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Decay kinematics in the aether: Pion decay, kaon decay, and superluminal Cherenkov radiation

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Abstract – Pion and kaon decay, $\pi \to \mu + \nu_\mu$, $K \to \mu + \nu_\mu$, into muons and GeV neutrinos is investigated with regard to a possible superluminal neutrino speed. Photonic Cherenkov emission by superluminal high-energy charges is shown to be forbidden by causality violation. A proper causality interpretation of decay processes outside the lightcone involving superluminal particles requires an absolute spacetime conception based on a distinguished frame of reference (aether frame). The universal reference frame is physically manifested as the rest frame of the cosmic microwave background (CMB) radiation. The propagation of particles and radiation modes in the CMB rest frame is determined by dispersive wave equations coupled to isotropic permeability tensors. The decay of ∼50 GeV pions and ∼85 GeV kaons generating the CERN neutrino beam to Gran Sasso (CNGS) is analyzed in this context. Causality constraints on the group velocity of the ∼17 GeV muon neutrinos produced in the decay are derived and compared to recent experimental bounds on the neutrino speed.

Introduction. – We investigate pion and kaon decay into muons and muon neutrinos, which is the source of the CNGS neutrino beam [1]. This is motivated by upper bounds on a superluminal neutrino velocity recently established by the OPERA [2,3], BOREXINO [4], LVD [5] and ICARUS [6] experiments. Outside the lightcone, a proper causality interpretation of decay processes requires an absolute spacetime conception, as the time order of spacelike connections established by superluminal signals can be overturned in the rest frames of the interacting subluminal constituents [7,8]. The absolute spacetime, the aether, is manifested by dispersive permeability tensors, which affect the wave propagation of particles and radiation modes, in particular their group velocity. The permeability tensors are isotropic in a distinguished frame of reference defined by vanishing temperature dipole anisotropy of the CMB [9–12]. The coupling of Dirac and gauge fields to frequency-dependent permeability tensors has been explained in [13,14]. Here, we focus on specific decay and emission processes outside the lightcone.

We study the decays $\pi \to \mu + \nu_\mu$ and $K \to \mu + \nu_\mu$ as well as the hypothetical Cherenkov emission of photons by superluminal charges [15–17]. Susceptibility functions are introduced, which allow to treat sub- and superluminal group velocities on equal footing and to develop the decay kinematics irrespectively of whether the particles and radiation quanta are sub- or superluminal. We obtain causality constraints on the group velocities in the CMB rest frame to be satisfied in addition to energy-momentum conservation. In the high-energy limit, these causality conditions can be made explicit as linear inequalities for the frequency-dependent susceptibility functions of the respective particles. These conditions are always met if all constituents of the interaction are subluminal, but they prohibit the often invoked [16,17] photonic Cherenkov radiation by superluminal charges.

Dispersive two-particle decay in a permeable spacetime: energy-momentum conservation, refractive indices and susceptibility functions. – We focus on pion and kaon decay as well as on Cherenkov radiation, but the formalism developed is applicable to two-particle decay in general. We start with energy-momentum conservation in the CMB rest frame (aether frame), $\omega_{\rm in} = \omega_{\rm out} + \omega_\nu$, $\mathbf{k}_{\rm in} = \mathbf{k}_{\rm out} + \mathbf{k}_\nu$, where $(\omega_{\rm in}, \mathbf{k}_{\rm in})$ denote the energy and momentum variables of the incoming pion or kaon, and $(\omega_{\rm out}, \mathbf{k}_{\rm out})$ the variables of the muon (or the outgoing pion in the case of Cherenkov

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radiation $\pi \to \pi + \gamma$). The frequency and wave vector of the neutrino (or photon) are denoted by $(\omega_\nu, \mathbf{k}_\nu)$. The outgoing momenta are split into longitudinal and transversal components,

$$\mathbf{k}_{\rm out} = \lambda_\parallel \mathbf{k}_{\rm in,0} + \lambda_\perp \mathbf{k}_{\perp,0}, \quad \mathbf{k}_\nu = \lambda_{\rm L} \mathbf{k}_{\rm in,0} - \lambda_\perp \mathbf{k}_{\perp,0}, \quad (1)$$

so that $\mathbf{k}_{\rm in,0}\mathbf{k}_{\perp,0} = 0$. Subscript zeros denote unit vectors, $\mathbf{k}_{\rm in} = k_{\rm in}(\omega_{\rm in})\mathbf{k}_{\rm in,0}$, $\mathbf{k}_{\rm out} = k_{\rm out}(\omega_{\rm out})\mathbf{k}_{\rm out,0}$, $\mathbf{k}_\nu = k_\nu(\omega_\nu)\mathbf{k}_{\nu,0}$, where the wave numbers are determined by dispersion relations. We square $\mathbf{k}_{\rm out}$ and $\mathbf{k}_\nu$ in (1), $\lambda_\parallel^2 + \lambda_\perp^2 = k_{\rm out}^2(\omega_{\rm out})$, $\lambda_{\rm L}^2 + \lambda_\perp^2 = k_\nu^2(\omega_\nu)$, and substitute $\omega_{\rm out} = \omega_{\rm in} - \omega_\nu$ into $k_{\rm out}(\omega_{\rm out})$. Subtracting these two equations, factorizing, and using momentum conservation $\lambda_{\rm L} + \lambda_\parallel = k_{\rm in}(\omega_{\rm in})$, we obtain

$$\begin{aligned} \lambda_{\rm L} &= \frac{k_{\rm in}^2 - k_{\rm out}^2 + k_\nu^2}{2k_{\rm in}}, \quad \lambda_\parallel = \frac{k_{\rm in}^2 + k_{\rm out}^2 - k_\nu^2}{2k_{\rm in}}, \\ \lambda_\perp^2 &= (k_\nu + \lambda_{\rm L})(k_\nu - \lambda_{\rm L}). \end{aligned} \quad (2)$$

