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Mass generation in the aether: Neutrino oscillations and massive gauge fields

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ABSTRACT

Neutrino mixing is studied in an absolute spacetime conception based on a dispersive aether. The effect of the frequency-dependent permeability of the aether on the interference phase of neutrino mass eigenstates is analyzed. Neutrinos are treated as massless Dirac spinors, and mass eigenstates are due to the neutrino permeability of spacetime. The aether can also generate effective gauge masses, resulting in massive dispersion relations preserving the gauge symmetry. The propagators of gauge and spinor fields are derived, illustrating mass generation by isotropic permeability tensors in the aether frame, the rest frame of the cosmic background radiation.

1. Introduction

We investigate the generation of mass in the dispersion relations of Dirac particles and gauge fields. Neutrino masses required for flavor oscillations as well as massive non-Abelian gauge fields are obtained by coupling the massless wave equations to permeability tensors. Flavor mixing [1–5] is compatible with sub- as well as superluminal neutrino velocities [6–9], and arises in a dispersive spacetime due to the frequency-dependent permeabilities of the aether [10,11].

The aether defines a distinguished frame of reference, physically manifested as the rest frame of the isotropic cosmic microwave background radiation [12–16], and wave fields couple to the aether with isotropic permeability tensors in this frame [17–20]. We discuss the permeability tensors of Dirac fermions and gauge bosons, the interference phase of neutrino mass eigenstates [1,5,21], as well as gauge fixing and propagators in the dispersive aether, and potential Michelson–Morley experiments with neutrino beams.

Dirac particles freely propagating in the aether are described by the Dirac equation

$$\gamma_\mu g^{\mu\nu}\psi_{,\nu} + m\psi = 0 \tag{1.1}$$

coupled to a real symmetric permeability tensor $g^{\mu\nu}(\omega)$, which depends on the frequency of the spinor modes $\psi \propto \exp(ik_\mu(\omega)x^\mu)$ [10]. The Dirac matrices satisfy $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\eta_{\mu\nu}$, where γ_0 is anti-Hermitian, the γ_i are Hermitian, and the sign convention for the Minkowski metric is $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. Latin indices run from one to three, Greek ones from zero to three; the latter are raised and lowered by the Minkowski metric. The wave 4-vector is $k_\mu = (-\omega, \mathbf{k})$, $\mathbf{k} = k(\omega)\mathbf{k}_0$, where $\mathbf{k}_0$ is the unit wave vector and $k(\omega)$ the wave number depending on the permeability tensor. In the aether frame, the permeability tensor is isotropic,

$$g^{00} = -\varepsilon(\omega), \qquad g^{ik} = \delta^{ik}/\mu(\omega), \qquad g^{0k} = 0, \tag{1.2}$$

where ε and μ are positive frequency-dependent permeabilities.

In Section 2, we discuss neutrino oscillations based on a massive Dirac equation (1.1) coupled by a permeability tensor (1.2) to the aether. In Section 3, we demonstrate that the interference phase can be generated by dispersive permeabilities (1.2) without a mass term in the wave equation. In Section 4, we study gauge and Proca fields in a permeable spacetime, in analogy to spinor fields. Each gauge component admits a permeability tensor which produces a massive dispersion relation without breaking the gauge invariance. In Section 5, we explain how permeability tensors generate effective mass in the propagators of massless gauge and Dirac fields. In Section 6, we study Dirac neutrinos in moving inertial frames coupled to an anisotropic permeability tensor and discuss Michelson–Morley neutrino experiments. In Section 7, we present our conclusions.

2. Neutrino mixing in a dispersive spacetime

We start with the dispersion relations [10]

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$$k_{(j)}(\omega)=\mu_{(j)}(\omega)\sqrt{\varepsilon_{(j)}^2(\omega)\omega^2-m_{(j)}^2}, \tag{2.1}$$

derived by squaring the Dirac equation, where j labels three mass eigenstates ψ_j, each characterized by permeabilities $\varepsilon_{(j)}(\omega)$, $\mu_{(j)}(\omega)$, and a neutrino mass $m_{(j)}$; the $k_{(j)}(\omega)$ are the respective wave numbers. Each eigenstate ψ_j satisfies a Dirac equation with mass $m_{(j)}$ and a permeability tensor $g_{(j)}^{\mu\nu}(\omega)$ defined by dispersive permeabilities $\varepsilon_{(j)}(\omega)$ and $\mu_{(j)}(\omega)$. We use the shortcuts $k_{(j)}=k_{(j)}(\omega_j)$, $\varepsilon_{(j)}=\varepsilon_{(j)}(\omega_j)$ and $\mu_{(j)}=\mu_{(j)}(\omega_j)$, where ω_j is the frequency of the eigenstate plane wave ψ_j with wave 4-vector $k_\mu^j=(-\omega_j,k_{(j)}\mathbf{k}_0)$. The flavor transition amplitude reads [1]

$$A(\nu_l\to\nu_{l'})=\sum_{j=1}^{3}U_{lj}D_jU_{jl'}^\dagger, \tag{2.2}$$

where $l,l'=e,\mu,\tau$ are the flavor indices of Dirac neutrinos and j is the eigenstate index. U_{lj} is a unitary matrix parametrized with Eulerian mixing angles and a CP violating phase. The D_j denote phase factors of the plane waves ψ_j,

$$D_j=\exp\bigl(\mathrm{i}(k_{(j)}L-\omega_jT)\bigr),$$
$$L=\mathbf{k}_0(\mathbf{x}_{\mathrm{abs}}-\mathbf{x}_{\mathrm{em}}),\qquad T=t_{\mathrm{abs}}-t_{\mathrm{em}}, \tag{2.3}$$

where the subscripts refer to emission and absorption. The neutrino wave vector is $\mathbf{k}_{(j)}=k_{(j)}\mathbf{k}_0$, L is the path length between source and detector, and T the time of flight.

The transition probability $|A(\nu_l\to\nu_{l'})|^2$ depends on the phase factors $D_iD_j^*=\exp(-\mathrm{i}\delta\varphi_{ij})$, defined by the phase difference

$$\delta\varphi_{ij}=\delta\omega_{ij}T-\delta k_{ij}L,\qquad \delta\omega_{ij}=\omega_i-\omega_j,$$
$$\delta k_{ij}=k_{(i)}(\omega_i)-k_{(j)}(\omega_j). \tag{2.4}$$

The increments

$$\Delta m_{ij}^2=m_{(i)}^2-m_{(j)}^2,$$
$$\Delta\varepsilon_{ij}(\omega)=\varepsilon_{(i)}(\omega)-\varepsilon_{(j)}(\omega),$$
$$\Delta\mu_{ij}(\omega)=\mu_{(i)}(\omega)-\mu_{(j)}(\omega), \tag{2.5}$$

as well as $\delta\omega_{ij}$ in (2.4) are treated as small variations, and we expand δk_{ij} in linear order,

$$\delta k_{ij}\sim\frac{\partial k_j}{\partial\omega_j}\delta\omega_{ij}+\frac{\partial k_j}{\partial\mu_j}\Delta\mu_{ij}(\omega_j)+\frac{\partial k_j}{\partial\varepsilon_j}\Delta\varepsilon_{ij}(\omega_j)+\frac{\partial k_j}{\partial m_j^2}\Delta m_{ij}^2. \tag{2.6}$$

The $k_{(j)}(\omega_j)$ are the eigenstate wave numbers (2.1), whose frequency derivative is the reciprocal group velocity, $\partial k_{(j)}/\partial\omega_j=1/\upsilon_{\mathrm{gr}(j)}$ [11]. We thus find

$$\delta k_{ij}\sim\frac{1}{\upsilon_{\mathrm{gr}(j)}}\delta\omega_{ij}+\frac{k_{(j)}}{\mu_{(j)}}\Delta\mu_{ij}(\omega_j)+\mu_{(j)}^2\varepsilon_{(j)}\frac{\omega_j^2}{k_{(j)}}\Delta\varepsilon_{ij}(\omega_j)-\frac{1}{2}\frac{\mu_{(j)}^2}{k_{(j)}}\Delta m_{ij}^2. \tag{2.7}$$

In the phase difference $\delta\varphi_{ij}$ in (2.4), we put $T=L/\upsilon_{\mathrm{gr}(j)}$ and substitute δk_{ij},

$$\frac{\delta\varphi_{ij}}{L}\sim\frac{1}{2}\frac{\mu_{(j)}^2}{k_{(j)}}\Delta m_{ij}^2-\frac{k_{(j)}}{\mu_{(j)}}\Delta\mu_{ij}(\omega_j)-\frac{\mu_{(j)}^2\varepsilon_{(j)}\omega_j^2}{k_{(j)}}\Delta\varepsilon_{ij}(\omega_j). \tag{2.8}$$

If the time of flight T is defined by $\upsilon_{\mathrm{gr}(i)}$ instead of $\upsilon_{\mathrm{gr}(j)}$, this does not affect $\delta\varphi_{ij}$ in linear order, as these two velocities only differ by delta increments. For the same reason, we can drop the index j from ω. Similarly, if we replace the eigenstate index j by i in the three ratios on the right-hand side of (2.8), this does not affect the indicated linear order.

