COSMOLOGICAL CP VIOLATION IN THE OPEN UNIVERSE

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ABSTRACT

An account on the physical impact of a topologically and metrically evolving space-time is given. We discuss the appearance of a chaotic nucleus in the infinite and unbounded 3-space, topologically induced *CP* violation, the existence of faster-than-light particles, and their topological localization.

1. The chaotic center of the Universe

Our basic assumptions are that the Universe is open and that its spacelike slices are multiply connected and negatively curved (extended Robertson-Walker cosmologies). Under these conditions there exists a finite region in the infinite 3-space in which the world lines are chaotic. On more elaboration on that see Refs. 1 and 2 and the caption of Fig. 1. It is beyond any doubt that some mechanism to generate chaos is needed to achieve the remarkable uniformity of the galactic background. In these cosmologies it is the local instability of the world lines and the global topology which induce chaos in a finite domain, the Center of the Universe, whose size scales with the expansion factor. Moreover there are regular trajectories which are shadowed over long times by chaotic ones. This could provide an explanation that perfect equidistribution of the galaxies has not really been attained[5,6].

2. The violation of the space-reflection symmetry by topological self-interference

The classical geodesic equations are still reflection invariant in a multiply connected universe, but the situation is quite different concerning quantum mechanics. A space-reflected wave packet can wrap around a tiny geodesic loop and overlap with itself. This gives rise to self-interference, and the unitarity of the parity operator is lost. In particular *CP* and *CPT* are already broken in the free Dirac equation, cf. Refs. 1, 5 and 6. Self-interference is quite an inevitable phenomenon in multiply connected spaces, cf. Refs. 7 and 5.

The three-space is multiply connected, and two points can be joined through various topological channels by geodesics, which are now local minima of a global variational problem. In the simply connected universal covering space of the 3-manifold, the space-reflection P_C is of course unique for a given center of reflection C, which lies in the middle of the geodesic arc joining a point X and $P_C(X)$. This is in fact the definition of the space reflection, in analogy to Euclidean space. The covering space is here

Group Theoretical Methods in Physics, XX Int. Colloq., pp. 479-482,
edited by A. Arima *et al.*, World Scientific, Singapore, 1995.

hyperbolic space H^3, which is geodesically complete. A space reflection in the 3-manifold we define via the universal covering projection π, $P_C^\pi := \pi \circ P_C$. The center of reflection C is now a point in the fundamental polyhedron F which represents the 3-manifold in the covering space. More generally, we can define space reflections $P_C^\gamma := P_C^\pi \circ \gamma$, $\gamma \in \Gamma$ (the covering group). If the connectivity of the manifold is finite, only finitely many of these reflections P_C^γ are really different from each other. All of them have the property that the center of reflection C lies in the middle of a geodesic joining an arbitrary point X in F with $P_C^\gamma(X)$.

