COSMIC TIME VARIATION OF THE GRAVITATIONAL CONSTANT

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Abstract. A pre-relativistic cosmological approach to electromagnetism and gravitation is explored that leads to a cosmic time variation of the fundamental constants. Space itself is supposed to have physical substance, which manifests by its permeability. The scale factors of the permeability tensor induce a time variation of the fundamental constants. Atomic radii, periods, and energy levels scale in cosmic time, which results in dispersionless redshifts without invoking a space expansion. Hubble constant and deceleration parameter are reviewed in this context. The time variation of the gravitational constant at the present epoch can be expressed in terms of these quantities. This provides a completely new way to restrain the deceleration parameter from laboratory bounds on the time variation of the gravitational constant. This variation also affects the redshift dependence of angular diameters and the surface brightness, and we study in some detail the redshift scaling of the linear sizes of radio sources. The effect of the varying constants on source counts is discussed, and an estimate on the curvature radius of the hyperbolic 3-space is inferred from the peak in the quasar distribution. The background radiation in this dispersionless, permeable space-time stays perfectly Planckian. Cosmic time is discussed in terms of atomic and gravitational clocks, as well as cosmological age dating, in particular how the age of the Universe relates to the age of the Galaxy in a permeable space-time.

1. Introduction

If the speed of light is varying in the cosmic evolution, it is tempting to model this variation by a permeability tensor (Whittaker, 1951; Tomaschitz, 1998a-c). We assume that cosmic space as generated by the expanding galatic grid is not a mere geometric construct, but has itself substance. This substance, the ether, manifests by its permeability.

The speed of electromagnetic waves is determined, like in a dielectric medium, by a permeability tensor which, with the usual appeal to cosmic homogeneity and isotropy, takes the form $ds_P^2 = -c^2h^2(\tau)d\tau^2 + b^2(\tau)d\sigma^2$, with two scale factors $h(\tau)$ and $b(\tau)$. This complements the Robertson-Walker (RW) metric $ds^2 = -c^2d\tau^2 + a^2(\tau)d\sigma^2$. (The line element $d\sigma^2$ on the 3-space induces constant curvature.) Maxwell's equations read

$$H^{\mu\nu}{}_{;\nu} = c^{-1}j^{\mu}, \quad (-g)^{-1/2}\varepsilon^{\lambda\alpha\beta\gamma}F_{\alpha\beta;\gamma} = 0, \tag{1.1}$$

with $H^{\mu\nu} := g^{P-1\mu\alpha}g^{P-1\nu\beta}F_{\alpha\beta}$. The permeability tensor $g^P_{\mu\nu}$ and the space-time metric $g_{\mu\nu}$ are defined by the line elements ds_P^2 and ds^2, respectively. The tensor

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$g^{P-1\mu\nu}$ denotes the inverse of $g^P_{\mu\nu}$. Covariant derivatives (;) are defined with the metric $g_{\mu\nu}$.

There are, however, two important differences compared to electrodynamics in a dielectric medium. The speed of light, being a function of cosmic time, scales as $\hat{c} = c/n(\tau)$, with the refractive index $n = \sqrt{\varepsilon\mu}$, $\varepsilon := a^2b^{-2}h^{-2}$ and $\mu := b^4a^{-4}$. For the ether to be dispersion free, electric and magnetic permeability must be proportional (Tomaschitz, 1998c), which requires h to be proportional to a^3/b^3. In the following we will require

$$h(\tau) = a^3(\tau)/b^3(\tau), \quad a(\tau_0) = b(\tau_0) = 1; \tag{1.2}$$

τ_0 is the present epoch. If (1.2) holds, then there is no dispersion in the direction of propagation, which could lead to a dimming of spectral lines (Sandage, 1988). The time evolution of the classical electromagnetic energy in the ether is proportional to the time scaling of frequency, $\omega \sim h(\tau)b^{-1}(\tau)$. Thus, if we take the Einstein relation $E = \hbar\omega$ for granted, Planck's constant is independent of cosmic time, $\hbar \propto 1$. Moreover, $\lambda\!\!\!^{-} \sim a(\tau)$, and $\hat{c} = \omega\lambda\!\!\!^{-}$. At τ_0, ds_P^2 and ds^2 coincide, so that $\hat{c}(\tau_0) = c$, the presently measured speed of light.