We split the eigenstate permeabilities and mass-squares in (2.1) as

$$m_{(j)}^2=m^2+\delta m_{(j)}^2,$$
$$\varepsilon_{(j)}(\omega)=\varepsilon(\omega)+\delta\varepsilon_{(j)}(\omega),$$
$$\mu_{(j)}(\omega)=\mu(\omega)+\delta\mu_{(j)}(\omega), \tag{2.9}$$

where the increments $\delta m_{(j)}^2$, $\delta\varepsilon_{(j)}(\omega)$ and $\delta\mu_{(j)}(\omega)$ are small deviations from base values m^2 (which can be zero) and $\varepsilon(\omega)$, $\mu(\omega)$ (both close to 1, see after (3.3)). In the ratios of Eq. (2.8), we replace the permeabilities and the mass-squares in the wave numbers by their base values, since the expansion is linear in the delta increments. We thus arrive at the interference phase

$$\delta\varphi_{ij}(\omega)\sim\left(\frac{\mu^2}{2k}\Delta m_{ij}^2-\frac{k}{\mu}\Delta\mu_{ij}(\omega)-\frac{\mu^2\varepsilon\omega^2}{k}\Delta\varepsilon_{ij}(\omega)\right)L, \tag{2.10}$$

where the wave number $k=\mu(\omega)\sqrt{\varepsilon^2(\omega)\omega^2-m^2}$ is independent of the eigenstate index.

3. Mass eigenstates generated by permeability tensors

If the mass eigenstates admit the same permeabilities, $\varepsilon_{(j)}(\omega)=\varepsilon(\omega)$, $\mu_{(j)}(\omega)=\mu(\omega)$, cf. (2.9), we can put $\Delta\mu_{ij}=\Delta\varepsilon_{ij}=0$ in the interference phase (2.10). Neglecting the mass-square m^2 in the wave number k, cf. (2.9) and after (2.10), we find

$$\delta\varphi_{ij}(\omega)\sim\frac{1}{2}\frac{\mu(\omega)}{\varepsilon(\omega)\omega}\Delta m_{ij}^2L. \tag{3.1}$$

Alternatively, we may assume that the mass eigenstates ψ_j admit the same mass-square $m_{(j)}^2=m^2$, so that $\Delta m_{ij}^2=0$. In this case, the interference phase (2.10) stems from the permeability increments (2.5). We put $m^2=0$ in (2.9), so that $k=\mu(\omega)\varepsilon(\omega)\omega$ and

$$\delta\varphi_{ij}(\omega)\sim-\bigl(\varepsilon(\omega)\Delta\mu_{ij}+\mu(\omega)\Delta\varepsilon_{ij}\bigr)\omega L, \tag{3.2}$$

where $\Delta\mu_{ij}(\omega)=\delta\mu_{(i)}(\omega)-\delta\mu_{(j)}(\omega)$ and analogously $\Delta\varepsilon_{ij}(\omega)$. The interference phase (3.1) is recovered from (3.2) by specifying the increments $\delta\varepsilon_{(j)}(\omega)$ and $\delta\mu_{(j)}(\omega)$ of the eigenstate permeabilities (2.9) as

$$\delta\varepsilon_{(j)}(\omega)=-\frac{a(\omega)}{2}\frac{1}{\varepsilon(\omega)}\frac{m_{(j)}^2}{\omega^2},$$
$$\delta\mu_{(j)}(\omega)=\frac{a(\omega)-1}{2}\frac{\mu(\omega)}{\varepsilon^2(\omega)}\frac{m_{(j)}^2}{\omega^2}. \tag{3.3}$$

Here, $a(\omega)$ is an arbitrary real function that drops out in the phase difference (3.2); we may conveniently choose $a=0,1$, or $a=\mu/(\varepsilon+\mu)\approx 1/2$. Thus we have shown that the interference phase (3.1) is reproduced by the eigenstate permeabilities $\varepsilon_{(j)}(\omega)$ and $\mu_{(j)}(\omega)$ defined in (2.9) and (3.3), without inserting mass terms $m_{(j)}\psi_j$ in the Dirac equations of the eigenstate plane waves. The refractive indices of the eigenstates ψ_j read, in leading order in $m_{(j)}^2/\omega^2$, cf. (2.9) and (3.3),

$$n_{\mathrm{r}(j)}(\omega)=\mu_{(j)}(\omega)\varepsilon_{(j)}(\omega)\sim\varepsilon(\omega)\mu(\omega)-\frac{1}{2}\frac{\mu(\omega)}{\varepsilon(\omega)}\frac{m_{(j)}^2}{\omega^2}, \tag{3.4}$$

and the wave numbers $k_{(j)}(\omega)=n_{\mathrm{r}(j)}(\omega)\omega$ admit massive dispersion relations $k_{(j)}^2(\omega)\sim\mu^2(\varepsilon^2\omega^2-m_{(j)}^2)$ with an effective neutrino mass $m_{(j)}$ generated by the permeability increments (3.3).

A bound on the neutrino refractive index $n_r \sim 1/\upsilon_{gr} \sim k(\omega)/\omega \sim \mu(\omega)\varepsilon(\omega)$ [11] at $\omega \sim 17$ GeV is $|1 - n_r| < 3 \times 10^{-6}$, based on recent measurements of the neutrino speed by the OPERA, BOREXINO, LVD and ICARUS Collaborations [6–9]. We use the $n_r(\omega)$ estimate also for the factors $\mu(\omega)$ and $\varepsilon(\omega)$. The neutrino flux from supernova SN1987A gives the bound $|\upsilon_{gr} - 1| < 2 \times 10^{-9}$ at 10 MeV [22], so that we put $\mu(\omega) \approx \varepsilon(\omega) \approx 1$ in the low MeV region. An upper bound on the eigenstate masses $m_{(j)}$ from tritium β decay is 2 eV [23], assuming vacuum permeabilities in the eV range. The neutrino frequency ω is in the low MeV region (for reactor and solar ν's) or low GeV region (accelerator, atmospheric ν's). The path length L is of order 10^3 km and 10^4 km for accelerator and atmospheric ν's. The differences Δm_{ij}^2 of the eigenstate mass-squares are of order 10^{-4} eV^2 or 10^{-3} eV^2 [1,5]. To generate flavor oscillations, a phase difference $|\delta\varphi_{ij}| \geqslant 1$ is required for substantial interference in the squared amplitudes (2.2). As for the neutrino speed determined by the refractive index $n_{r(j)}$ in (3.4), we can ignore terms depending on the tiny (compared to $O(10^{-6})$) ratios $m_{(j)}^2/\omega^2$, at least at length scales relevant for terrestrial experiments and solar ν's. Accordingly, $n_r \sim \mu(\omega)\varepsilon(\omega)$ is independent of the eigenstate index, so that all neutrino flavors admit the same group velocity $\upsilon_{gr} = 1/(\omega n_r(\omega))' \sim 1/n_r$.

4. Gauge and Proca fields in the aether: Massive dispersion relations preserving gauge invariance

The Lagrangian of a Proca field with negative mass-square reads

$$L = -\frac{1}{4} F_{\alpha\beta} g_F^{\alpha\beta\mu\nu} F_{\mu\nu} + \frac{1}{2} m_t^2 A_\mu g_A^{\mu\nu} A_\nu - \frac{1}{2\xi} \left(g_A^{\mu\nu} A_{\mu,\nu}\right)^2 + A_\mu g_J^{\mu\nu} j_\nu, \tag{4.1}$$

where $\eta^{\alpha\mu}\eta^{\beta\nu} \to g_F^{\alpha\beta\mu\nu}$, $\eta^{\mu\nu} \to g_A^{\mu\nu}$, and $\eta^{\mu\nu} \to g_J^{\mu\nu}$ are permeability tensors replacing the Minkowski metric $\eta^{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ in the vacuum Lagrangian [24,25]. The signs in (4.1) are chosen in a way that $m_t^2 > 0$ is the negative mass-square of the tachyonic Proca field A_μ, and $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ is the field tensor. A gauge-fixing term $\propto 1/\xi$ is included as Lagrange multiplier. Massive Proca fields are obtained by replacing $m_t^2 \to -m^2$. The 3D field strengths are $E_i = F_{i0}$, $B^k = \varepsilon^{kij} F_{ij}/2$, and inversely $F_{ij} = \varepsilon_{ijk} B^k$. Thus, $E_i = A_{0,i} - A_{i,0}$, $B^k = \varepsilon^{kij} A_{j,i}$, where ε_{ijk} is the totally antisymmetric tensor. The field is coupled to a current $j_\mu = (-\rho, \mathbf{j})$. The permeability tensors $g_F^{\alpha\beta\mu\nu}(\omega)$ and $g_{A,J}^{\mu\nu}(\omega)$ are real and frequency dependent. The $g_{A,J}^{\mu\nu}$ are symmetric, and $g_F^{\alpha\beta\mu\nu}$ is antisymmetric in the first as well as second index pair, and symmetric regarding the interchange $\alpha\beta \leftrightarrow \mu\nu$ of the index pairs. We define the inductive potential, field tensor and current as

$$C^\mu = g_A^{\mu\nu} A_\nu, \qquad H^{\alpha\beta} = g_F^{\alpha\beta\mu\nu} F_{\mu\nu}, \qquad j_\Omega^\mu = g_J^{\mu\nu} j_\nu, \tag{4.2}$$

and write $j_\Omega^\mu = (\rho_\Omega, \mathbf{j}_\Omega)$, so that $j_{\Omega 0} = -\rho_\Omega$. Greek indices are raised and lowered with the Minkowski metric. The Lagrangian (4.1) can thus be written as

$$L = -\frac{1}{4} F_{\alpha\beta} H^{\alpha\beta} + \frac{1}{2} m_t^2 A_\mu C^\mu - \frac{1}{2\xi} \left(C^\mu_{,\mu}\right)^2 + A_\mu j_\Omega^\mu. \tag{4.3}$$