We factorize the wave numbers of the in- and outgoing particles and the neutrino (or photon),

$$\begin{aligned} k_{\rm in} &= \omega_{\rm in} n_{\rm in}(\omega_{\rm in}), \quad k_{\rm out} = \omega_{\rm out} n_{\rm out}(\omega_{\rm out}), \\ k_\nu &= \omega_\nu n_\nu(\omega_\nu), \end{aligned} \quad (3)$$

where the refractive indices $n_{\rm in,out,\nu}$ read [8]

$$n_{\rm in} = \mu_{\rm in}\varepsilon_{\rm in}\sqrt{1 - \frac{m_{\rm in}^2}{\varepsilon_{\rm in}^2\omega_{\rm in}^2}}, \quad n_\nu = \mu_\nu\varepsilon_\nu\sqrt{1 - \frac{m_\nu^2}{\varepsilon_\nu^2\omega_\nu^2}}, \quad (4)$$

and analogously for $n_{\rm out}$. The permeabilities $(\varepsilon_{\rm in}(\omega_{\rm in}), \mu_{\rm in}(\omega_{\rm in}))$, $(\varepsilon_{\rm out}(\omega_{\rm out}), \mu_{\rm out}(\omega_{\rm out}))$ and $(\varepsilon_\nu(\omega_\nu), \mu_\nu(\omega_\nu))$ are positive functions of the indicated variables, defining isotropic permeability tensors $h^{\alpha\beta}_{\rm in,out,\nu,}$ in the CMB rest frame [14],

$$h^{00}_{\rm in}(\omega_{\rm in}) = -\varepsilon_{\rm in}^2(\omega_{\rm in}), \quad h^{ik}_{\rm in}(\omega_{\rm in}) = \frac{\delta^{ik}}{\mu_{\rm in}^2(\omega_{\rm in})}, \quad h^{0k}_{\rm in} = 0, \quad (5)$$

and analogously for the tensors $h^{\alpha\beta}_{\rm out}(\omega_{\rm out})$ and $h^{\alpha\beta}_\nu(\omega_\nu)$. The subscript ν labels the neutrino or photon variables, and is not to be confused with a tensor index. The dispersion relations are derived from Klein-Gordon equations such as $(h^{\alpha\beta}_{\rm in}\partial_\alpha\partial_\beta - m_{\rm in}^2)\psi_{\rm in} = 0$, obtained by squaring the Dirac equation of the respective particle [8]. We find $h^{\alpha\beta}_{\rm in}k_{{\rm in},\alpha}k_{{\rm in},\beta} + m_{\rm in}^2 = 0$, $h^{\alpha\beta}_\nu k_{\nu,\alpha}k_{\nu,\beta} + m_\nu^2 = 0$, and similarly for $k_{{\rm out},\alpha}$, where $k_{{\rm in},\alpha} = (-\omega_{\rm in}, \mathbf{k}_{\rm in})$, $k_{{\rm out},\alpha} = (-\omega_{\rm out}, \mathbf{k}_{\rm out})$ and $k_{\nu,\alpha} = (-\omega_\nu, \mathbf{k}_\nu)$ are the 4-momenta of the in- and outgoing particles and the neutrino.

In high-energy interactions, where the speed of the sub- and superluminal particles is close to the speed of light, it is efficient to define susceptibility functions for each particle species, in analogy to dielectrics, which serve as expansion parameters. The electric and magnetic susceptibilities of the incoming particle are denoted by $\chi_{\rm e,in} = \varepsilon_{\rm in} - 1$ and $\chi_{\rm m,in} = \mu_{\rm in} - 1$, and analogously for the out-state. The neutrino (or photonic) susceptibilities are $\chi_{{\rm e},\nu}(\omega_\nu) = \varepsilon_\nu - 1$ and $\chi_{{\rm m},\nu}(\omega_\nu) = \mu_\nu - 1$. We expand the neutrino (photon) refractive index $n_\nu(\omega_\nu)$ in linear order in $\chi_{{\rm e},\nu}$, $\chi_{{\rm m},\nu}$ and m_ν^2/ω_ν^2, cf. (4), and analogously the refractive indices of the in- and out-states. This triple Taylor expansion is possible if the velocities of all particles involved are close to the speed of light, so that the permeability tensors (5) are close to the Minkowski metric $\eta^{\alpha\beta} = {\rm diag}(-1, 1, 1, 1)$, with electric and magnetic susceptibilities close to zero. In the case of pion and kaon decay, the squared mass/energy ratios in the GeV region are small as well, ensuring refractive indices close to 1 and thus small index variations

$$\delta n_{\rm in} = n_{\rm in} - 1 \sim \chi_{\rm in} - \frac{m_{\rm in}^2}{2\omega_{\rm in}^2}, \quad \delta n'_{\rm in} \sim \chi'_{\rm in} + \frac{m_{\rm in}^2}{\omega_{\rm in}^3}, \quad (6)$$

and analogously $\delta n_{\rm out}$ and δn_ν. Here, we expanded in linear order in the enumerated parameters, and defined the shortcut $\chi_{\rm in} = \chi_{\rm e,in} + \chi_{\rm m,in}$, and analogously for $\chi_{\rm out}$ and χ_ν. (That is, $n_{\rm in}$ in (6) is linearized in $\chi_{\rm e,in}$, $\chi_{\rm m,in}$ and $m_{\rm in}^2/\omega_{\rm in}^2$.) The index variations $\delta n_{\rm in,out,\nu}$ and susceptibilities $\chi_{\rm in,out,\nu}$ can have either sign, being small parameters close to zero like the squared mass/energy ratios. If not indicated otherwise, the energy dependence of the susceptibility functions is $\chi_{\rm in}(\omega_{\rm in})$, $\chi_\nu(\omega_\nu)$, and $\chi_{\rm out}(\omega_{\rm out})$, and similarly for $\delta n_{\rm in,out,\nu}$. The primes on $\delta n'_{\rm in,out,\nu}$ denote frequency derivatives; $\delta n'_{\rm in}$ in (6) stands for $(\delta n_{\rm in})'$ taken at $\omega_{\rm in}$.

