\mathbf{j}} = -(q/\mu)\hat{\bar{\varphi}}\gamma_n\hat{\psi}$ stated above for constant permeabilities. In the first term, we substitute the adjoint Dirac equation (10), and use (9) in the second term, to decompose the current into a convection and a spin current, $\hat{\mathbf{j}} = \hat{\mathbf{j}}_{\rm C} + \hat{\mathbf{j}}_{\rm S}$, cf. [13,14],

$$\hat{\mathbf{j}}_{\rm C} = -\frac{q}{2m}\left(\frac{\hat{\bar{\varphi}}_{,k}\hat{\psi}}{\mu^2(\omega')} - \frac{\hat{\bar{\varphi}}\hat{\psi}_{,k}}{\mu^2(\omega)}\right), \qquad (12)$$

$$\hat{\mathbf{j}}_{\rm S} = -\frac{q}{2m}\left[\frac{\hat{\bar{\varphi}}_{,i}\sigma_{ik}\hat{\psi}}{\mu^2(\omega')} - \frac{\hat{\bar{\varphi}}\sigma_{ki}\hat{\psi}_{,i}}{\mu^2(\omega)} - {\rm i}\left(\frac{\varepsilon(\omega')\omega'}{\mu(\omega')} - \frac{\varepsilon(\omega)\omega}{\mu(\omega)}\right)\hat{\bar{\varphi}}\sigma_{0k}\hat{\psi}\right], \qquad (13)$$

where $\sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$, so that $\gamma_\mu\gamma_\nu = \eta_{\mu\nu} + \sigma_{\mu\nu}$. We also split the charge density into a convection and a spin density, $\hat{\rho} = \hat{\rho}_{\rm C} + \hat{\rho}_{\rm S}$, and use the time separation $\rho_{\rm C,S} = \hat{\rho}_{\rm C,S}{\rm e}^{{\rm i}(\omega'-\omega)t}$, $\mathbf{j}_{\rm C,S} = \hat{\mathbf{j}}_{\rm C,S}{\rm e}^{{\rm i}(\omega'-\omega)t}$. The time derivatives of $\hat{\rho}_{\rm C}$ and $\hat{\rho}_{\rm S}$ are defined by separate continuity equations, $\partial\rho_{\rm C}/\partial t = -{\rm div}\,\mathbf{j}_{\rm C}$, $\partial\rho_{\rm S}/\partial t = -{\rm div}\,\mathbf{j}_{\rm S}$, so that a time integration gives

$$\hat{\rho}_{\rm C,S} = {\rm i}\frac{{\rm div}\,\hat{\mathbf{j}}_{\rm C,S}}{\omega' - \omega}. \qquad (14)$$

Here, we substitute the current component (12), and perform the limit $\omega' \to \omega$ by expanding in powers of $\omega' - \omega$, to find the convection density

$$\hat{\rho}_{\rm C} = \frac{{\rm i}q}{2m}\left(\hat{\bar{\varphi}}\hat{\psi}\frac{{\rm d}}{{\rm d}\omega}\frac{k^2}{\mu^2} - \hat{\bar{\varphi}}_{,k}\hat{\psi}_{,k}\frac{{\rm d}}{{\rm d}\omega}\frac{1}{\mu^2}\right), \qquad (15)$$

where k^2 is defined in (8), and both fields $\hat{\psi}$ and $\hat{\bar{\varphi}}$ are taken at ω. The second term in (15) can be rewritten via the Klein-Gordon equation (6),

$$\hat{\rho}_{\rm C} = \frac{{\rm i}q}{m}\frac{k}{\mu^2 \upsilon_{\rm gr}}\hat{\bar{\varphi}}\hat{\psi} - \frac{{\rm i}q}{2m}(\hat{\bar{\varphi}}_{,k}\hat{\psi})_{,k}\frac{{\rm d}}{{\rm d}\omega}\frac{1}{\mu^2}, \qquad (16)$$

$$\frac{1}{\upsilon_{\rm gr}} = \frac{\mu^2}{k}\left[\omega\varepsilon^2\left(1 + \frac{\varepsilon'}{\varepsilon}\omega + \frac{\mu'}{\mu}\omega\right) - \frac{\mu'}{\mu}m^2\right], \qquad (17)$$

where $\upsilon_{\rm gr}$ is the group velocity

$$\upsilon_{\rm gr} = \frac{{\rm d}\omega}{{\rm d}k} = \frac{2k}{{\rm d}k^2/{\rm d}\omega}, \qquad (18)$$

with $k(\omega)$ in (8). The second term in (16) is a divergence, which drops out in a spatial integration or for plane waves. The phase velocity is $\upsilon_{\rm ph} = \omega/k$, and the refractive index is defined as the inverse phase velocity, $n_{\rm r} = k/\omega$.