The 3D inductions are $D^i = H^{0i}$, $H_i = \varepsilon_{imn} H^{mn}/2$, and inversely $H^{mn} = H_j \varepsilon^{jmn}$, so that $F_{\alpha\beta} H^{\alpha\beta} = 2(\mathbf{BH}–\mathbf{ED})$. The constitutive equations relating the 3D inductions to the field strengths read

$$D^i = \varepsilon^{ij} E_j + \kappa_k^i B^k, \qquad H_i = \mu_{ij} B^j - \kappa_i^j E_j, \tag{4.4}$$

where the permeabilities $\varepsilon^{ij}(\omega)$ and $\mu_{ij}(\omega)$ are real symmetric 3-tensors, and $\kappa_k^i(\omega)$ is a mixed tensor. We need not assume different tensors κ_k^i in the two relations (4.4), as the Lagrangian would only depend on their sum via $F_{\alpha\beta} H^{\alpha\beta}$. The permeability tensor $g_F^{\alpha\beta\mu\nu}$ in (4.1) is defined by these 3-tensors,

$$g_F^{0i0j} = -\frac{1}{2}\varepsilon^{ij}, \qquad g_F^{0iab} = \frac{1}{2}\kappa_k^i \varepsilon^{kab},$$
$$g_F^{abcd} = \frac{1}{2}\varepsilon^{abi}\mu_{ik}\varepsilon^{kcd}, \qquad g_F^{mnk0} = -\frac{1}{2}\kappa_i^k \varepsilon^{imn}. \tag{4.5}$$

All other components of $g_F^{\alpha\beta\mu\nu}$ follow from the mentioned symmetries or vanish. Inversely,

$$\varepsilon^{ij} = -2 g_F^{0i0j}, \qquad \mu_{ik} = \frac{1}{2}\varepsilon_{iab} g_F^{abmn} \varepsilon_{mnk},$$
$$\kappa_j^i = g_F^{0imn} \varepsilon_{mnj}. \tag{4.6}$$

In the isotropic aether frame, we can put $\kappa_i^j = 0$, cf. after (4.11), but in moving frames the tensor κ_i^j does not vanish, cf. after (5.4).

Euler variation of Lagrangian (4.3) gives the field equations and gauge condition

$$H^{\mu\nu}_{,\nu} - m_t^2 C^\mu - \frac{1}{\xi} g_A^{\mu\nu} C^\kappa_{,\kappa,\nu} = j_\Omega^\mu, \tag{4.7}$$

$$\frac{1}{\xi} g_A^{\mu\nu} C^\kappa_{,\kappa,\mu,\nu} + m_t^2 C^\kappa_{,\kappa} = -j^\mu_{\Omega,\mu}. \tag{4.8}$$

For instance, the Lorentz condition $C^\kappa_{,\kappa} = 0$ follows from current conservation $j^\mu_{\Omega,\mu} = 0$ if the mass-square does not vanish and the gauge-fixing term in the Lagrangian is dropped, $\xi = \infty$. Eqs. (4.7) are equivalent to

$$D^i_{,i} + m_t^2 C_0 - \frac{1}{\xi} g_A^{0\nu} C^\kappa_{,\kappa,\nu} = -j_{\Omega 0},$$
$$\varepsilon^{ikn} H_{n,k} - D^i_{,0} - m_t^2 C^i - \frac{1}{\xi} g_A^{i\nu} C^\kappa_{,\kappa,\nu} = j_\Omega^i, \tag{4.9}$$

where $j_{\Omega 0} = -\rho_\Omega$, and the constitutive relations (4.4) apply. In the massless case $m_t^2 = 0$, the gauge-invariant Maxwell equations in a permeable spacetime are recovered by imposing the Lorentz condition $C^\kappa_{,\kappa} = 0$ on the vector potential, also see (4.11).

We consider isotropic permeability tensors, cf. (4.2), (4.4) and (4.5),

$$\varepsilon^{ik} = \varepsilon\delta^{ik}, \qquad \mu_{ik} = (1/\mu)\delta_{ik}, \qquad \kappa_i^j = \kappa\delta_i^j,$$
$$g_A^{00} = -\varepsilon_0, \qquad g_A^{ij} = \delta^{ij}/\mu_0, \qquad g_A^{k0} = 0,$$
$$g_J^{00} = -\Omega_0, \qquad g_J^{mn} = \delta^{mn}/\Omega, \qquad g_J^{k0} = 0, \tag{4.10}$$

with permeabilities (ε, μ), (ε_0, μ_0) and (Ω_0, Ω) depending on the frequency ω of the wave modes in the aether frame [11]. The tensor $g_J^{\alpha\beta}$ couples the current to the wave modes, cf. (4.1) and (4.2), and amounts to a varying coupling constant if $\Omega_0(\omega)$ coincides with $1/\Omega(\omega)$ [20]. We note $H^k = \varepsilon^{kij} A_{j,i}/\mu$ and $D_k = \varepsilon(A_{0,k} - A_{k,0})$, cf. after (4.1) and (4.4). The isotropic field equations for the vector potential read, cf. (4.9),

$$A_{0,k,k} - A_{k,k,0} + m_t^2 \frac{\varepsilon_0}{\varepsilon} A_0 + \frac{1}{\xi}\frac{\varepsilon_0}{\varepsilon} C^\kappa_{,\kappa,0} = -\frac{1}{\varepsilon} j_{\Omega 0},$$
$$A_{i,k,k} - A_{k,k,i} + \varepsilon\mu(A_{0,0,i} - A_{i,0,0}) + m_t^2 \frac{\mu}{\mu_0} A_i + \frac{1}{\xi}\frac{\mu}{\mu_0} C^\kappa_{,\kappa,i}$$
$$= -\mu j_{\Omega i}, \tag{4.11}$$

where $C^\kappa_{,\kappa} = -\varepsilon_0 A_{0,0} + A_{k,k}/\mu_0$. The isotropic tensor $\kappa_i^j = \kappa\delta_i^j$ in the constitutive relations (4.4) drops out in the field equations, since the $\kappa\mathbf{EB}$ term in the Lagrangian (see after (4.3)) is a divergence, $2E_k B^k = (\varepsilon^{\alpha\beta\mu\nu} A_\beta A_{\mu,\nu})_{,\alpha}$. In [19,20], we studied the field Eqs. (4.11) in the limit $\xi = \infty$. In the following, we consider the opposite limit, $m_t^2 = 0$ with finite ξ. That is, we drop the mass term in Lagrangian (4.1) and generate a positive or negative mass-square in the dispersion relation by way of the permeability tensors (4.10).