We substitute the wave numbers (3) and $\omega_{\rm out} = \omega_{\rm in} - \omega_\nu$ into the longitudinal momentum coefficients (2),

$$\lambda_{{\rm L},\parallel} = \frac{\omega_{\rm in}^2 n_{\rm in}^2(\omega_{\rm in}) \mp (\omega_{\rm in} - \omega_\nu)^2 n_{\rm out}^2(\omega_{\rm out}) \pm \omega_\nu^2 n_\nu^2(\omega_\nu)}{2\omega_{\rm in} n_{\rm in}(\omega_{\rm in})}, \quad (7)$$

where the upper sign refers to $\lambda_{\rm L}$. The squared transversal coefficient $\lambda_\perp^2$ in (2) is the product of $(\omega_\nu n_\nu(\omega_\nu) \pm \lambda_{\rm L})$. We expand (7) in linear order in the index variations $\delta n_{\rm in,out,\nu}$, cf. (6),

$$\begin{aligned} \lambda_{\rm L} &\sim \omega_\nu + \omega_{\rm in}\left(1 - \frac{\omega_\nu}{\omega_{\rm in}}\right)\delta n_{\rm in} - \omega_{\rm in}\left(1 - \frac{\omega_\nu}{\omega_{\rm in}}\right)^2 \delta n_{\rm out} + \frac{\omega_\nu^2}{\omega_{\rm in}}\delta n_\nu, \\ \lambda_\parallel &\sim \omega_{\rm in}\left(1 - \frac{\omega_\nu}{\omega_{\rm in}}\right) + \omega_\nu\delta n_{\rm in} + \omega_{\rm in}\left(1 - \frac{\omega_\nu}{\omega_{\rm in}}\right)^2 \delta n_{\rm out} - \frac{\omega_\nu^2}{\omega_{\rm in}}\delta n_\nu, \\ \lambda_\perp^2 &\sim 2\omega_\nu(\omega_\nu n_\nu - \lambda_{\rm L}) \sim 2\omega_\nu\omega_{\rm in}\left(1 - \frac{\omega_\nu}{\omega_{\rm in}}\right)\Delta n, \end{aligned} \quad (8)$$

where Δn denotes the refractive-index increment

$$\Delta n = \left(1 - \frac{\omega_\nu}{\omega_{\rm in}}\right)\delta n_{\rm out} + \frac{\omega_\nu}{\omega_{\rm in}}\delta n_\nu - \delta n_{\rm in}. \quad (9)$$

The only approximation in (8) and (9) is linearization in the index variations $\delta n_{\rm in,out,\nu}$. The condition $\lambda_\perp^2 > 0$ thus means $\Delta n > 0$. All three frequencies in $\omega_{\rm out} = \omega_{\rm in} - \omega_\nu$ are positive, and positivity of the index increment Δn in (9) is a necessary condition for momentum conservation, cf. (8).

Decay angles and group velocities in the CMB rest frame. – To find the decay (or emission) angles, we write the wave vectors (1) as, cf. (3) and (6),

$$(\omega_{\rm in}-\omega_\nu)(1+\delta n_{\rm out})\mathbf{k}_{{\rm out},0}=\lambda_\|\mathbf{k}_{{\rm in},0}+\lambda_\perp\mathbf{k}_{\perp,0},\\ \omega_\nu(1+\delta n_\nu)\mathbf{k}_{\nu,0}=\lambda_{\rm L}\mathbf{k}_{{\rm in},0}-\lambda_\perp\mathbf{k}_{\perp,0}, \tag{10}$$

and multiply these identities by $\mathbf{k}_{{\rm in},0}$, using $\mathbf{k}_{{\rm in},0}\mathbf{k}_{\perp,0}=0$. Defining the decay angles by $\cos\theta_{\rm out}=\mathbf{k}_{{\rm in},0}\mathbf{k}_{{\rm out},0}$ and $\cos\theta_\nu=\mathbf{k}_{{\rm in},0}\mathbf{k}_{\nu,0}$, we obtain

$$\cos\theta_{\rm out}\sim\frac{\lambda_\|(1-\delta n_{\rm out})}{\omega_{\rm in}(1-\omega_\nu/\omega_{\rm in})}\sim1-\frac{\omega_\nu}{\omega_{\rm in}}\frac{\Delta n}{1-\omega_\nu/\omega_{\rm in}},\\ \cos\theta_\nu\sim\frac{\lambda_{\rm L}}{\omega_\nu}(1-\delta n_\nu)\sim1-\frac{\omega_{\rm in}}{\omega_\nu}\left(1-\frac{\omega_\nu}{\omega_{\rm in}}\right)\Delta n, \tag{11}$$

with the longitudinal momentum coefficients $\lambda_\|$ and $\lambda_{\rm L}$ in (8) and the refractive-index increment Δn in (9). The only approximation in (11) is systematic linearization in the index variations $\delta n_{{\rm in,out},\nu}$, cf. (6) and (9). Expanding the cosines, we find the decay angles

$$\theta_{\rm out}\sim\frac{\sqrt{2\Delta n}}{\sqrt{\omega_{\rm in}/\omega_\nu-1}},\qquad\theta_\nu\sim\left(\frac{\omega_{\rm in}}{\omega_\nu}-1\right)\theta_{\rm out}. \tag{12}$$

The ratio $\theta_\nu/\theta_{\rm out}$ does not depend on the refractive indices, and the angle between the wave vectors $\mathbf{k}_{{\rm out},0}$ and $\mathbf{k}_{\nu,0}$ of the outgoing particles is $\theta_\nu+\theta_{\rm out}\sim\theta_{\rm out}\omega_{\rm in}/\omega_\nu$. Thus,

$$\cos(\theta_\nu+\theta_{\rm out})\sim1-\frac{\omega_{\rm in}}{\omega_\nu}\frac{\Delta n}{1-\omega_\nu/\omega_{\rm in}}, \tag{13}$$

with $\cos(\theta_\nu+\theta_{\rm out})=\mathbf{k}_{{\rm out},0}\mathbf{k}_{\nu,0}$.