We consider plane-wave solutions of the Dirac equations (9) and (10) at $\omega' = \omega$: $\hat{\psi}(\omega) = \tilde{\psi}{\rm e}^{{\rm i}\mathbf{kx}}$ and $\hat{\bar{\varphi}}(\omega) = \tilde{\bar{\varphi}}{\rm e}^{-{\rm i}\mathbf{kx}}$, where $\mathbf{k} = k(\omega)\mathbf{k}_0$, cf. (8). We find, cf. (12) and (16),

$$\hat{\mathbf{j}}_{\rm C} = \frac{{\rm i}qk}{m\mu^2}\tilde{\bar{\varphi}}\tilde{\psi}\mathbf{k}_0, \quad \hat{\rho}_{\rm C} = \frac{{\rm i}q}{m\mu^2}\frac{k}{\upsilon_{\rm gr}}\tilde{\bar{\varphi}}\tilde{\psi}, \qquad (19)$$

and $\hat{\mathbf{j}}_{\rm C} = \upsilon_{\rm gr}\hat{\rho}_{\rm C}\mathbf{k}_0$, so that the charge transfer happens at group velocity.

The charge density of the spin current $\hat{\mathbf{j}}_{\rm S}$ in (13) is obtained in like manner, via the time integration (14),

$$\hat{\rho}_{\rm S} = -\frac{{\rm i}q}{2m}\left[\hat{\bar{\varphi}}_{,k}\sigma_{ki}\hat{\psi}_{,i}\frac{{\rm d}}{{\rm d}\omega}\frac{1}{\mu^2} - {\rm i}(\hat{\bar{\varphi}}\sigma_{0k}\hat{\psi})_{,k}\frac{{\rm d}}{{\rm d}\omega}\frac{\omega\varepsilon}{\mu}\right], \qquad (20)$$

where the adjoint field $\hat{\bar{\varphi}}(\omega)$ is taken at $\omega' = \omega$. We write $\hat{\bar{\varphi}}_{,k}\sigma_{ki}\hat{\psi}_{,i}$ as a divergence, symmetrize, and use the Dirac equations (9) and (10) to obtain

$$\hat{\rho}_{\rm S} = \frac{{\rm i}q}{2m}\left[\frac{1}{2}(\hat{\bar{\varphi}}\hat{\psi}_{,k} + \hat{\bar{\varphi}}_{,k}\hat{\psi})_{,k}\frac{{\rm d}}{{\rm d}\omega}\frac{1}{\mu^2} - {\rm i}(\hat{\bar{\varphi}}\sigma_{0k}\hat{\psi})_{,k}\left(\omega\mu\varepsilon\frac{{\rm d}}{{\rm d}\omega}\frac{1}{\mu^2} - \frac{{\rm d}}{{\rm d}\omega}\frac{\omega\varepsilon}{\mu}\right)\right]. \qquad (21)$$

The spin density $\hat{\rho}_{\rm S}$ is the divergence of a polarization field, $\hat{\rho}_{\rm S} = -{\rm div}\,\mathbf{P}$,

$$\mathbf{P} = \frac{q}{2m}\frac{\varepsilon}{\mu}\left(1 + \omega\frac{\varepsilon'}{\varepsilon} + \omega\frac{\mu'}{\mu}\right)\hat{\bar{\varphi}}\sigma_{0k}\hat{\psi} + \frac{{\rm i}q}{2m}\frac{\mu'}{\mu^3}(\hat{\bar{\varphi}}\hat{\psi}_{,k} + \hat{\bar{\varphi}}_{,k}\hat{\psi}). \qquad (22)$$

The spin current $\hat{\mathbf{j}}_{\rm S}$ in (13) at $\omega' = \omega$ is a magnetization current,

$$\hat{\mathbf{j}}_{\rm S} = \frac{q}{2m\mu^2}(\hat{\bar{\varphi}}\sigma_{ki}\hat{\psi})_{,i} = {\rm rot}\,\mathbf{M}, \qquad (23)$$

where the magnetization field reads

$$\mathbf{M} = \frac{q}{2m\mu^2}\hat{\bar{\varphi}}\boldsymbol{\sigma}\hat{\psi}, \quad \sigma_k = \frac{1}{2}\varepsilon_{kij}\sigma_{ij}. \qquad (24)$$

Here, we defined a spin vector $\boldsymbol{\sigma} = \sigma_k = \varepsilon_{kij}\gamma_i\gamma_j/2$, where ε_{kij} is the Levi-Civita symbol. $\hat{\mathbf{j}}_{\rm S}$ vanishes for plane waves, and so does $\hat{\rho}_{\rm S}$. If we refrain from performing the limit $\omega' \to \omega$ in (14), a polarization current $\partial\mathbf{P}/\partial t$ has to be added to rot$\mathbf{M}$ in (23); at $\omega' = \omega$ the polarization field (22) is time independent.