The isotropic field equations (4.11) (with $m_t^2 = 0$, $j_\Omega^\mu = 0$) admit the plane-wave solutions

$$A_0(\mathbf{x}, t) = -\frac{\omega}{k_L}\hat{\mathbf{A}}\mathbf{k}_0 e^{i(k_L(\omega)\mathbf{k}_0\mathbf{x} - \omega t)} + \text{c.c.},$$
$$\mathbf{A}(\mathbf{x}, t) = \hat{\mathbf{A}}_\perp e^{i(k_T(\omega)\mathbf{k}_0\mathbf{x} - \omega t)} + (\hat{\mathbf{A}}\mathbf{k}_0)\mathbf{k}_0 e^{i(k_L(\omega)\mathbf{k}_0\mathbf{x} - \omega t)} + \text{c.c.}, \tag{4.12}$$

where we have split the field into a transversal and longitudinal component, $\hat{\mathbf{A}} = \hat{\mathbf{A}}_\perp + (\hat{\mathbf{A}}\mathbf{k}_0)\mathbf{k}_0$, $\hat{\mathbf{A}}_\perp\mathbf{k}_0 = 0$, with wave number $k_T^2 = \varepsilon\mu\omega^2$ and $k_L^2 = \varepsilon_0\mu_0\omega^2$, respectively. The gauge-fixing constant ξ does not explicitly show in (4.12). If the gauge-fixing term is dropped by putting $\xi = \infty$, the general plane-wave solution of frequency ω is still (4.12), but with $k_L(\omega)$ unspecified. A finite ξ gives a well-defined wave number to the longitudinal component $\propto \hat{\mathbf{A}}\mathbf{k}_0$, which is a gauge transformation, $A_\mu = A_\mu^\perp + \lambda_{,\mu}$, $\lambda = \hat{\mathbf{A}}\mathbf{k}_0 e^{i(k_L\mathbf{k}_0\mathbf{x} - \omega t)}/(ik_L) + \text{c.c.}$

The permeability tensor $g_A^{\mu\nu}$ defines the gauge-fixing term in Lagrangian (4.3) ($m_t^2 = 0$). We identify $\varepsilon_0 = \varepsilon$ and $\mu_0 = \mu$, so that $g_A^{\mu\nu}$ in (4.10) reads $g_A^{00} = -\varepsilon$, $g_A^{ij} = \delta^{ij}/\mu$, $g_A^{k0} = 0$. The transversal and longitudinal components of the vector potential (4.12) thus admit the same wave number, $k^2 = k_{T,L}^2 = \varepsilon\mu\omega^2$, where $\varepsilon(\omega)$ and $\mu(\omega)$ are the permeabilities defining $g_F^{\kappa\lambda\mu\nu}$, cf. (4.5) and (4.10). A massive dispersion relation $k^2 = \omega^2 - m^2$ is induced by permeabilities satisfying $\varepsilon\mu = 1 - m^2/\omega^2$, without a mass term in the Lagrangian, and longitudinal field components remain gauge transformations. If we impose the Lorentz condition $C^\kappa_{,\kappa} = 0$, the gauge-fixing term drops out in the field equations (4.11) (with $m_t^2 = 0$), and they become gauge invariant despite of the massive dispersion relation induced by the permeabilities. [A negative mass-square in the dispersion relation requires permeabilities related by $\varepsilon\mu = 1 + m_t^2/\omega^2$. A tachyonic dispersion relation preserving the hermiticity of the Dirac Hamiltonian is obtained by replacing $m_{(j)}^2 \to -m_{t(j)}^2$ in (5.14).] As for electromagnetic fields, we use vacuum permeabilities, $\varepsilon_0 = \varepsilon = 1$ and $\mu_0 = \mu = 1$, defining the constant speed in the Lorentz boosts, so that the electromagnetic Michelson–Morley isotropy is preserved in moving frames.

5. Effective mass-squares in propagators of dispersive gauge and Dirac fields

The foregoing admits generalization to non-Abelian gauge fields A_μ^α, each component being coupled to the aether by permeabilities $\varepsilon_{(\alpha)}(\omega)$ and $\mu_{(\alpha)}(\omega)$, $\alpha = 1, \ldots, N$. We consider the Lagrangian, cf. (4.3),

$$L = -\frac{1}{4}F^\alpha_{\kappa\lambda}H_\alpha^{\kappa\lambda} - \frac{1}{2\xi}\left(C^\mu_{\alpha,\mu}\right)^2 + A_\mu^\alpha j^\mu_{\Omega\alpha}, \tag{5.1}$$

with field tensor $F^\alpha_{\mu\nu} = A^\alpha_{\nu,\mu} - A^\alpha_{\mu,\nu} - gf^\alpha_{\beta\gamma}A^\beta_\mu A^\gamma_\nu$, inductions $H_\alpha^{\kappa\lambda} = g_{F(\alpha)}^{\kappa\lambda\mu\nu}F^\alpha_{\mu\nu}$, $C^\mu_\alpha = g^{\mu\nu}_{A(\alpha)}A^\alpha_\nu$, and inductive current $j^\mu_{\Omega\alpha} = g^{\mu\nu}_{J(\alpha)}j^\alpha_\nu$. We write the gauge index α of the permeability tensors $g_{F(\alpha)}^{\kappa\lambda\mu\nu}$, $g^{\mu\nu}_{A(\alpha)}$ and $g^{\mu\nu}_{J(\alpha)}$ in parentheses, to indicate that it is not a summation index unless in a product where α appears twice without parentheses. When calculating the propagator, it suffices to use the quadratic component of the Lagrangian and to put the coupling constant g in the field tensor to zero, so that the linearized field equations are independent of the structure constants $f^\alpha_{\beta\gamma}$, $H^{\mu\nu}_{\alpha,\nu} - C^\kappa_{\alpha,\kappa,\nu}g^{\nu\mu}_{A(\alpha)}/\xi = j^\mu_{\Omega\alpha}$, cf. (4.7). The tensors $g^{\mu\nu}_{A(\alpha)}$ and $g^{\mu\nu}_{J(\alpha)}$ are isotropic in the aether frame, cf. (4.8), (4.10) and after (4.12):

$$g^{00}_{A(\alpha)} = -\varepsilon_{(\alpha)}(\omega), \qquad g^{ij}_{A(\alpha)} = \delta^{ij}/\mu_{(\alpha)}(\omega), \qquad g^{0i}_{A(\alpha)} = 0,$$
$$g^{00}_{J(\alpha)} = -\Omega_{0(\alpha)}(\omega), \qquad g^{ij}_{J(\alpha)} = \delta^{ij}/\Omega_{(\alpha)}(\omega), \qquad g^{0i}_{J(\alpha)} = 0. \tag{5.2}$$

The isotropic permeability tensor $g_{F(\alpha)}^{\kappa\lambda\mu\nu}$ reads, cf. (4.5) and (4.10),

$$g^{0i0j}_{F(\alpha)} = -\frac{1}{2}\varepsilon_{(\alpha)}\delta^{ij}, \qquad g^{0imn}_{F(\alpha)} = \frac{1}{2}\kappa_{(\alpha)}\varepsilon^{imn},$$
$$g^{klmn}_{F(\alpha)} = \frac{\delta^{km}\delta^{ln} - \delta^{kn}\delta^{lm}}{2\mu_{(\alpha)}(\omega)}, \tag{5.3}$$

with all other components following from symmetry properties, cf. after (4.1). Without loss of generality, we can put $\kappa_{(\alpha)}(\omega) = 0$, cf. after (4.11), and reduce $g_{F(\alpha)}^{\kappa\lambda\mu\nu}$ to a symmetric tensor $g^{\mu\nu}_{(\alpha)} = \mu^{1/2}_{(\alpha)}g^{\mu\nu}_{A(\alpha)}$:

$$g_{F(\alpha)}^{\kappa\lambda\mu\nu} = \frac{1}{2}\left(g^{\kappa\mu}_{(\alpha)}g^{\lambda\nu}_{(\alpha)} - g^{\kappa\nu}_{(\alpha)}g^{\lambda\mu}_{(\alpha)}\right), \qquad H_\alpha^{\kappa\lambda} = g^{\kappa\mu}_{(\alpha)}g^{\lambda\nu}_{(\alpha)}F^\alpha_{\mu\nu}. \tag{5.4}$$

The refractive indices $n_{r(\alpha)} = \sqrt{\varepsilon_{(\alpha)}\mu_{(\alpha)}} = \sqrt{1 - m^2_{(\alpha)}/\omega^2}$ give rise to massive dispersion relations $k_{(\alpha)} = \omega n_{r(\alpha)}$ with an effective gauge mass $m_{(\alpha)}$ for each gauge component $A^\alpha_\mu = (A^\alpha_0, \mathbf{A}^\alpha)$, cf. (4.12). The tensors $g^{\mu\nu}_{(\alpha)}$ transform contravariantly under Lorentz boosts, $g'^{\mu\nu}_{(\alpha)} = g^{\kappa\lambda}_{(\alpha)}\Lambda^{(-1)\mu}_\kappa\Lambda^{(-1)\nu}_\lambda$ cf. [11] and Section 6.