The particle velocities are group velocities obtained as reciprocal frequency derivative of the wave number (3): $1/\upsilon_\nu=n_\nu+\omega_\nu n_\nu'$, where $\boldsymbol{\upsilon}_\nu=\upsilon_\nu\mathbf{k}_{\nu,0}$, cf. after (1), and analogously for $\upsilon_{\rm in,out}$. We parametrize the absolute values $\upsilon_{{\rm in,out},\nu}$ with $n_{{\rm in,out},\nu}=1+\delta n_{{\rm in,out},\nu}$, cf. (6), and expand in linear order,

$$\upsilon_\nu\sim1-\delta n_\nu-\omega_\nu\delta n_\nu',\\ \upsilon_{\rm in,out}\sim1-\delta n_{\rm in,out}-\omega_{\rm in,out}\delta n_{\rm in,out}'. \tag{14}$$

The scalar products of the group velocities linearized in the index variations read

$$\boldsymbol{\upsilon}_\nu\boldsymbol{\upsilon}_{\rm in}=\upsilon_\nu\upsilon_{\rm in}\cos\theta_\nu\sim1-\Delta\upsilon_{\nu\times{\rm in}},\\ \boldsymbol{\upsilon}_{\rm out}\boldsymbol{\upsilon}_{\rm in}=\upsilon_{\rm out}\upsilon_{\rm in}\cos\theta_{\rm out}\sim1-\Delta\upsilon_{{\rm out}\times{\rm in}},\\ \boldsymbol{\upsilon}_{\rm out}\boldsymbol{\upsilon}_\nu=\upsilon_{\rm out}\upsilon_\nu\cos(\theta_\nu+\theta_{\rm out})\sim1-\Delta\upsilon_{{\rm out}\times\nu}, \tag{15}$$

where, cf. (9),

$$\Delta\upsilon_{\nu\times{\rm in}}=\delta n_\nu+\omega_\nu\delta n_\nu'+\delta n_{\rm in}+\omega_{\rm in}\delta n_{\rm in}'+\left(\frac{\omega_{\rm in}}{\omega_\nu}-1\right)\Delta n, \tag{16}$$

$$\Delta\upsilon_{{\rm out}\times{\rm in}}=\delta n_{\rm in}+\omega_{\rm in}\delta n_{\rm in}'+\delta n_{\rm out}+\omega_{\rm out}\delta n_{\rm out}'\\ +\frac{\omega_\nu}{\omega_{\rm in}}\frac{\Delta n}{1-\omega_\nu/\omega_{\rm in}}, \tag{17}$$

$$\Delta\upsilon_{{\rm out}\times\nu}=\delta n_\nu+\omega_\nu\delta n_\nu'+\delta n_{\rm out}+\omega_{\rm out}\delta n_{\rm out}'\\ +\frac{\omega_{\rm in}}{\omega_\nu}\frac{\Delta n}{1-\omega_\nu/\omega_{\rm in}}. \tag{18}$$

The causality constraints are $\boldsymbol{\upsilon}_i\boldsymbol{\upsilon}_j<1$, where the subscript i labels the subluminal velocities and j the superluminal ones in the aether frame [7]. These kinematic constraints are thus tantamount to positivity of the respective increments $\Delta\upsilon_{i\times j}$ of the velocity products in (16)–(18). For instance, if the pion velocity $\upsilon_{\rm in}$ is superluminal and the muon velocity $\upsilon_{\rm out}$ subluminal, the pion trajectory appears time inverted in the proper time of the muon if $\boldsymbol{\upsilon}_{\rm out}\boldsymbol{\upsilon}_{\rm in}>1$. In this case, the pion reemerges during the muon's proper lifetime, which is causality violating, as the pion was annihilated by decay at the time the muon was created. The velocity constraints $\boldsymbol{\upsilon}_i\boldsymbol{\upsilon}_j<1$ in the aether frame are necessary and sufficient to exclude causality violating time inversions in the rest frames of the subluminal particles. The velocities refer to asymptotic in- and out-states of the interacting particles and radiation modes. The inertial frames and proper times of subluminal in- and out-states are linked by Lorentz boosts to the aether frame [8], which is the universal frame of reference, manifested as the homogeneous and isotropic CMB rest frame [18].

Causality conditions on the susceptibility functions of the aether. – We parametrize the velocity increments $\Delta\upsilon_{i\times j}$ in (16)–(18) with the susceptibility functions (6), starting with

$$\Delta n\sim\left(1-\frac{\omega_\nu}{\omega_{\rm in}}\right)\chi_{\rm out}+\frac{\omega_\nu}{\omega_{\rm in}}\chi_\nu-\chi_{\rm in}\\ +\frac{1}{2}\left(\frac{m_{\rm in}^2}{\omega_{\rm in}^2}-\frac{m_{\rm out}^2/\omega_{\rm in}^2}{1-\omega_\nu/\omega_{\rm in}}-\frac{m_\nu^2}{\omega_{\rm in}\omega_\nu}\right), \tag{19}$$

where we invoked energy conservation $\omega_{\rm out}=\omega_{\rm in}(1-\omega_\nu/\omega_{\rm in})$. We find

$$\Delta\upsilon_{\nu\times{\rm in}}\sim\frac{\omega_{\rm in}}{\omega_\nu}\left(1-\frac{\omega_\nu}{\omega_{\rm in}}\right)^2\chi_{\rm out}+\left(2-\frac{\omega_\nu}{\omega_{\rm in}}\right)\chi_\nu\\ +\left(2-\frac{\omega_{\rm in}}{\omega_\nu}\right)\chi_{\rm in}+\omega_\nu\chi_\nu'+\omega_{\rm in}\chi_{\rm in}'\\ +\frac{m_{\rm in}^2-m_{\rm out}^2+m_\nu^2}{2\omega_{\rm in}\omega_\nu}, \tag{20}$$

$$\Delta\upsilon_{{\rm out}\times{\rm in}}\sim\left(1+\frac{\omega_\nu}{\omega_{\rm in}}\right)\chi_{\rm out}\\ +\frac{\omega_\nu^2/\omega_{\rm in}^2}{1-\omega_\nu/\omega_{\rm in}}\chi_\nu+\frac{1-2\omega_\nu/\omega_{\rm in}}{1-\omega_\nu/\omega_{\rm in}}\chi_{\rm in}\\ +\omega_{\rm in}\chi_{\rm in}'+\omega_{\rm out}\chi_{\rm out}'+\frac{m_{\rm in}^2+m_{\rm out}^2-m_\nu^2}{2\omega_{\rm in}^2(1-\omega_\nu/\omega_{\rm in})}. \tag{21}$$