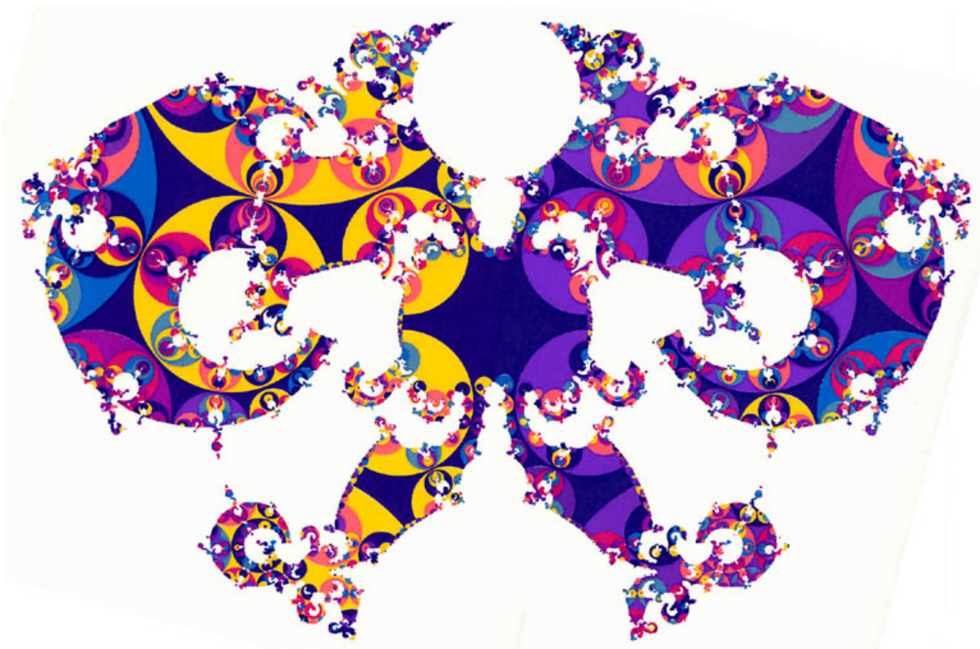
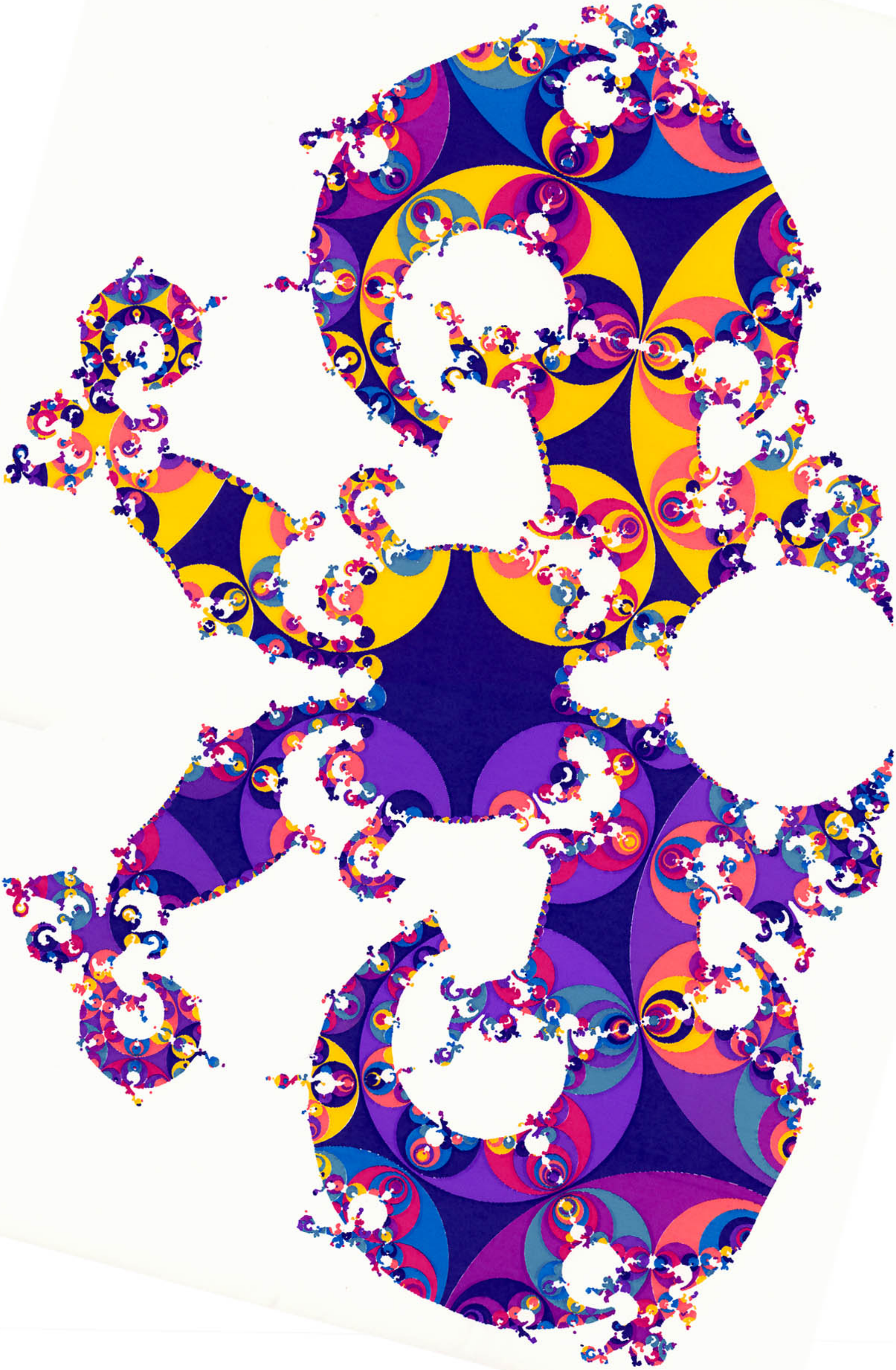