We substitute the plane waves $A^\alpha_\mu = \hat{A}^\alpha_\mu e^{i(\mathbf{kx} - \omega t)} + \text{c.c.}$ and $j^\mu_{\Omega\alpha} = \hat{j}^\mu_{\Omega\alpha}e^{i(\mathbf{kx} - \omega t)} + \text{c.c.}$ into the linearized isotropic Lagrangian (5.1),

$$L = -\frac{1}{4}\frac{1}{\mu_{(\alpha)}}F^\alpha_{ml}F^\alpha_{ml} + \frac{1}{2}\varepsilon_{(\alpha)}F^\alpha_{0n}F^\alpha_{0n} - \frac{1}{2\xi}\left(A^\alpha_{i,i}/\mu_{(\alpha)} - \varepsilon_{(\alpha)}A^\alpha_{0,0}\right)^2 + A^\alpha_\mu j^\mu_{\Omega\alpha}, \tag{5.5}$$

and perform a time average over a period to find

$$L = -\hat{A}^\alpha_\mu\hat{A}^{\beta*}_\nu\hat{G}^{(-1)\mu\nu}_{\alpha\beta} + \hat{A}^\alpha_\mu\hat{j}^{\mu*}_{\Omega\alpha} + \hat{A}^{\alpha*}_\mu\hat{j}^\mu_{\Omega\alpha}, \tag{5.6}$$

where $\hat{G}^{(-1)\mu\nu}_{\alpha\beta}$ denotes the Gaussian kernel

$$\hat{G}^{(-1)00}_{\alpha\beta} = -\delta_{\alpha\beta}\varepsilon_{(\alpha)}\left(\mathbf{k}^2 - \frac{\varepsilon_{(\alpha)}}{\xi}\omega^2\right),$$
$$\hat{G}^{(-1)m0}_{\alpha\beta} = \delta_{\alpha\beta}\varepsilon_{(\alpha)}\left(\frac{1}{\xi\mu_{(\alpha)}} - 1\right)\omega k_m,$$
$$\hat{G}^{(-1)mn}_{\alpha\beta} = \frac{\delta_{\alpha\beta}}{\mu_{(\alpha)}}\left[\left(\mathbf{k}^2 - \varepsilon_{(\alpha)}\mu_{(\alpha)}\omega^2\right)\delta_{mn} + \left(\frac{1}{\xi\mu_{(\alpha)}} - 1\right)k_mk_n\right]. \tag{5.7}$$

Euler variation of L with respect to $\hat{A}^{\alpha*}_\mu$ gives $\hat{G}^{(-1)\mu\nu}_{\alpha\beta}\hat{A}^\alpha_\mu = \hat{j}^\nu_{\Omega\beta}$. The inverse kernel is the propagator $\hat{G}^{\alpha\beta}_{\mu\nu}$,

$$\hat{G}^{\alpha\beta}_{00} = -\frac{\delta^{\alpha\beta}}{\varepsilon_{(\alpha)}}\frac{\mathbf{k}^2 - \xi\mu^2_{(\alpha)}\varepsilon_{(\alpha)}\omega^2}{(\mathbf{k}^2 - \mu_{(\alpha)}\varepsilon_{(\alpha)}\omega^2)^2},$$
$$\hat{G}^{\alpha\beta}_{m0} = \delta^{\alpha\beta}\frac{\mu_{(\alpha)}(1 - \xi\mu_{(\alpha)})\omega k_m}{(\mathbf{k}^2 - \mu_{(\alpha)}\varepsilon_{(\alpha)}\omega^2)^2},$$
$$\hat{G}^{\alpha\beta}_{mn} = \frac{\delta^{\alpha\beta}\mu_{(\alpha)}}{\mathbf{k}^2 - \varepsilon_{(\alpha)}\mu_{(\alpha)}\omega^2}\left(\delta_{mn} + \frac{(\xi\mu_{(\alpha)} - 1)k_mk_n}{\mathbf{k}^2 - \varepsilon_{(\alpha)}\mu_{(\alpha)}\omega^2}\right). \tag{5.8}$$

At $\xi = 0$, this propagator satisfies the Lorentz condition $g^{\mu\nu}_{A(\alpha)}k_\mu\hat{G}^{\alpha\beta}_{\nu\lambda} = 0$, where $k_\mu = (-\omega, \mathbf{k})$. A frequency-dependent gauge parameter is also admissible, which can even be different for each gauge component α; for instance, we may substitute $\xi \to \xi_{(\alpha)} = 1/\mu_{(\alpha)}$, which diagonalizes the propagator. Observable gauge-invariant quantities such as the linear inductions $H^k_\alpha = \varepsilon^{kij} A^\alpha_{j,i}/\mu_{(\alpha)}$ and $D^\alpha_k = \varepsilon_{(\alpha)}(A^\alpha_{0,k} - A^\alpha_{k,0})$ obtained from the solution $\hat{A}^\alpha_\mu = \hat{G}^{\alpha\beta}_{\mu\nu} \hat{j}^\nu_{\Omega\beta}$ are independent of the gauge parameter ξ, and the same holds true for the field strengths $E^\alpha_k = D^\alpha_k/\varepsilon_{(\alpha)}$ and $B^k_\alpha = \mu_{(\alpha)} H^k_\alpha$:

$$E^\alpha_n = \hat{E}^\alpha_n e^{i(\mathbf{kx}-\omega t)} + \text{c.c.}, \qquad \hat{E}^\alpha_n = i\frac{\omega\mu_{(\alpha)}\varepsilon_{(\alpha)}\hat{j}_{\Omega\alpha n} - k_n \hat{j}^0_{\Omega\alpha}}{\varepsilon_{(\alpha)}(\mathbf{k}^2 - \mu_{(\alpha)}\varepsilon_{(\alpha)}\omega^2)},$$

$$B^n_\alpha = \hat{B}^n_\alpha e^{i(\mathbf{kx}-\omega t)} + \text{c.c.}, \qquad \hat{B}^n_\alpha = \frac{i\mu_{(\alpha)}\varepsilon^{nij} k_i \hat{j}_{\Omega\alpha j}}{\mathbf{k}^2 - \varepsilon_{(\alpha)}\mu_{(\alpha)}\omega^2}. \tag{5.9}$$

A more compact representation of kernel (5.7) and propagator (5.8) is

$$\hat{G}^{(-1)\mu\nu}_{\alpha\beta} = \delta_{\alpha\beta}\left[g^{\mu\nu}_{(\alpha)} g^{\kappa\lambda}_{(\alpha)} k_\kappa k_\lambda + \left(\frac{1}{\xi\mu_{(\alpha)}} - 1\right) g^{\mu\kappa}_{(\alpha)} g^{\nu\lambda}_{(\alpha)} k_\kappa k_\lambda\right], \tag{5.10}$$

$$\hat{G}^{\alpha\beta}_{\mu\nu} = \frac{\delta^{\alpha\beta}}{g^{\kappa\lambda}_{(\alpha)} k_\kappa k_\lambda}\left[g^{(-1)}_{(\alpha)\mu\nu} + (\xi\mu_{(\alpha)} - 1)\frac{k_\mu k_\nu}{g^{\kappa\lambda}_{(\alpha)} k_\kappa k_\lambda}\right], \tag{5.11}$$

where $g^{(-1)}_{(\alpha)\mu\nu}$ denotes the inverse of the diagonal matrix $g^{\mu\nu}_{(\alpha)} = \mu^{1/2}_{(\alpha)} g^{\mu\nu}_{A(\alpha)}$, cf. (5.2) and (5.4). We can replace $g^{\mu\nu}_{A(\alpha)} \to g^{\mu\nu}_{(\alpha)}$ in the gauge-fixing term of Lagrangian (5.1), so that the inductive field is $C^\mu_\alpha = g^{\mu\nu}_{(\alpha)} A^\alpha_\nu$. This amounts to a conformal rescaling $\xi \to \xi/\mu_{(\alpha)}$ of the gauge parameter in propagator (5.11), which becomes manifestly covariant in this gauge, $g^{\mu\nu}_{(\alpha)} C^\kappa_{\alpha,\kappa,\mu,\nu} + \xi j^\mu_{\Omega\alpha,\mu} = 0$, cf. (4.8) and after (5.8).