As a consistency check, we may interchange the indices $\nu \leftrightarrow \mathrm{out}$ in (20), and use $\omega_{\rm out} = \omega_{\rm in} - \omega_\nu$ to recover (21). Similarly, cf. (18),

$$\Delta \upsilon_{\mathrm{out}\times\nu} \sim \left(1+\frac{\omega_{\rm in}}{\omega_\nu}\right)\chi_{\rm out} + \frac{2-\omega_\nu/\omega_{\rm in}}{1-\omega_\nu/\omega_{\rm in}}\chi_\nu - \frac{\omega_{\rm in}/\omega_\nu}{1-\omega_\nu/\omega_{\rm in}}\chi_{\rm in} + \omega_\nu\chi'_\nu + \omega_{\rm out}\chi'_{\rm out} + \frac{m_{\rm in}^2 - m_{\rm out}^2 - m_\nu^2}{2\omega_{\rm in}\omega_\nu(1-\omega_\nu/\omega_{\rm in})}. \tag{22}$$

By interchanging $\nu \leftrightarrow \mathrm{in}$, we recover (21).

For instance, the decay of a superluminal pion, $\pi \to \mu + \nu_\mu$, into a subluminal muon and a superluminal neutrino requires two kinematic causality conditions, $\Delta\upsilon_{\mathrm{out}\times\mathrm{in}} > 0$ and $\Delta\upsilon_{\mathrm{out}\times\nu} > 0$. The third condition, $\Delta\upsilon_{\nu\times\mathrm{in}} > 0$, need not be satisfied, as both the outgoing neutrino and the incoming pion are superluminal, so that neither of them admits a rest frame where a time inversion could occur. All causality conditions refer to group velocities in the CMB rest frame. The third condition for this decay is $\Delta n > 0$ in (19), required by momentum conservation, cf. (8). In brief, the causality condition $\Delta\upsilon_{i\times j} > 0$ has to be satisfied in the aether frame if one of the indices labels a superluminal particle or radiation mode and the other a subluminal one. If this condition is violated, the trajectory of the superluminal particle is time inverted in the rest frame of the subluminal particle, so that absorption happens prior to emission in the proper time of the subluminal particle.

Pion decay $\pi \to \mu + \nu_\mu$ generating the CNGS beam: constraints on the neutrino velocity. – The mass squares in the refractive indices (6) of pion and muon read $m_{\rm in}^2 = m_\pi^2 \approx 0.0195\,\mathrm{GeV}^2$ and $m_{\rm out}^2 = m_\mu^2 \approx 0.0112\,\mathrm{GeV}^2$ [19]. A neutrino mass of below 2 eV [20] in the neutrino refractive index is negligible in the GeV range, $m_\nu^2 \approx 0$, cf. (4) and (6). As for the CNGS neutrino beam, the energy of the incoming pions is about $\omega_{\rm in} \approx 50\,\mathrm{GeV}$ [1], and the average energy of the neutrinos $\omega_\nu \approx 17\,\mathrm{GeV}$ [2–6]. The neutrino refractive index (6) is $n_\nu(\omega_\nu) = 1 + \delta n_\nu$, with $\delta n_\nu = n_\nu - 1 \sim \chi_\nu$. The frequency derivatives of the susceptibilities are put to zero, $\chi'_{\nu,\mathrm{in,out}} \approx 0$. The pion refractive index is parametrized by $\delta n_{\rm in} \sim \chi_{\rm in} - m_{\rm in}^2/(2\omega_{\rm in}^2)$, with derivative $\delta n'_{\rm in} \sim m_{\rm in}^2/\omega_{\rm in}^3$, cf. (6), and analogously for the outgoing muon, $\delta n_{\rm out} \sim \chi_{\rm out} - m_{\rm out}^2/(2\omega_{\rm out}^2)$, and $\delta n'_{\rm out} \sim m_{\rm out}^2/\omega_{\rm out}^3$, taken at $\omega_{\rm out} = \omega_{\rm in} - \omega_\nu \approx 33\,\mathrm{GeV}$. The refractive-index increment (19) gives the constraint

$$\Delta n \sim 0.66\chi_{\rm out} + 0.34\chi_\nu - \chi_{\rm in} + 5.06\times 10^{-7} > 0, \tag{23}$$

required by energy-momentum conservation. The causality conditions for this decay are obtained from the velocity increments (20)–(22), with $\chi'_{\nu,\mathrm{in,out}} \approx 0$ and $m_\nu^2 \approx 0$:

$$\Delta\upsilon_{\nu\times\mathrm{in}} \sim 1.28\chi_{\rm out} + 1.66\chi_\nu - 0.94\chi_{\rm in} + 4.88\times 10^{-6} > 0, \tag{24}$$

$$\Delta\upsilon_{\mathrm{out}\times\mathrm{in}}\left(1-\frac{\omega_\nu}{\omega_{\rm in}}\right) \sim 1.34\chi_{\rm out} + 0.175\chi_\nu + 0.485\chi_{\rm in} + 9.30\times 10^{-6} > 0, \tag{25}$$

$$\Delta\upsilon_{\mathrm{out}\times\nu}\left(1-\frac{\omega_\nu}{\omega_{\rm in}}\right) \sim 2.60\chi_{\rm out} + 1.66\chi_\nu - 2.94\chi_{\rm in} + 4.88\times 10^{-6} > 0. \tag{26}$$