Conservation of energy in the aether. – In the case of a constant permeability tensor $g^{\mu\nu}$, the energy-momentum tensor of the Dirac field is obtained by standard Lagrange formalism [15,16],

$$T_\mu^\nu = \frac{1}{2{\rm i}}(\bar{\varphi}\gamma_\alpha g^{\alpha\nu}\psi_{,\mu} - \bar{\varphi}_{,\mu}\gamma_\alpha g^{\alpha\nu}\psi). \qquad (25)$$

This tensor satisfies the conservation law $T^\nu_{\mu,\nu} = 0$, provided that the fields ψ and $\bar{\varphi}$ solve the Dirac equation and the permeability tensor $g^{\alpha\nu}$ is constant. The components T_0^ν define the energy density T_0^0 and energy flux T_0^i. We specify the permeability tensor as in (3) (frequency independent) and split off time, writing $\psi = \hat{\psi}(\omega){\rm e}^{-{\rm i}\omega t}$, $\bar{\varphi} = \hat{\bar{\varphi}}(\omega'){\rm e}^{{\rm i}\omega' t}$ and $T_\mu^\nu = \hat{T}_\mu^\nu{\rm e}^{{\rm i}(\omega'-\omega)t}$. The fields $\hat{\psi}(\omega)$ and

$\bar{\hat{\varphi}}(\omega')$ satisfy the time-separated Dirac equations (9) and (10). We thus find

$$\hat{T}_0^0 = \varepsilon \frac{\omega+\omega'}{2} \bar{\hat{\varphi}}(\omega')\gamma_0 \hat{\psi}(\omega), \qquad \hat{T}_0^n = -\frac{\omega+\omega'}{2\mu} \bar{\hat{\varphi}}(\omega')\gamma_n \hat{\psi}(\omega). \tag{26}$$

Next, we consider a dispersive spacetime with frequency-dependent permeabilities $\varepsilon(\omega)$ and $\mu(\omega)$, and define the flux vector as

$$\hat{T}_0^n = -\frac{1}{2}\left(\frac{\omega'}{\mu(\omega')} + \frac{\omega}{\mu(\omega)}\right) \bar{\hat{\varphi}}(\omega')\gamma_n \hat{\psi}(\omega). \tag{27}$$

At $\omega' = \omega$, we may write $\hat{T}_0^n = (\omega/q)\hat{\mathbf{j}}$, with current $\hat{\mathbf{j}}$ in (11). Here, we substitute the Gordon decomposition $\hat{\mathbf{j}} = \hat{\mathbf{j}}_\mathrm{C} + \hat{\mathbf{j}}_\mathrm{S}$, cf. (12) and (13). We obtain the energy density $\hat{T}_0^0$ by applying the time integration (14) and performing the limit $\omega' \to \omega$,

$$\hat{T}_0^0 = \frac{\omega}{q}(\hat{\rho}_\mathrm{C} + \hat{\rho}_\mathrm{S}), \quad \hat{T}_0^n = \frac{\omega}{q}(\hat{\mathbf{j}}_\mathrm{C} + \hat{\mathbf{j}}_\mathrm{S}), \tag{28}$$

with the convection and spin densities $\hat{\rho}_\mathrm{C,S}$ in (16) and (21). On substituting plane-wave solutions as in (19), we find $\hat{\rho}_\mathrm{S} = 0$, $\hat{\mathbf{j}}_\mathrm{S} = 0$, as well as $\hat{T}_0^n = \upsilon_\mathrm{gr}\hat{T}_0^0 \mathbf{k}_0$, where $\mathbf{k}_0$ is the unit wave vector. The energy flux propagates in the dispersive aether with group velocity υ_gr, cf. (17).

Spinorial wave propagation in the aether with zero rest mass. – In the massless limit, the convection and spin densities in (16) and (21) become singular, and we have to proceed differently. We put $m = 0$ in the Dirac equations (9) and (10), and multiply by γ_0. The spinors $\hat{\psi}(\omega)$ and $\bar{\hat{\varphi}}(\omega')$ satisfy the Klein-Gordon equation (6) with dispersion relation $k^2 = \varepsilon^2(\omega)\mu^2(\omega)\omega^2$, cf. (8).