We specify the permeabilities as $\varepsilon_{(\alpha)}\mu_{(\alpha)} \sim 1 - (m^2_{(\alpha)} - i\tilde{\varepsilon}_{(\alpha)})/\omega^2$, with positive infinitesimal constants $\tilde{\varepsilon}_{(\alpha)} \to 0$ regularizing the denominators of $\hat{G}^{\alpha\beta}_{\mu\nu}$ in (5.8) as $\mathbf{k}^2 - \omega^2 + m^2_{(\alpha)} - i\tilde{\varepsilon}_{(\alpha)}$. The wave numbers thus admit an infinitesimal imaginary part and a massive dispersion relation $k_{(\alpha)} \sim \sqrt{\omega^2 - m^2_{(\alpha)}} + i/(2\delta_{(\alpha)})$, where $\delta_{(\alpha)} \sim \sqrt{\omega^2 - m^2_{(\alpha)}}/\tilde{\varepsilon}_{(\alpha)}$ is the diverging attenuation length of the field component α in the infinitesimally dissipative aether [20]. To find the lifetime associated with this attenuation length, we put $\omega = E - i\Gamma_{(\alpha)}(E)/2$, and determine the decay rate $\Gamma_{(\alpha)}$ by solving $\operatorname{Im} k_{(\alpha)}(\omega) = 0$, obtaining $\Gamma_{(\alpha)} \sim \tilde{\varepsilon}_{(\alpha)}/E$ in linear order in the infinitesimal $\tilde{\varepsilon}_{(\alpha)}$. In this way, one can switch between complex space and time components of the plane-wave phase, using either $k_{(\alpha)}\mathbf{k}_0\mathbf{x} - \omega t$ or $\sqrt{E^2 - m^2_{(\alpha)}}\mathbf{k}_0\mathbf{x} - (E - i\tilde{\varepsilon}_{(\alpha)}/(2E))t$ (where we may again write ω for E). Thus the squared wave function scales with $\exp(-\mathbf{k}_0\mathbf{x}/\delta_{(\alpha)})$ or $\exp(-\tilde{\varepsilon}_{(\alpha)}t/\omega)$. The lifetime $1/\Gamma_{(\alpha)} \sim \omega/\tilde{\varepsilon}_{(\alpha)}$ of a mode in the aether is related to its attenuation length by $\delta_{(\alpha)} \sim \upsilon_{\text{gr}(\alpha)}/\Gamma_{(\alpha)}$, where $\upsilon_{\text{gr}(\alpha)} = \sqrt{\omega^2 - m^2_{(\alpha)}}/\omega$ is the group velocity. At $\tilde{\varepsilon}_{(\alpha)} = 0$, the permeabilities are real and symmetric on the real axis, satisfying $\varepsilon^*_{(\alpha)}(\omega) = \varepsilon_{(\alpha)}(-\omega^*)$ and $\mu^*_{(\alpha)}(\omega) = \mu_{(\alpha)}(-\omega^*)$ in the complex plane. The antipropagator $\hat{G}^{\alpha\beta*}_{\mu\nu}$ is found by $\delta\hat{A}^\alpha_\mu$ variation of L in (5.6).

To obtain the Dirac propagator, we start with the Lagrangian of a spinor multiplet ψ_j,

$$L = \frac{1}{i}\left(\frac{1}{2}\bar{\psi}_j \gamma_\mu g^{\mu\nu}_{(j)} \psi_{j,\nu} - \frac{1}{2}\bar{\psi}_{j,\mu} g^{\mu\nu}_{(j)} \gamma_\nu \psi_j + m_{(j)}\bar{\psi}_j \psi_j\right), \tag{5.12}$$

where $\bar{\psi}_j = -\psi^\dagger_j \gamma_0$ is the adjoint spinor, and $g^{\mu\nu}_{(j)}(\omega)$ the permeability tensor with components $g^{00}_{(j)} = -\varepsilon_{(j)}$, $g^{mn}_{(j)} = \delta^{mn}/\mu_{(j)}$, $g^{0n}_{(j)} = 0$, see after (2.1). The Dirac equation reads $g^{\mu\nu}_{(j)}\gamma_\mu \psi_{j,\nu} + m_{(j)}\psi_j = 0$, where j is a multi-index labeling flavor, color, etc. We write j in parentheses if it is not meant as a summation index, cf. after (5.1). On substituting plane waves $\psi_j = \hat{\psi}_j e^{i(\mathbf{kx}-\omega t)}$, we find $L = -i\bar{\hat{\psi}}_i \hat{G}^{(-1)}_{ij} \hat{\psi}_j$, with kernel $\hat{G}^{(-1)}_{ij} = \delta^{ij}(i g^{\mu\nu}_{(j)} k_\mu \gamma_\nu + m_{(j)})$. To invert $\hat{G}^{(-1)}_{ij}$, we note $\gamma_\mu\gamma_\nu = \eta_{\mu\nu} + \sigma_{\mu\nu}$, where $\sigma_{\mu\nu}$ is antisymmetric, so that the product of $i g^{\mu\nu}_{(j)} k_\mu \gamma_\nu \pm m_{(j)}$ is $-(h^{\alpha\beta}_{(j)} k_\alpha k_\beta + m^2_{(j)})$, with the squared permeability tensor $h^{\alpha\beta}_{(j)} = g^{\alpha\mu}_{(j)} g^{\beta\nu}_{(j)} \eta_{\mu\nu}$ [10]. Hence,

$$\hat{G}^{ij} = -\delta^{ij}\frac{i g^{\mu\nu}_{(j)} k_\mu \gamma_\nu - m_{(j)}}{h^{\alpha\beta}_{(j)} k_\alpha k_\beta + m^2_{(j)}} = -i\delta^{ij}\mu_{(j)}\omega\frac{\varepsilon_{(j)}\mu_{(j)}\gamma_0 + \gamma_n k_n/\omega + i\mu_{(j)} m_{(j)}/\omega}{\mathbf{k}^2 - \varepsilon^2_{(j)}\mu^2_{(j)}\omega^2 + \mu^2_{(j)} m^2_{(j)}}, \tag{5.13}$$

where we used the isotropic permeability tensor $g^{\mu\nu}_{(j)}$ stated after (5.12). We note that $k_n/\omega = \mathbf{k}_0 \varepsilon_{(j)}\mu_{(j)}(1 + O(m^2_{(j)}/\omega^2))$, cf. (2.1).

We drop all mass terms in (5.12) and (5.13), that is we use the massless Dirac equation, cf. after (5.12), and specify the refractive indices $n_{r(j)} = \varepsilon_{(j)}\mu_{(j)}$ of the multiplet ψ_j as, cf. (3.4),

$$n_{r(j)} = \varepsilon_{(j)}\mu_{(j)} = \varepsilon(\omega)\mu(\omega)\sqrt{1 - \frac{m^2_{(j)} - i\tilde{\varepsilon}_{(j)}}{\varepsilon^2(\omega)\omega^2}}. \tag{5.14}$$

We also put $\mu_{(j)} = \mu(\omega)$ and assume $\varepsilon(\omega)$ and $\mu(\omega)$ to be close to one, infinitesimal $\tilde{\varepsilon}_{(j)}$, as well as $m_{(j)}/\omega \ll 1$, so that the permeability tensor $g^{\mu\nu}_{(j)}$ (stated after (5.12)) is close to the Minkowski metric $\eta^{\mu\nu}$. We thus find the propagator $\hat{G}^{ij}_0$ of the massless Dirac equation as, cf. (5.13),

$$\hat{G}^{ij}_0 = -\delta^{ij}\frac{i g^{\mu\nu}_{(j)} k_\mu \gamma_\nu}{h^{\alpha\beta}_{(j)} k_\alpha k_\beta} = -\delta^{ij}\frac{i\mu\omega(\gamma_0 n_{r(j)} + \gamma_n k_n/\omega)}{\mathbf{k}^2 - \mu^2(\varepsilon^2\omega^2 - m^2_{(j)} + i\tilde{\varepsilon}_{(j)})}. \tag{5.15}$$

In the nominator of (5.15), we can put $\tilde{\varepsilon}_{(j)} = 0$ in $n_{r(j)}$. The $\tilde{\varepsilon}_{(j)}$ are positive infinitesimal constants defining the Green function (here the propagator), i.e. the integration path around its pole; the complex wave numbers $k_{(j)} = \omega n_{r(j)}$ result in infinitesimally damped wave fields, cf. before (5.12).

In propagator $\hat{G}^{ij}$ of the massive Dirac equation, cf. (5.13), we put $\varepsilon_{(j)} = \varepsilon(\omega)$, $\mu_{(j)} = \mu(\omega)$, and use the integration path prescription $i\tilde{\varepsilon}_{(j)}$ as in (5.15). Since the neutrino mass is much smaller than the neutrino energy, cf. the end of Section 3, we can drop the $i\mu_{(j)} m_{(j)}/\omega$ term in (5.13) and expand $n_{r(j)} = \varepsilon\mu(1 + O(m^2_{(j)}/\omega^2))$ in (5.14), so that the propagators (5.13) and (5.15) coincide in leading order in an $m_{(j)}/\omega$ expansion. The mass-square determining the pole singularity of $\hat{G}^{ij}$ is recovered in propagator $\hat{G}^{ij}_0$ without invoking a mass term in the Dirac Lagrangian (5.12).