The susceptibilities refer to different frequencies, $\chi_{\rm out}(\omega_{\rm out})$, $\chi_\nu(\omega_\nu)$ and $\chi_{\rm in}(\omega_{\rm in})$, and the frequency variation is neglected, assuming vanishing derivatives at the respective energies. The neutrino group velocity and refractive index read, cf. (6) and (14),

$$\upsilon_\nu - 1 \sim -\frac{m_\nu^2}{2\omega_\nu^2} - \chi_\nu - \omega_\nu\chi'_\nu, \qquad 1 - n_\nu \sim \upsilon_\nu - 1 + \frac{m_\nu^2}{\omega_\nu^2} + \omega_\nu\chi'_\nu, \tag{27}$$

and analogously for the pionic and muonic group velocities $\upsilon_{\rm in,out}$ and their refractive indices $n_{\rm in,out}$. The susceptibilities $\chi_{\nu,\mathrm{in,out}}$ are close to zero and can have either sign, and the same holds true for $\upsilon_{\nu,\mathrm{in,out}} - 1$ and $1 - n_{\nu,\mathrm{in,out}}$ in (27). In deriving (27), we used $(\omega n_\nu)' = 1/\upsilon_\nu$. We also put $\chi'_{\nu,\mathrm{in,out}} \approx 0$ and $m_\nu^2 \approx 0$ as in (23)–(26), so that $1 - n_\nu \sim \upsilon_\nu - 1 \sim -\chi_\nu$. A neutrino index $n_\nu < 1$ or a negative susceptibility is thus tantamount to a superluminal neutrino speed $\upsilon_\nu > 1$. The pion and muon velocities are related to their susceptibilities by $\upsilon_{\rm in} - 1 \sim -3.90\times 10^{-6} - \chi_{\rm in}$ and $\upsilon_{\rm out} - 1 \sim -5.14\times 10^{-6} - \chi_{\rm out}$.

The OPERA Collaboration derived the bound $\upsilon_\nu - 1 = (2.7 \pm 6.5)\times 10^{-6}$ on the neutrino velocity, based on ∼15200 events collected in 2009–2011 [2]. BOREXINO obtained $\upsilon_\nu - 1 = 2.7 \pm 5.4\times 10^{-6}$ in the Oct./Nov. 2011 run, and $|\upsilon_\nu - 1| < 2.1\times 10^{-6}$ in May 2012 [4]. The OPERA upper bounds inferred from the May 2012 data are $\upsilon_\nu - 1 < 2.3\times 10^{-6}$ and $\upsilon_{\bar\nu} - 1 < 3.0\times 10^{-6}$ for antineutrinos [3]. The LVD experiment obtained $|\upsilon_\nu - 1| < 3.5\times 10^{-6}$ [5], and the ICARUS bound from the May 2012 run is $|\upsilon_\nu - 1| < 1.6\times 10^{-6}$ [6]. All bounds refer to an averaged neutrino energy of 17 GeV in the CNGS beam. The OPERA bound from the 2009–2011 data [2] is based on a much larger sample than the other experiments (< 100 events). The current MINOS estimate, $\upsilon_\nu - 1 = 6 \pm 13\times 10^{-6}$ [21], is in line with the quoted CNGS results; all these experiments report a positive neutrino excess velocity with an error bound safely consistent with the speed of light.

Given the low relative speed $\upsilon_{\rm r} \approx 1.2\times 10^{-3}$ of the Solar system barycenter in the CMB rest frame [11,12], we can use the linearized addition law for velocities, $\upsilon_{\rm CMB} - 1 = (\upsilon_\nu - 1)(1 + \mathrm{O}(\upsilon_{\rm r}))$, where $\upsilon_{\rm CMB}$ is the neutrino speed in the CMB rest frame and υ_ν the speed measured in the baseline frame (rest frame of source and detector) [8]. Hence, $\upsilon_{\rm CMB} - 1 \sim \upsilon_\nu - 1 \sim 1 - n_\nu$, cf. after (27).

In the following, we assume that the incoming pion and the outgoing muon are subluminal, and the neutrino is superluminal. Furthermore, we assume that pion and muon have similar susceptibilities, so that we can equate $\chi_{\rm in} \approx \chi_{\rm out} \approx \chi$ in the energy-momentum and causality conditions (23)–(26). (We will later drop this assumption, cf. after (30).) The velocity condition on pion and muon is $1-\upsilon_{\rm in,out} > 0$, which gives the bound $-3.90\times10^{-6} < \chi$, cf. the estimates stated after (27). Since the neutrino is superluminal, we have $\chi_\nu < 0$. The energy-momentum constraint (23) combined with the velocity condition gives

$$-3.90\times10^{-6} < \chi < \chi_\nu + 1.49\times10^{-6}. \tag{28}$$

The causality conditions (24) and (26) read

$$-(4.88\chi_\nu + 1.435\times10^{-5}) < \chi < 4.88\chi_\nu + 1.435\times10^{-5}, \tag{29}$$

which can only be satisfied if $\chi_\nu > -2.94\times10^{-6}$. In fact, for (28) to be consistent with (29), an even stronger lower bound on χ_ν is required, $\chi_\nu > -2.694\times10^{-6}$. The causality condition (25) does not apply, as the pion as well as the muon are subluminal, cf. after (22). If $\chi_\nu \approx -2.694\times10^{-6}$, we find the unique solution $\chi\approx -1.204\times10^{-6}$. (At the opposite edge $\chi_\nu\approx 0$ of the allowed χ_ν interval, the admissible χ range is given by (28).) As we have put $\chi_{\rm in}\approx\chi_{\rm out}\approx\chi$, we find the pion and muon velocities $1-\upsilon_{\rm in}\sim 2.7\times10^{-6}$ and $1-\upsilon_{\rm out}\sim 3.9\times10^{-6}$, respectively. The neutrino excess velocity is $\upsilon_\nu - 1 \sim -\chi_\nu \sim 2.7\times10^{-6}$, which coincides with the quoted OPERA and BOREXINO 2011 upper bounds [2,4].