We start with current $\hat{\mathbf{j}}$ in (11). In the first term, we replace $\bar{\hat{\varphi}}(\omega')$ by way of the adjoint Dirac equation (10), and in the second term we substitute (9) for $\hat{\psi}(\omega)$, to split the current into a convection and spin component, $\hat{\mathbf{j}} = \hat{\mathbf{j}}_\mathrm{C} + \hat{\mathbf{j}}_\mathrm{S}$, $\hat{\rho} = \hat{\rho}_\mathrm{C} + \hat{\rho}_\mathrm{S}$, where, cf. (12) and (13),

$$\hat{\mathbf{j}}_\mathrm{C} = \mathrm{i}\frac{q}{2}\left(\frac{\bar{\hat{\varphi}}_{,k}\gamma_0\hat{\psi}}{\varepsilon(\omega')\mu^2(\omega')\omega'} - \frac{\bar{\hat{\varphi}}\gamma_0\hat{\psi}_{,k}}{\varepsilon(\omega)\mu^2(\omega)\omega}\right), \tag{29}$$

$$\hat{\mathbf{j}}_\mathrm{S} = \mathrm{i}\frac{q}{2}\left(\frac{\bar{\hat{\varphi}}_{,i}\gamma_0\sigma_{ik}\hat{\psi}}{\varepsilon(\omega')\mu^2(\omega')\omega'} - \frac{\bar{\hat{\varphi}}\gamma_0\sigma_{ki}\hat{\psi}_{,i}}{\varepsilon(\omega)\mu^2(\omega)\omega}\right). \tag{30}$$

The charge densities $\hat{\rho}_\mathrm{C,S}$ are calculated via the time integration (14) and by performing the limit $\omega' \to \omega$ and making use of the Klein-Gordon equation (6). The convection density reads

$$\hat{\rho}_\mathrm{C} = \frac{q}{\mu}\frac{1}{\upsilon_\mathrm{gr}}\bar{\hat{\varphi}}\gamma_0\hat{\psi} - \frac{q}{2}(\bar{\hat{\varphi}}_{,k}\gamma_0\hat{\psi})_{,k}\frac{\mathrm{d}}{\mathrm{d}\omega}\frac{1}{\omega\varepsilon\mu^2}, \tag{31}$$

where υ_gr is the group velocity in (17) taken at $m = 0$, so that $1/\upsilon_\mathrm{gr} = \varepsilon\mu + (\varepsilon\mu)'\omega$. Using plane waves $\bar{\hat{\varphi}} = \bar{\tilde{\varphi}}\mathrm{e}^{-\mathrm{i}\mathbf{kx}}$, $\hat{\psi} = \tilde{\psi}\mathrm{e}^{\mathrm{i}\mathbf{kx}}$, we find

$$\hat{\mathbf{j}}_\mathrm{C} = \frac{q}{\mu}\bar{\tilde{\varphi}}\gamma_0\tilde{\psi}\mathbf{k}_0, \qquad \hat{\rho}_\mathrm{C} = \frac{q}{\mu}\frac{1}{\upsilon_\mathrm{gr}}\bar{\tilde{\varphi}}\gamma_0\tilde{\psi}, \tag{32}$$

and thus $\hat{\mathbf{j}}_\mathrm{C} = \upsilon_\mathrm{gr}\hat{\rho}_\mathrm{C}\mathbf{k}_0$. We may replace $\bar{\hat{\varphi}}\gamma_0 = \hat{\varphi}^\dagger$, where $\varphi^\dagger$ denotes the transposed and conjugated spinor. The charge transfer happens with group velocity, and the refractive index for zero rest mass is $n_\mathrm{r} = k/\omega = \varepsilon\mu$, cf. [3,17].

The charge density of the spin current $\hat{\mathbf{j}}_\mathrm{S}$ in (30) reads, cf. (14),

$$\hat{\rho}_\mathrm{S} = -\frac{q}{2}\bar{\hat{\varphi}}_{,k}\gamma_0\sigma_{ki}\hat{\psi}_{,i}\frac{\mathrm{d}}{\mathrm{d}\omega}\frac{1}{\omega\varepsilon\mu^2}. \tag{33}$$

We symmetrize the divergence $(\bar{\hat{\varphi}}_{,k}\gamma_0\sigma_{ki}\hat{\psi})_{,i}$, and use the massless Dirac equations (9) and (10) as well as $(\bar{\hat{\varphi}}\gamma_i\hat{\psi})_{,i} = 0$, to write this as a polarization density $\hat{\rho}_\mathrm{S} = -\mathrm{div}\,\mathbf{P}$,

$$\mathbf{P} = \frac{q}{4}\frac{1}{\varepsilon\mu^2\omega^2}\left(1 + \frac{\varepsilon'}{\varepsilon}\omega + 2\frac{\mu'}{\mu}\omega\right)(\bar{\hat{\varphi}}_{,i}\gamma_0\hat{\psi} + \bar{\hat{\varphi}}\gamma_0\hat{\psi}_{,i}). \tag{34}$$

The spin current $\hat{\mathbf{j}}_\mathrm{S}$ in (30) is a magnetization current,

$$\hat{\mathbf{j}}_\mathrm{S} = -\mathrm{i}\frac{q}{2}\frac{(\hat{\varphi}^\dagger\sigma_{ki}\hat{\psi})_{,i}}{\omega\varepsilon\mu^2} = \mathrm{rot}\,\mathbf{M}, \quad \mathbf{M} = -\mathrm{i}\frac{q}{2}\frac{\hat{\varphi}^\dagger\boldsymbol{\sigma}\hat{\psi}}{\omega\varepsilon\mu^2}, \tag{35}$$

with spin vector $\boldsymbol{\sigma}$ defined in (24).