6. Dirac equation and permeability tensor in moving inertial frames: Michelson–Morley experiments with neutrino beams

We consider an inertial frame uniformly moving in the aether with velocity $\boldsymbol{\upsilon}_r = \upsilon_r \boldsymbol{\upsilon}_{r,0}$. Unit vectors are denoted by a zero subscript. The spacetime coordinates in the rest frame of the aether (aether frame) are denoted by $x^\mu = (t, \mathbf{x})$ and in the inertial frame by $y^\mu = (\tau, \mathbf{y})$. The proper orthochronous Lorentz boost $x^\mu = \Lambda^\mu_\nu y^\nu$ relating the frames is

$$\Lambda^0_0 = \gamma_r, \qquad \Lambda^0_k = \sqrt{\gamma^2_r - 1}\,\boldsymbol{\upsilon}^k_{r,0}, \qquad \Lambda^i_0 = \sqrt{\gamma^2_r - 1}\,\boldsymbol{\upsilon}^i_{r,0},$$
$$\Lambda^i_k = \delta_{ik} + (\gamma_r - 1)\boldsymbol{\upsilon}^i_{r,0}\boldsymbol{\upsilon}^k_{r,0}, \qquad \gamma_r = \left(1 - \upsilon^2_r\right)^{-1/2}. \tag{6.1}$$

The inverse transformation $y^\mu = \Lambda_\nu^{(-1)\mu} x^\nu$ is found by changing the sign of the relative velocity $\boldsymbol{\upsilon}_{\mathrm{r},0}$. Contravariant quantities transform from the aether frame to a moving inertial frame like $g^{\mu\nu}_{\mathrm{inertial}} = g^{\alpha\beta}_{\mathrm{aether}} \Lambda_\alpha^{(-1)\mu} \Lambda_\beta^{(-1)\nu}$, covariant ones like $g^{\mathrm{in}}_{\mu\nu} = g^{\mathrm{ae}}_{\alpha\beta} \Lambda^\alpha_\mu \Lambda^\beta_\nu$, and mixed quantities according to this pattern. Indices are raised and lowered with the invariant Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. Energy and momentum variables are denoted by $k_\mu = (-\omega, \mathbf{k})$ in the aether frame and by $p_\mu = (-E, \mathbf{p})$ in the inertial frame, and $p_\mu = k_\alpha \Lambda^\alpha_\mu$.

In the aether frame, the Dirac equation (1.1) reads

$$\left(\gamma_\mu g^{\mu\nu}_{\mathrm{ae}}(\omega) \frac{\partial}{\partial x^\nu} + m\right) \psi_{\mathrm{ae}}(\omega; x) = 0, \tag{6.2}$$

coupled to the isotropic permeability tensor $g^{\mu\nu}_{\mathrm{ae}}(\omega)$ in (1.2). We consider plane waves $\psi_{\mathrm{ae}}(\omega; x) = \tilde{\psi}_{\mathrm{ae}} \mathrm{e}^{\mathrm{i}k_\mu x^\mu}$, where $\tilde{\psi}_{\mathrm{ae}}$ is a constant 4-spinor. The wave vector $k_\mu = (-\omega, \mathbf{k})$, $\mathbf{k} = k(\omega)\mathbf{k}_0$, satisfies the dispersion relation (2.1), $k^2 = \mu^2(\varepsilon^2\omega^2 - m^2)$.

In the moving inertial frame related by the Lorentz boost (6.1) to the aether frame, the permeability tensor is anisotropic, $g^{\mu\nu}_{\mathrm{in}}(\omega) = g^{\alpha\beta}_{\mathrm{ae}}(\omega) \Lambda_\alpha^{(-1)\mu} \Lambda_\beta^{(-1)\nu}$. The tensors $g^{\mu\nu}_{\mathrm{ae,in}}(\omega)$ are independent of space and time coordinates; they only depend on the frequency ω of the wave modes in the aether frame, which is the universal frame of reference. We denote the boosts (6.1) by $x = \Lambda(y)$ and $y = \Lambda^{-1}(x)$. To find the Dirac equation in the inertial frame, we write Eq. (6.2) as

$$\left(\gamma_\mu \Lambda^\mu_\alpha g^{\alpha\beta}_{\mathrm{in}}(\omega) \frac{\partial}{\partial y^\beta} + m\right) \psi_{\mathrm{ae}}\big(\omega; \Lambda(y)\big) = 0, \tag{6.3}$$

and consider a spinor field $\psi_{\mathrm{in}}(\omega; y)$ in the inertial frame, related to a plane-wave solution $\psi_{\mathrm{ae}}(\omega; x)$ of Eq. (6.2) by a similarity S_Λ, so that $\psi_{\mathrm{in}}(\omega; y) = S_\Lambda \psi_{\mathrm{ae}}(\omega; \Lambda(y))$ and $\psi_{\mathrm{ae}}(\omega; x) = S_\Lambda^{-1} \psi_{\mathrm{in}}(\omega; y)$. We may thus write Eq. (6.3) as

$$\left(S_\Lambda \gamma_\mu S_\Lambda^{-1} \Lambda^\mu_\alpha g^{\alpha\beta}_{\mathrm{in}}(\omega) \frac{\partial}{\partial y^\beta} + m\right) \psi_{\mathrm{in}}(\omega; y) = 0. \tag{6.4}$$

By choosing the matrix S_Λ as a solution of $S_\Lambda \gamma_\mu S_\Lambda^{-1} = \gamma_\alpha \Lambda_\mu^{(-1)\alpha}$,

$$S_\Lambda = \frac{1}{\sqrt{2}\sqrt{\gamma_{\mathrm{r}} - 1}} \left(\sqrt{\gamma_{\mathrm{r}}^2 - 1} + (1 - \gamma_{\mathrm{r}}) \gamma_0 \gamma_i \boldsymbol{\upsilon}^i_{\mathrm{r},0}\right), \tag{6.5}$$

where γ_{r} denotes the Lorentz factor, cf. (6.1), we find the Dirac equation in the inertial frame,

$$\left(\gamma_\alpha g^{\alpha\beta}_{\mathrm{in}}(\omega) \frac{\partial}{\partial y^\beta} + m\right) \psi_{\mathrm{in}}(\omega; y) = 0. \tag{6.6}$$

The inverse similarity S_Λ^{-1} is obtained by changing the sign of the velocity unit vector in (6.5), so that $S_\Lambda^{-1} = S_{\Lambda^{-1}}$. The Dirac matrices in (6.5) satisfy $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\eta_{\mu\nu}$, where γ_0 is anti-Hermitian and the γ_n and S_Λ are Hermitian, and $\gamma_0 S_\Lambda = S_\Lambda^{-1} \gamma_0$. A plane-wave solution $\psi_{\mathrm{ae}}(\omega; x) = \tilde{\psi}_{\mathrm{ae}} \mathrm{e}^{\mathrm{i}k_\mu x^\mu}$ of (6.2) in the aether frame is transformed into $\psi_{\mathrm{in}}(\omega; y) = \tilde{\psi}_{\mathrm{in}} \mathrm{e}^{\mathrm{i}p_\mu y^\mu}$ solving (6.6), where $\tilde{\psi}_{\mathrm{in}} = S_\Lambda \tilde{\psi}_{\mathrm{ae}}$. The dispersion relations in the aether frame and the inertial frame can be written as $h^{\mu\nu}_{\mathrm{ae}} k_\mu k_\nu + m^2 = 0$ and $h^{\mu\nu}_{\mathrm{in}} p_\mu p_\nu + m^2 = 0$, respectively, where $h^{\mu\nu}_{\mathrm{ae,in}}(\omega) = g^{\mu\alpha}_{\mathrm{ae,in}} \eta_{\alpha\beta} g^{\beta\nu}_{\mathrm{ae,in}}$, cf. (5.13). The Dirac equation in (6.2) and (6.6) has a manifestly covariant appearance as well, but is not covariant at all, since the anisotropic permeability tensor $g^{\mu\nu}_{\mathrm{in}}(\omega)$ of the inertial frame depends on the energy ω of the wave field in the aether frame, which is the universal frame of reference; $g^{\mu\nu}_{\mathrm{in}}$ is obtained by applying a Lorentz boost to the isotropic reference tensor $g^{\mu\nu}_{\mathrm{ae}}$ of the aether frame.

As for Michelson–Morley experiments, we consider neutrinos propagating with constant (group) velocity $\boldsymbol{\upsilon}_{\mathrm{gr}} = \upsilon_{\mathrm{gr}} \boldsymbol{\upsilon}_{\mathrm{gr},0}$ in the aether frame where the permeability tensor is isotropic. An inertial laboratory frame moves with constant relative speed $\boldsymbol{\upsilon}_{\mathrm{r}} = \upsilon_{\mathrm{r}} \boldsymbol{\upsilon}_{\mathrm{r},0}$ in the aether. The frames are related by the Lorentz boost (6.1). In the context of neutrino velocity experiments [6–9], the inertial frame is the neutrino baseline frame (rest frame of neutrino source and detector), slowly moving with a relative velocity of order $\upsilon_{\mathrm{r}} = \mathrm{O}(10^{-3})$ in the aether, cf. the discussion after (6.10). In the moving inertial frame, the neutrino velocity is denoted by $\boldsymbol{\upsilon}_{\mathrm{in}} = \upsilon_{\mathrm{in}} \boldsymbol{\upsilon}_{\mathrm{in},0}$ and related to the group velocity in the aether frame by the relativistic addition law (linearized in the relative speed υ_{r}),

$$\begin{aligned} \upsilon_{\mathrm{in}} &\sim \upsilon_{\mathrm{gr}} \left(1 + \left(\upsilon_{\mathrm{gr}}^2 - 1\right) \frac{\upsilon_{\mathrm{r}}}{\upsilon_{\mathrm{gr}}} \cos\theta_{\mathrm{gr}}\right), \\ \cos\theta_{\mathrm{in}} &\sim \cos\theta_{\mathrm{gr}} - \frac{\upsilon_{\mathrm{r}}}{\upsilon_{\mathrm{gr}}} \sin^2\theta_{\mathrm{gr}}, \end{aligned} \tag{6.7}$$