We return to the basic energy-momentum and causality conditions (23), (24) and (26), substitute $\chi_\nu\approx -2.7\times10^{-6}$, and drop the assumption of equal pion and muon susceptibilities $\chi_{\rm in}\approx\chi_{\rm out}$ to obtain

$$\Delta n \sim 0.66\chi_{\rm out} - \chi_{\rm in} - 4.1\times10^{-7} > 0, \tag{30}$$

$$\Delta\upsilon_{\nu\times{\rm in}} \propto 1.28\chi_{\rm out} - 0.94\chi_{\rm in} + 4.1\times10^{-7} > 0, \tag{31}$$

$$\Delta\upsilon_{{\rm out}\times\nu} \propto 2.60\chi_{\rm out} - 2.94\chi_{\rm in} + 4.1\times10^{-7} > 0. \tag{32}$$

Adding the first to the second and third of these inequalities, we find $\chi_{\rm in} < \chi_{\rm out}$ and $\chi_{\rm in} < 0.83\chi_{\rm out}$, the latter is weaker and can be ignored if we consider negative susceptibilities $\chi_{\rm in,out} < 0$. The constraints (30)–(32) thus reduce to $\chi_{\rm in} < 0.66\chi_{\rm out} - 4.1\times10^{-7}$ and $\chi_{\rm in} < \chi_{\rm out}$ for $\chi_{\rm in,out}$ in the range $-3.90\times10^{-6} < \chi_{\rm in} < 0$ and $-5.14\times10^{-6} < \chi_{\rm out} < 0$. The latter two lower bounds on the pion and muon susceptibilities are required by a subluminal particle velocity, cf. after (27). These constraints are based on the neutrino susceptibility $\chi_\nu \approx -2.7\times10^{-6}$; a possible solution is $\chi_{\rm in}\approx\chi_{\rm out}\approx -1.2\times10^{-6}$ as discussed after (29).

Kaon decay $K\to\mu+\nu_\mu$ and superluminal muon neutrinos. – The reasoning is analogous to that of pion decay, cf. (30)–(32). The pionic mass square is replaced by the kaon mass, $m_{\rm in}^2 = m_K^2 \approx 0.244\,{\rm GeV}^2$ [19], and the energy of the incoming kaon is $\omega_{\rm in}\approx 85\,{\rm GeV}$ [1], so that $\omega_{\rm out} = \omega_{\rm in} - \omega_\nu \approx 68\,{\rm GeV}$ for the outgoing muon. The group velocities of kaon and muon are calculated as in (27), $1-\upsilon_{\rm in}\sim 1.7\times10^{-5}+\chi_{\rm in}$, and $1-\upsilon_{\rm out}\sim 1.2\times10^{-6}+\chi_{\rm out}$. The energy-momentum and causality constraints (19), (20) and (22) read

$$\Delta n \sim 0.8\chi_{\rm out} + 0.2\chi_\nu - \chi_{\rm in} + 1.6\times10^{-5} > 0, \tag{33}$$

$$\Delta\upsilon_{\nu\times{\rm in}} \propto 1.1\chi_{\rm out} + 0.6\chi_\nu - \chi_{\rm in} + 2.7\times10^{-5} > 0, \tag{34}$$

$$\Delta\upsilon_{{\rm out}\times\nu} \propto 1.0\chi_{\rm out} + 0.36\chi_\nu - \chi_{\rm in} + 1.6\times10^{-5} > 0. \tag{35}$$

These conditions can readily be satisfied with a neutrino susceptibility in the interval $0 > \chi_\nu > -2.7\times10^{-6}$, since in this case the χ_ν terms are negligible; $\chi_\nu \approx -2.7\times10^{-6}$ is the neutrino susceptibility defined by the quoted OPERA [2] and BOREXINO [4] upper bounds on the neutrino excess velocity. Subluminal kaon and muon velocities require the constraints $-1.7\times10^{-5} < \chi_{\rm in}$ and $-1.2\times10^{-6} < \chi_{\rm out}$. Conditions (33)–(35) are satisfied by negative susceptibilities $\chi_{\rm in,out}$ subject to these velocity bounds. We do not assume $\chi_{\rm in}\approx\chi_{\rm out}$, as the kaon and muon mass squares substantially differ. $\chi_{\rm out}(\omega_{\rm out})$ refers to a muon energy of $\omega_{\rm out}\approx 68\,{\rm GeV}$, as compared to 33 GeV in the case of pion decay. The neutrino susceptibility $\chi_\nu(\omega_\nu)$ is taken at a neutrino energy of 17 GeV in both cases.