Refractive index and neutrino excess velocity. – The goal is to relate the OPERA excess velocity to the refractive index n_r of the aether, cf. after (18) and (32). In the massless limit, the group velocity (17) reads

$$\frac{1}{\upsilon_\mathrm{gr}(\omega)} = n_\mathrm{r}(\omega) + \omega n_\mathrm{r}'(\omega), \tag{36}$$

where $n_\mathrm{r} = \varepsilon(\omega)\mu(\omega)$, cf. (3) and (8). A possible neutrino rest mass of a few eV is negligible given the GeV energy range of the OPERA experiment. I also mention that a negative mass-square is not an option in the Dirac equation, as mass enters linearly and an imaginary mass destroys the hermiticity of the energy operator [4].

At 28.2 GeV, an excess speed of $\upsilon_\mathrm{gr} - 1 \approx (2.51 \pm 0.6) \times 10^{-5}$ was measured for muon neutrinos. At 13.8 GeV and 40.7 GeV, the excess speed was $\upsilon_\mathrm{gr} - 1 \approx 2.25 \times 10^{-5}$ and $\upsilon_\mathrm{gr} - 1 \approx 2.80 \times 10^{-5}$, respectively, inferred from the time-of-flight differences $\delta t_{28.2} \approx 61.1$ ns, $\delta t_{13.8} \approx 54.7$ ns, and $\delta t_{40.7} \approx 68.1$ ns, where $\delta t = \tau_c - \tau_\nu$ is the difference of the photon and neutrino time of flight [1]. A baseline estimate of 730.085 km was used, resulting in a photon time of flight of $\tau_c \approx 2.43530 \times 10^6$ ns. The excess velocities are found via $\upsilon_\mathrm{gr} - 1 = \delta t/(\tau_c - \delta t)$. The independent MINOS experiment [18] measured a neutrino excess speed of $\upsilon_\mathrm{gr} - 1 \approx (5.1 \pm 2.9) \times 10^{-5}$ at a peak energy of 3 GeV. The neutrino flux from supernova SN1987A at 10 MeV did not exhibit a deviation from the vacuum speed of light [19–21].

We use the OPERA excess velocities to determine the frequency variation of the refractive index $n_\mathrm{r}(\omega)$ in the 10–50 GeV range. To this end, we expand $n_\mathrm{r}(\omega)$ at the average energy $\omega_0 \approx 28.2$ GeV into a truncated Taylor series (or Laurent series if needed),

$$n_\mathrm{r} \approx n_\mathrm{r}^{(0)} + n_\mathrm{r}^{(1)}(\omega - \omega_0) + \frac{n_\mathrm{r}^{(2)}}{2}(\omega - \omega_0)^2, \tag{37}$$

and substitute this into (36). We obtain three linear equations for the expansion coefficients $n_{\mathrm{r}}^{(0,1,2)}(\omega_0)$, if we use for ω the neutrino energies of 28.2 GeV, 13.8 GeV and 40.7 GeV, and for $v_{\mathrm{gr}}-1$ the respective excess velocities. Solving this system, we find $1-n_{\mathrm{r}}^{(0)}(\omega_0)\approx 2.27\times 10^{-5}$, as well as the derivatives $n_{\mathrm{r}}^{(1)}(\omega_0)\approx -8.61\times 10^{-8}\,\mathrm{GeV}^{-1}$ and $n_{\mathrm{r}}^{(2)}(\omega_0)\approx -1.27\times 10^{-9}\,\mathrm{GeV}^{-2}$. These small derivatives justify expansion (37) in the 10–50 GeV range. On substituting the coefficients $n_{\mathrm{r}}^{(0,1,2)}(\omega_0)$ into (37), we obtain the frequency variation of the refractive index in this interval. For instance, $1-n_{\mathrm{r}}(28.2\,\mathrm{GeV})=2.27\times 10^{-5}$, $1-n_{\mathrm{r}}(13.8\,\mathrm{GeV})=2.16\times 10^{-5}$, and $1-n_{\mathrm{r}}(40.7\,\mathrm{GeV})=2.38\times 10^{-5}$, in accordance with the mild variation of the excess velocities at these energies. If we extrapolate further down to 3 GeV, the energy of the MINOS experiment, we find, via (36) and (37), $v_{\mathrm{gr}}-1\approx 2.11\times 10^{-5}$ at 3 GeV. This is still marginally consistent with the MINOS excess speed $(5.1\pm 2.9)\times 10^{-5}$.