and inversely,

$$\begin{aligned} \upsilon_{\mathrm{gr}} &\sim \upsilon_{\mathrm{in}} \left(1 - \left(\upsilon_{\mathrm{in}}^2 - 1\right) \frac{\upsilon_{\mathrm{r}}}{\upsilon_{\mathrm{in}}} \cos\theta_{\mathrm{in}}\right), \\ \cos\theta_{\mathrm{gr}} &\sim \cos\theta_{\mathrm{in}} + \frac{\upsilon_{\mathrm{r}}}{\upsilon_{\mathrm{in}}} \sin^2\theta_{\mathrm{in}}, \end{aligned} \tag{6.8}$$

where $\cos\theta_{\mathrm{gr}} = \boldsymbol{\upsilon}_{\mathrm{gr},0} \boldsymbol{\upsilon}_{\mathrm{r},0}$ and $\cos\theta_{\mathrm{in}} = \boldsymbol{\upsilon}_{\mathrm{in},0} \boldsymbol{\upsilon}_{\mathrm{r},0}$, which applies in slowly moving inertial frames, $\upsilon_{\mathrm{r}} \ll 1$. The velocities υ_{gr} and υ_{in} are either both sub- or superluminal, which can be seen from the exact velocity addition law [10]. The transformation (6.7) and its inverse (6.8) are complemented by the Doppler shift connecting the neutrino energy E measured in the inertial frame to the energy ω of the wave mode in the aether frame according to $p_\mu = k_\alpha \Lambda^\alpha_\mu$, cf. after (6.1). The Doppler shift (linearized in υ_{r}) and its inverse read

$$\begin{aligned} E &\sim \omega\big(1 - n_{\mathrm{r}}(\omega) \upsilon_{\mathrm{r}} \cos\theta_{\mathrm{gr}}\big), \\ \omega &\sim E\big(1 + n_{\mathrm{r}}(E) \upsilon_{\mathrm{r}} \cos\theta_{\mathrm{in}}\big), \end{aligned} \tag{6.9}$$

where $n_{\mathrm{r}}(\omega)$ is the refractive index

$$n_{\mathrm{r}}(\omega) = \frac{k(\omega)}{\omega} = \mu\varepsilon \sqrt{1 - \frac{m^2}{\varepsilon^2\omega^2}}, \qquad \frac{1}{\upsilon_{\mathrm{gr}}} = \big(\omega n_{\mathrm{r}}(\omega)\big)', \tag{6.10}$$

defined by the dispersion relation in the aether frame, cf. after (6.2). $n_{\mathrm{r}}(E)$ in (6.9) means $n_{\mathrm{r}}(\omega)$ taken at $\omega = E$. (Terms quadratic in υ_{r} are neglected in (6.9).) The Doppler shift from the aether frame to a moving inertial frame depends not only on the relative velocity but also on the refractive index, which is a function of the neutrino frequency ω in the aether frame. Due to the dispersive permeabilities, the neutrino group velocity $\upsilon_{\mathrm{gr}}(\omega)$ is energy dependent even for massless neutrinos.

The group velocity υ_{gr} in the aether frame is obtained via transformation (6.8), once we have measured the neutrino velocity υ_{in} in the inertial laboratory frame and the angle $\cos\theta_{\mathrm{in}}$ between inertial velocity and relative speed. Assuming adiabatic frequency variation of the refractive index, $\omega n_{\mathrm{r}}'(\omega)/n_{\mathrm{r}} \ll 1$, we can identify $n_{\mathrm{r}} \sim 1/\upsilon_{\mathrm{gr}}$, cf. (6.10). The neutrino frequency ω in the aether frame is then found by measuring the neutrino energy E in the inertial frame, $\omega/E \sim 1 + (\upsilon_{\mathrm{r}}/\upsilon_{\mathrm{gr}}) \cos\theta_{\mathrm{in}}$, cf. (6.9). Adiabatic variation of $n_{\mathrm{r}}(\omega)$ requires adiabatic permeabilities, $\omega\varepsilon'(\omega)/\varepsilon \ll 1$ and $\omega\mu'(\omega)/\mu \ll 1$, as well as a small neutrino mass, $m/\omega \ll 1$, cf. (6.10).

The neutrino velocity υ_{in} in the laboratory frame depends on the angle $\cos\theta_{\mathrm{in}}$ between the neutrino velocity and the relative velocity of the inertial frame in the aether, cf. (6.7). The minimal (maximal) neutrino velocity υ_{in} is attained at $\theta_{\mathrm{in}} = 0$ or $\theta_{\mathrm{in}} = \pi$, when the velocity vectors are aligned or opposite, depending on whether the refractive index $n_{\mathrm{r}} \sim 1/\upsilon_{\mathrm{gr}}$ is larger or smaller than one (sub- or superluminal group velocity in the aether frame).

The maximal variation of the inertial neutrino speed in the baseline frame is thus $\delta\upsilon_{\rm in} \sim 2\upsilon_{\rm r}(1/n_{\rm r}^2 - 1)$, cf. (6.7). If the neutrino beam can arbitrarily be rotated and the neutrino velocity $\upsilon_{\rm in}$ is measured as a function of the angles parametrizing the rotation, one can in this way determine the relative speed $\boldsymbol{\upsilon}_{\rm r}$ of the inertial frame (here the local baseline rest frame of source and detector) in the aether, as attempted in the optical Michelson–Morley experiment. At $\theta_{\rm in} = \pi/2$, when the velocities $\boldsymbol{\upsilon}_{\rm r}$ and $\boldsymbol{\upsilon}_{\rm in}$ are orthogonal, the measured inertial neutrino speed $\upsilon_{\rm in}$ coincides with the neutrino group velocity $\upsilon_{\rm gr} \sim 1/n_{\rm r}$ in the aether frame (that is, the linear order in the relative speed $\upsilon_{\rm r}$ vanishes, cf. (6.8)). The OPERA Collaboration derived the bound $\upsilon_{\rm in} - 1 = (2.7 \pm 6.5) \times 10^{-6}$ on the neutrino velocity [6]. To illustrate the angular dependence of this velocity, we use $\upsilon_{\rm in} - 1 \approx 2.7 \times 10^{-6}$ and the relative speed $\upsilon_{\rm r} \approx 1.2 \times 10^{-3}$ of the Solar system barycenter in the cosmic microwave background. (Earth's relative velocities in the Solar system are by at least one order smaller.) We note $\upsilon_{\rm gr} - 1 = (\upsilon_{\rm in} - 1)(1 + {\rm O}(\upsilon_{\rm r}))$, cf. (6.8), and find a maximal angular variation of $\delta\upsilon_{\rm in} \approx 4\upsilon_{\rm r}(\upsilon_{\rm in} - 1) \approx 1.3 \times 10^{-8}$ for the neutrino velocity in the inertial frame, which is still by three orders smaller than the accuracy of current $\upsilon_{\rm in}$ measurements [7–9,26]. The isotropic aether frame is identified as the rest frame of the microwave background defined by vanishing temperature dipole anisotropy [12,13].

7. Conclusion

We demonstrated that effective mass-squares can be generated by permeability tensors in spinor as well as gauge fields, for fermions and bosons alike, familiar from the electrodynamics of dispersive dielectric media. This analogy is evident in each step, from the Lagrangians to the propagators, cf. Sections 4 and 5. Massless gauge fields coupled to the aether by dispersive permeability tensors admit massive dispersion relations preserving the gauge invariance. Mass generation by a permeability tensor is particularly attractive in the case of neutrino mixing, when small masses in the sub-eV range are needed [27,28], since such masses are generated by permeability tensors close to the Minkowski metric, cf. Sections 2 and 5. Interference factors in the flavor transition probabilities are due to phase differences of the mass-eigenstate plane waves, which result in oscillations if the neutrino path length is sufficiently large. All neutrino flavors propagate at the same group velocity, $\upsilon_{\rm gr} \approx 1/(\varepsilon\mu)$, as the refractive indices of high-energy mass eigenstates only differ by negligible terms of order $m_{(j)}^2/\omega^2$, cf. Section 3. Sub- and superluminal neutrino velocities close to the speed of light can be described on equal footing by permeability tensors; the permeabilities defining the dispersion relations are slightly larger than one in the case of subluminal neutrino speeds and smaller than one for superluminal neutrinos. The neutrino velocity measured in the baseline frame (rest frame of neutrino source and detector) depends on the angle between the neutrino beam (baseline) and its relative velocity in the aether. This angular dependence can be used to locally determine the velocity of source and detector in the aether, cf. Section 6, without invoking the temperature anisotropy of the cosmic microwave background in the neutrino baseline frame.

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