Causality violation prohibiting photon emission $\pi\to\pi+\gamma$ by superluminal high-energy charges. – In this section, the subscript index ν labels photon variables. The outgoing photon with frequency ω_ν has zero rest mass $m_\nu^2 = 0$ and a refractive index $n_\nu - 1 = \delta n_\nu \sim \chi_\nu$, cf. (6). We assume a nearly constant photon susceptibility $\chi_\nu \geqslant 0$, $\chi'_\nu \approx 0$, so that the photonic group velocity (14) is $\upsilon_\nu \approx 1-\chi_\nu \leqslant 1$. We use a refractive photon index that is slightly larger than one, so that a rest frame exists for the photon. The causality conditions are $\Delta\upsilon_{\nu\times{\rm in}} > 0$ and $\Delta\upsilon_{{\rm out}\times\nu} > 0$, cf. (16) and (18). The vacuum limit $\chi_\nu\to 0$ (photonic permeability tensor coinciding with Minkowski metric) is performed in the subsequent inequalities by putting $\delta n_\nu \approx 0$ and $\delta n'_\nu \approx 0$ in the causality constraints and the refractive-index increment (9). Energy conservation means $\omega_{\rm out} = \omega_{\rm in} - \omega_\nu$, with positive frequencies. The refractive indices of the in- and outgoing charges are $\delta n_{\rm in,out} \sim \chi_{\rm in,out} - m_{\rm in}^2/(2\omega_{\rm in,out}^2)$, cf. (6), with derivatives $\delta n'_{\rm in,out} \sim \chi'_{\rm in,out} + m_{\rm in}^2/\omega_{\rm in,out}^3$ at $\omega_{\rm in,out}$. The susceptibility functions $\chi_{\rm in,out}(\omega)$ are identical, but taken at different energies $\omega_{\rm in,out}$. Expanding $\chi_{\rm in}(\omega) \approx \chi_{\rm in} + (\omega - \omega_{\rm in})\chi'_{\rm in}$ in linear order at $\omega_{\rm in}$, and making use of energy conservation, we can approximate $\chi_{\rm out}(\omega_{\rm out}) \approx \chi_{\rm in}(\omega_{\rm in}) - \omega_\nu\chi'_{\rm in}(\omega_{\rm in})$ and $\chi'_{\rm out}(\omega_{\rm out}) \approx \chi'_{\rm in}(\omega_{\rm in})$. The refractive-index increment (19) reads in this case

$$\Delta n \sim -\frac{\omega_\nu/\omega_{\rm in}}{1-\omega_\nu/\omega_{\rm in}}\left[\left(1-\frac{\omega_\nu}{\omega_{\rm in}}\right)^2 \omega_{\rm in}\chi'_{\rm in} + \left(1-\frac{\omega_\nu}{\omega_{\rm in}}\right)\chi_{\rm in} + \frac{m_{\rm in}^2}{2\omega_{\rm in}^2}\right]. \tag{36}$$

First, we show that one of the causality conditions $\Delta\upsilon_{\nu\times\mathrm{in}} > 0$ and $\Delta\upsilon_{\mathrm{out}\times\nu} > 0$ is violated for negative χ_{in}. In fact, the velocity increment (20) reads

$$\Delta\upsilon_{\nu\times\mathrm{in}} \sim \chi_{\mathrm{in}} \left(\frac{\omega_\nu}{\omega_{\mathrm{in}}} + \left(2 - \frac{\omega_\nu}{\omega_{\mathrm{in}}} \right) \omega_\nu \frac{\chi'_{\mathrm{in}}}{\chi_{\mathrm{in}}} \right), \qquad (37)$$

so that $\Delta\upsilon_{\nu\times\mathrm{in}} < 0$ if both χ_{in} and χ'_{in} are negative. Increment (22) can be written as

$$\Delta\upsilon_{\mathrm{out}\times\nu} \sim \frac{\chi_{\mathrm{in}}}{1 + \omega_\nu/\omega_{\mathrm{in}}} \left(2 + \frac{\omega_\nu}{\omega_{\mathrm{in}}} - 2 \left(1 + \frac{\omega_\nu}{\omega_{\mathrm{in}}} \right) \omega_\nu \frac{\chi'_{\mathrm{in}}}{\chi_{\mathrm{in}}} \right), \qquad (38)$$

so that $\Delta\upsilon_{\mathrm{out}\times\nu} < 0$ if χ_{in} is negative and χ'_{in} positive. Thus the emission $\pi \to \pi\gamma$ is causality violating if the susceptibility $\chi_{\mathrm{in}}(\omega_{\mathrm{in}})$ is negative.

This emission process is also forbidden in the case of a positive susceptibility $\chi_{\mathrm{in}}(\omega_{\mathrm{in}})$. If both χ_{in} and χ'_{in} are positive, this implies a negative refractive-index increment Δn, cf. (36), so that momentum cannot be conserved. (In this case, the group velocity (27) of the incoming charge is subluminal.) If χ_{in} is positive and χ'_{in} negative, the conditions $\Delta n > 0$ and $\Delta\upsilon_{\nu\times\mathrm{in}} > 0$ cannot simultaneously be satisfied, cf. (36) and (37). In fact, we may drop the mass term in (36) and require $(1 - \omega_\nu/\omega_{\mathrm{in}})\omega_{\mathrm{in}}\chi'_{\mathrm{in}} + \chi_{\mathrm{in}} < 0$, which is necessary (but not sufficient) for $\Delta n > 0$. The causality condition $\Delta\upsilon_{\nu\times\mathrm{in}} > 0$ implies $(\omega_\nu/\omega_{\mathrm{in}} - 2)\omega_{\mathrm{in}}\chi'_{\mathrm{in}} - \chi_{\mathrm{in}} < 0$, cf. (37). Adding these inequalities, we obtain $-\omega_{\mathrm{in}}\chi'_{\mathrm{in}} < 0$, in contradiction to the assumed negative derivative χ'_{in}. We have thus demonstrated that photon emission by superluminal high-energy charges is forbidden since it is causality violating.

Conclusion. – We have studied two-particle decay in a dispersive spacetime, deriving bounds on a superluminal group velocity of the decay products. The nonlinear causality and energy-momentum constraints can be linearized in the high-energy regime by introducing frequency-dependent susceptibility functions for the in- and outgoing particles which serve as expansion parameters. In this way, analytically tractable causality conditions are obtained even in multi-channel interactions. These constraints on the susceptibility functions in the isotropic aether frame (identified as CMB rest frame [22]) prevent time inversions in the rest frames of the subluminal particles and radiation modes (inertial in- and out-states) of the decay process [7].

Specifically, we discussed the dispersive kinematics of pion and kaon decay in the aether, and calculated the susceptibility functions with input parameters of the CNGS neutrino beam. The causality conditions are linear inequalities to be satisfied by the susceptibilities of the respective particles. We employed these constraints to obtain velocity estimates for the muon and muon neutrino generated by the decay. Finally we used causality conditions on susceptibility functions to demonstrate, without the use of specific input parameters, that photonic Cherenkov radiation by superluminal high-energy charges is causality violating.

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