The mentioned SN result at 10 MeV is a null result consistent with the speed of light [21]. At this energy, the permeability tensor (3) coincides with the Minkowski metric. Accordingly, there is a substantial frequency variation of the permeabilities between 3 GeV and 10 MeV. As there are no data points available in this energy range, I do not specify the permeabilities in this range, which have to be inferred from an empirical fit. If $v_{\mathrm{gr}}(\omega)$ is known over an extended energy range, one can obtain the refractive index by a numerical quadrature of (36), $\omega n_{\mathrm{r}}(\omega)-\omega_0 n_{\mathrm{r}}(\omega_0)=\int_{\omega_0}^{\omega}\mathrm{d}\omega/v_{\mathrm{gr}}(\omega)$, where the integration constant $n_{\mathrm{r}}(\omega_0)$ is calculated locally as above, by means of a truncated Taylor expansion.

Conclusion. – The OPERA excess velocity amounts to superluminal signal transfer, which is causality violating in a relativistic spacetime view based on the relativistic interpretation of the Lorentz symmetry of Maxwell's equations [22,23]. Causality implies that every effect has a cause, that the cause precedes the effect, and that the distinction of cause and effect is unambiguous. The third condition means that every observer can arrive at the same conclusion as to what is cause and effect. If two events (cause and effect) are related by the emission and absorption of a superluminal signal, this establishes a spacelike connection. One can always find a Lorentz boost which overturns the time order of this connection. Two inertial observers related by this Lorentz boost will see a different time order in their proper time, and arrive at the opposite conclusion as to what is cause and effect. This does not depend on the specific physical mechanism of the signal transfer. Superluminal signal transfer is tantamount to a spacelike connection, and the time order in spacelike connections is ambiguous, as it can be altered by a Lorentz transformation. Thus, any kind of superluminal signal transfer is causality violating in a relativistic context, since, according to the relativity principle, the contradictory conclusions of the two inertial observers are equally valid. Therefore, a distinguished frame of reference is required, to which all uniformly moving inertial observers can relate and arrive at an unambiguous causality interpretation of their observations, even though the cosmic time order of events may be inverted in their proper time. We have identified this distinguished frame as the rest frame of the aether, in which the cosmic microwave background is isotropic, and which determines the absolute cosmic time order [5,6,24]. This discussion can be made quantitative quite easily, as the causality of superluminal signal transfer does not depend on the specific realization of the signal.

The coordinates of the rest frame of the aether are denoted by $(t,\mathbf{x})$. The world lines of source and detector read $\mathbf{x}_{\mathrm{S}}(t)=\tilde{\mathbf{x}}_0+\boldsymbol{v}t$ and $\mathbf{x}_{\mathrm{D}}(t)=\tilde{\mathbf{x}}_1+\boldsymbol{v}t$, respectively. The rest frame of source and detector is denoted by primed coordinates $(t',\mathbf{x}')$, and connected to the aether frame by a proper orthochronous Lorentz boost,

$$\begin{aligned} t' &= \gamma t-\sqrt{\gamma^2-1}\,\boldsymbol{v}_0\mathbf{x}, \\ \mathbf{x}' &= -\sqrt{\gamma^2-1}\,\boldsymbol{v}_0 t+\mathbf{x}+(\gamma-1)\boldsymbol{v}_0(\boldsymbol{v}_0\mathbf{x}), \end{aligned} \tag{38}$$

where $\boldsymbol{v}_0$ is the velocity unit vector of source and detector, $\boldsymbol{v}=v\boldsymbol{v}_0$, and $\gamma=(1-v^2)^{-1/2}$, so that $\gamma v=\sqrt{\gamma^2-1}$. The source and detector world lines $(t'_{\mathrm{S}},\mathbf{x}'_{\mathrm{S}})$ and $(t'_{\mathrm{D}},\mathbf{x}'_{\mathrm{D}})$ in the primed frame can be parametrized by

$$\begin{aligned} t'_{\mathrm{S,D}} &= \gamma(t-\boldsymbol{v}\mathbf{x}_{\mathrm{S,D}}(t)), \\ \mathbf{x}'_{\mathrm{S,D}} &= \tilde{\mathbf{x}}_{0,1}+(\gamma-1)\boldsymbol{v}_0(\boldsymbol{v}_0\tilde{\mathbf{x}}_{0,1}). \end{aligned} \tag{39}$$