Fig. 1. The horizon at infinity of the Poincaré half-space H^3. A spacelike slice (F, Γ) is realized in H^3 as a polyhedron F with a face-identification. The identifying transformations of F generate the covering group Γ which, applied to the polyhedron, gives a tessellation $\Gamma(F)$ of H^3 with polyhedral images. This tessellation induces by continuity also a tiling on the boundary of H^3, that is depicted here. The qualitative structure of the singular set depends on Γ, for quasi-Fuchsian groups like here it is a Jordan curve, for Schottky groups a Cantor set, cf. Refs. 3 and 4. ($(F, \Gamma) \approx I \times S$, S a Riemann surface, $g(S) = 19$, $\delta(\Lambda) \approx 1.45$.) From the fractal limit set $\Lambda(\Gamma)$ one can easily determine the chaotic or nearly chaotic trajectories, which shadow each other over long times. Their lifts have initial and end points in or close to $\Lambda(\Gamma)$. Projecting them into the 3-space (F, Γ) one obtains the chaotic nucleus. Once the tiling is generated it is easy to construct the covering projection, $\pi(x) := \gamma^{-1}(x)$, the point x lies in the tile $\gamma(F)$. Likewise, a geodesic arc crossing a tile $\gamma(F)$ is mapped by π via γ^{-1} into F.



3. Tachyonic dynamics in the open Universe

If the cosmic expansion factor has turning or inflexion points, it can happen that a tachyon moves within a finite time an infinite distance, with a finite though unbounded velocity. We start with the Lagrangian $L = g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}$ of a Robertson-Walker geometry with negatively curved 3-slices, more specifically, $g_{00} = -c^2$, $g_{ij} = a^2(\tau)R^2t^{-2}\delta_{ij}$, in the Poincaré half-space H^3 with coordinates $x^i := (z,t)$, $z \in C$, $t>0$, cf. Ref. 1.

We have immediately one integral of motion, $c^2\left(\frac{d\tau}{ds}\right)^2 - g_{ij}\frac{dx^i}{ds}\frac{dx^j}{ds} = \varepsilon$. Without loss of generality we can choose $\varepsilon = \pm 1$ or 0, by rescaling the parameter s. We have $\varepsilon = 1$ for particles, $\varepsilon = 0$ for rays, and $\varepsilon = -1$ for tachyons. We assume that tachyons have a positive mass. Then we define the energy-momentum vector as $p^{\mu} = mc\frac{dx^{\mu}}{ds} =: (c^{-2}E, \vec{p})$. Clearly we have $p^{\mu}p_{\mu} = -\varepsilon m^2c^2$, and we define a velocity $\vec{v}$ in analogy to Minkowski space that parametrizes this hyperboloid, cf. e. g. Refs. 8 and 9,

$\vec{p} = \frac{m\vec{v}}{\sqrt{\varepsilon\left(1-|\vec{v}|^2/c^2\right)}}$, $E = \frac{mc^2}{\sqrt{\varepsilon\left(1-|\vec{v}|^2/c^2\right)}}$. It is sufficient to study a trajectory perpendicular to the complex plane, all other trajectories we obtain by applying to it some transformation of the invariance group of H^3. Its time parametrization is given by

$$t(\tau) = t(\tau_0)\exp\left[\pm\frac{c}{R}\int_{\tau_0}^{\tau} d\tau\, a^{-1}(\tau)\left(1+\varepsilon v^{-2}c^{-2}a^2(\tau)\right)^{-1/2}\right].$$

We consider from now on only $\varepsilon=-1$, the tachyonic case. Moreover we assume that $a(\tau)$ is increasing in an interval $[\tau_0,\tau_\infty]$, and that $\dot{a}(\tau_\infty) = 0$. The integration constant v which determines the energy, we choose as $v = c^{-1}a(\tau_\infty)$. Then we have

$t(\tau) \sim \text{const.}\,|\tau_\infty-\tau|^{\pm\alpha}$, $\alpha := \frac{c}{Ra(\tau_\infty)}\left|\frac{\ddot{a}(\tau_\infty)}{a(\tau_\infty)}\right|^{-1/2}$, and $|\vec{v}| \sim c\left|\frac{\ddot{a}(\tau_\infty)}{a(\tau_\infty)}\right|^{-1/2}|\tau_\infty-\tau|^{-1}$, for $\tau\to\tau_\infty$. Similar formulas we get if in addition $\ddot{a}(\tau_\infty) = 0$. Therefore, if $a(\tau_\infty)$ is a turning or inflexion point, and if we choose the initial energy as we did, then we have always $t(\tau_\infty) = 0$ or ∞. This means that the tachyon, starting at some point x_0 in the interior of H^3, reaches within a finite time $\Delta\tau = \tau_\infty - \tau_0$ the boundary at infinity of H^3. The tachyon can reach within a time $\Delta\tau$ every point in H^3, however remote from x_0, and it arrives with a finite fraction of its initial energy there. Thus tachyons can transfer signals over an infinite distance within a finite time, which is a very useful property in an infinite universe. Contrary to Newtonian mechanics there is no external force needed to achieve this, since tachyons move freely and covariantly along geodesics in accordance with the principle of general relativity. Finally, a tachyon can get topologically localized. If the spacelike slices are multiply connected, it can get trapped like a particle or ray in their chaotic nucleus, cf. Ref. 6. It comes then within a time $\Delta\tau$ arbitrarily close to every point, it is dense there.

Acknowledgments

The author acknowledges the support of the 'Japanese Society for the Promotion of Science' , as well as a Monbusho Grant-in-Aid, contract no. P-93006, and in particular the stimulating atmosphere and kind hospitality of the Particle Physics Group here at Hiroshima University.

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Discussion

N. Sakai: Should we expect that your mechanism is going to explain the *CP*-violation in kaon systems ?

R. T. : Yes, you should. I think *CP*-violation appears quite generically on the microscopic level, but usually weakly enough that it escapes detection. I think it is a basic quantum mechanical interference phenomenon, related to the microscopic topology of space-time. The traditional deliberate use of symmetry breaking interactions in the Lagrangians I regard as a heuristic description.

E. Elizalde: Is your definition of the 'Center of the Universe' covariant ?

R. T. : Yes, I define it as the region in which the world lines are mixing. Liapounov instability is defined with respect to special coordinate frames, but 'mixing' or the Bernoulli property are generally covariant concepts. The limit set of the covering group (as defined in the caption of Fig. 1) is likewise a covariant construct. The mixing phenomenon can be understood in terms of the local hyperbolicity of the 3-space metric, and the fact that the global topology can confine trajectories which would otherwise, in the simply connected universal covering space, tend to infinity.