In the aether frame, a superluminal signal is emitted at $(t_0,\mathbf{x}_{\mathrm{S}}(t_0))$, and absorbed at a later instant $t_1>t_0$ at $(t_1,\mathbf{x}_{\mathrm{D}}(t_1))$. The speed of this signal is assumed to be constant, $\boldsymbol{v}_{\mathrm{T}}=(\mathbf{x}_{\mathrm{D}}(t_1)-\mathbf{x}_{\mathrm{S}}(t_0))/(t_1-t_0)$, and tachyonic, $v_{\mathrm{T}}>1$, so that a spacelike connection is established between emission and absorption. In the primed frame, the emission and absorption events take place at $(t'_0,\mathbf{x}'_{\mathrm{S}})$ and $(t'_1,\mathbf{x}'_{\mathrm{D}})$, respectively, where $t'_0=t'_{\mathrm{S}}(t_0)$ and $t'_1=t'_{\mathrm{D}}(t_1)$, cf. (39). The time interval between emission t'_0 and absorption t'_1 in the rest frame of source and detector is

$$t'_1-t'_0=(t_1-t_0)(1-\boldsymbol{v}\boldsymbol{v}_{\mathrm{T}})\gamma. \tag{40}$$

We may write $\boldsymbol{v}\boldsymbol{v}_{\mathrm{T}}=vv_{\mathrm{T}}\cos\theta$, where $\boldsymbol{v}$ and $\boldsymbol{v}_{\mathrm{T}}$ denote velocities in the aether frame, $\boldsymbol{v}$ being the velocity of the source and the detector and $\boldsymbol{v}_{\mathrm{T}}$ the tachyonic (superluminal) signal speed. v_{T} corresponds to v_{gr} in (36). A subluminal source moving with velocity $\boldsymbol{v}$ in the aether frame can only emit superluminal signals of velocity $\boldsymbol{v}_{\mathrm{T}}$ if the condition $\boldsymbol{v}\boldsymbol{v}_{\mathrm{T}}<1$ is satisfied. This condition is required by causality, in addition to kinematic constraints due to energy-momentum conservation, and is also necessary for absorption. If an observer moves with velocity $\boldsymbol{v}_{\mathrm{obs}}$ in the aether frame, he will see the time order of emission and absorption inverted in his proper time, $\mathrm{d}t_{\mathrm{obs}}=\mathrm{d}t(1-\boldsymbol{v}_{\mathrm{obs}}\boldsymbol{v}_{\mathrm{T}})\gamma$, if the angle between $\boldsymbol{v}_{\mathrm{obs}}$ and

$\boldsymbol{\upsilon}_{\rm T}$ is acute and the superluminal signal speed sufficiently high.

The causality of superluminal signal transfer is solely determined by the time order in the aether frame, in particular it does not depend on the energy concept. In the rest frame of the aether, sub- as well as superluminal particles or radiation quanta have positive energy, and energy-momentum conservation applies. The positivity of energy of a subluminal particle (with positive mass square) is preserved under a Lorentz transformation, which is not the case for tachyonic quanta, so that superluminal emission can result in energy gain in the rest frame of the emitting subluminal particle, if causality is violated [25].

The speed of the superluminal signal in the primed rest frame of source and detector is $\boldsymbol{\upsilon}'_{\rm T} = (\mathbf{x}'_{\rm D} - \mathbf{x}'_{\rm S})/(t'_1 - t'_0)$, where, cf. (39),

$$\mathbf{x}'_{\rm D} - \mathbf{x}'_{\rm S} = [\boldsymbol{\upsilon}_{\rm T} + ((\gamma-1)\boldsymbol{\upsilon}_0\boldsymbol{\upsilon}_{\rm T} - \sqrt{\gamma^2-1})\boldsymbol{\upsilon}_0](t_1 - t_0). \tag{41}$$

Here, we split the superluminal speed $\boldsymbol{\upsilon}_{\rm T} = \upsilon_{{\rm T},\parallel}\boldsymbol{\upsilon}_0 + \boldsymbol{\upsilon}_{{\rm T},\perp}$ into components parallel and orthogonal to the unit vector $\boldsymbol{\upsilon}_0$, and substitute (40) for $t_1 - t_0$ to obtain

$$\boldsymbol{\upsilon}'_{\rm T} = \frac{\boldsymbol{\upsilon}_{{\rm T},\perp} + (\upsilon_{{\rm T},\parallel} - \upsilon)\gamma\boldsymbol{\upsilon}_0}{(1 - \upsilon\upsilon_{{\rm T},\parallel})\gamma}. \tag{42}$$

We multiply with $\boldsymbol{\upsilon}_0$ to find $\upsilon_{{\rm T},\parallel} = (\upsilon + \boldsymbol{\upsilon}_0\boldsymbol{\upsilon}'_{\rm T})/(1 + \boldsymbol{\upsilon}\boldsymbol{\upsilon}'_{\rm T})$, substitute this into (42) and solve for $\boldsymbol{\upsilon}_{{\rm T},\perp}$. Thus, $\boldsymbol{\upsilon}_{\rm T} = \boldsymbol{\upsilon}_{{\rm T},\parallel} + \boldsymbol{\upsilon}_{{\rm T},\perp}$, where

$$\boldsymbol{\upsilon}_{{\rm T},\parallel} = \frac{\boldsymbol{\upsilon} + \boldsymbol{\upsilon}_0(\boldsymbol{\upsilon}_0\boldsymbol{\upsilon}'_{\rm T})}{1 + \boldsymbol{\upsilon}\boldsymbol{\upsilon}'_{\rm T}}, \quad \boldsymbol{\upsilon}_{{\rm T},\perp} = \frac{\boldsymbol{\upsilon}'_{\rm T} - \boldsymbol{\upsilon}_0(\boldsymbol{\upsilon}_0\boldsymbol{\upsilon}'_{\rm T})}{(1 + \boldsymbol{\upsilon}\boldsymbol{\upsilon}'_{\rm T})\gamma}. \tag{43}$$

By means of this addition law, we can infer the superluminal speed $\boldsymbol{\upsilon}_{\rm T}$ in the aether frame from the measured speed $\boldsymbol{\upsilon}'_{\rm T}$ in the rest frame of the source (and detector) and the measured speed $\boldsymbol{\upsilon}$ of this rest frame in the aether. Expanding (43) in powers of υ, we find

$$\begin{aligned}\boldsymbol{\upsilon}_{\rm T} \sim\ & \boldsymbol{\upsilon}'_{\rm T} + \upsilon[\boldsymbol{\upsilon}_0 - \boldsymbol{\upsilon}'_{\rm T}(\boldsymbol{\upsilon}_0\boldsymbol{\upsilon}'_{\rm T})] \\ & + \upsilon^2[\boldsymbol{\upsilon}'_{\rm T}(\boldsymbol{\upsilon}_0\boldsymbol{\upsilon}'_{\rm T})^2 - \frac{1}{2}\boldsymbol{\upsilon}_0(\boldsymbol{\upsilon}_0\boldsymbol{\upsilon}'_{\rm T}) - \frac{1}{2}\boldsymbol{\upsilon}'_{\rm T}],\end{aligned} \tag{44}$$

up to terms of $O(\upsilon^3)$. We square this and substitute $\boldsymbol{\upsilon}_0\boldsymbol{\upsilon}'_{{\rm T},0} = \cos\theta'$ to obtain

$$\begin{aligned}\upsilon_{\rm T} \sim\ & \upsilon'_{\rm T}\Bigg[1 - \frac{\upsilon}{\upsilon'_{\rm T}}(\upsilon'^2_{\rm T} - 1)\cos\theta' \\ & - \frac{1}{2}\frac{\upsilon^2}{\upsilon'^2_{\rm T}}(\upsilon'^2_{\rm T} - 1)(1 - (2\upsilon'^2_{\rm T} + 1)\cos^2\theta')\Bigg],\end{aligned} \tag{45}$$

where $\boldsymbol{\upsilon}'_{\rm T} = \upsilon'_{\rm T}\boldsymbol{\upsilon}'_{{\rm T},0}$. In the example discussed in this article, $\upsilon'_{\rm T}$ is the measured neutrino speed, in units of the speed of light, and υ is the speed of the Solar system barycenter or neutrino baseline (neglecting the Earth's relative motions, which are at least one order of magnitude smaller than υ) in the aether. Since $\upsilon'_{\rm T} - 1 \approx 2.5 \times 10^{-5}$ and $\upsilon \approx 369\,{\rm km\,s^{-1}}/c \approx 1.2 \times 10^{-3}$, we find $\upsilon_{\rm T} \approx \upsilon'_{\rm T}(1 - 6 \times 10^{-8}\cos\theta')$. Thus, in view of the current error bound on $\upsilon'_{\rm T} - 1 \approx (2.5 \pm 0.6) \times 10^{-5}$, the superluminal speed $\upsilon_{\rm T}$ in the aether frame can be identified with the actually measured speed $\upsilon'_{\rm T}$, irrespectively of the periodically varying angle $\cos\theta'$ between the neutrino baseline and the velocity of the barycenter. However, this angle becomes significant if the error bound is reduced by two orders. In any case, if $\upsilon'_{\rm T} >> 1$ or $\upsilon \sim 1$, the velocities $\boldsymbol{\upsilon}_{\rm T}$ and $\boldsymbol{\upsilon}'_{\rm T}$ can substantially differ depending on this angle, cf. (45).

Wave fields propagating in the aether are coupled by permeability tensors, which are a manifestation of the microscopic space structure [3,22]. Here, we have discussed the permeability tensor for neutrinos, and explained the coupling of the Dirac equation to this tensor. We have studied freely propagating spinorial wave modes in the aether, and shown that the frequency-dependent refractive index gives rise to superluminal energy transfer. The OPERA excess velocities measured at GeV energies allow to derive a quantitative estimate of the refractive index of the aether for superluminal neutrinos in this energy range.

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