The second difference to a dielectric medium is, that the ether does not only affect electromagnetic waves but also massive particles and the gravitational field; hence, not only the speed of light scales, but also the measuring rods. The ether is the medium of electromagnetic wave propagation (Whittaker, 1951), and the same holds for matter waves, and therefore it affects classical point particles as well, as a consequence of the semiclassical limit. The dynamics of point particles in the ether is defined by extending the eikonal equation, $g^{P-1\mu\nu}\psi_{,\mu}\psi_{,\nu} = 0$, to a Hamilton-Jacobi equation, $g^{P-1\mu\nu}S_{,\mu}S_{,\nu} = -c^2m^2$.

The potential of a static electric point charge e_s in the ether was calculated in Tomaschitz (1998c). We derived from the Hamilton-Jacobi equation ($S_{,\mu} \to S_{,\mu} - eA_\mu$) the energy of a charge e moving in this potential,

$$E = \frac{\hat{m}\hat{c}^2}{\sqrt{1 - \mathbf{v}^2/\hat{c}^2}} + \frac{\hat{e}\hat{e}_s}{4\pi}\frac{1}{r}, \tag{1.3}$$

$$\hat{c}(\tau) = ch^{4/3}, \quad \hat{m}(\tau) = mh^{-5/3}, \quad \hat{e}(\tau) = eh^{2/3}. \tag{1.4}$$

Equations (1.4) constitute the scaling laws for the speed of light, mass, and charge. [c, m, and e are the presently measured values, as $h(\tau_0) = 1$.] It follows from (1.2) and (1.4) that the fine structure constant $\alpha = \hat{e}^2/(4\pi\hbar\hat{c})$ does not scale in cosmic time, if the ether is dispersion free. The universality of the scaling law for mass is supported by the fact that the ratios of electron, proton, and neutron masses are apparently constant (Varshalovich and Potekhin, 1995), and there are also tight bounds on the variation of the fine structure constant ($10^{-13} - 10^{-17}$ yr^{-1} for the logarithmic time derivative), derived from atomic clocks (Prestage *et al.*, 1995), quasar spectra (Varshalovich and Potekhin, 1995), and the Oklo natural reactor (Shlyakhter, 1976; Irvine, 1983).

By means of the Bohr quantization rules for the hydrogen atom, one can readily write down the scaling laws for the energy levels, the Bohr radii, and the orbital velocity and period (Tomaschitz, 1998c),

$$E_n = -\frac{\hat{m}}{2\hbar^2}\left(\frac{\hat{e}\hat{e}_s}{4\pi}\right)^2\frac{1}{n^2} \propto h(\tau), \quad r_n = -\frac{4\pi}{\hat{e}\hat{e}_s}\frac{\hbar^2 n^2}{\hat{m}} \propto h^{1/3}(\tau),$$

$$\mathbf{v}_n = \frac{\hbar n}{\hat{m} r_n} \propto h^{4/3}(\tau), \quad T_n = 2\pi\frac{r_n^2\hat{m}}{\hbar n} \propto h^{-1}(\tau). \tag{1.5}$$

We use the Poincaré half-space representation of hyperbolic geometry; the spatial line element in the permeability tensor is then defined by $d\sigma^2 = R^2 t^{-2}(|d\xi|^2 + dt^2)$ in the half-space H^3, (ξ, t), $\xi \in \mathbb{C}$, $t > 0$, $x^\mu = (\tau, \xi, t)$. H^3 endowed with $d\sigma^2$ has constant sectional curvature $-1/R^2$, so that the cosmic 3-space has the curvature radius $a(\tau)R$. Clearly, atomic radii and periods vary in time, and if we measure the speed of light in these varying units, it stays constant, that is, as long as we can neglect the gravitational potential. In fact, if we disregard gravitation for the moment, then the cosmology defined by the two line elements ds_P^2, ds^2 is equivalent to a standard RW cosmology

$$ds_{RW}^2 = -c^2 dt^2 + a_{RW}^2(t) d\sigma^2, \quad a_{RW}(t) := b(\tau(t)), \quad t = \int_{const}^{\tau} h(\tau) d\tau. \tag{1.6}$$

The Hubble parameter then reads $H^{RW}(t) = \dot{b}(\tau)/(hb) =: H(\tau)/h(\tau)$. Because $dt = h(\tau)d\tau$, we can identify t with atomic time, counting periods as defined in (1.5), cf. the end of Section 2. As for the Hubble constant, we find $H_0 := H^{RW}(t_0) = H(\tau_0)$, since $dt = d\tau$ at the present epoch, cf. (1.2). Conservation of the galaxy number requires $\rho_{RW}(t) \propto a_{RW}^{-3}(t)$, and thus $\rho(\tau) \propto b^{-3}(\tau)h^{-1}(\tau)$; in the steady state case, $\rho_{RW}(t) \propto 1$ and $\rho(\tau) \propto h^{-1}$. $\rho_{RW}(t)$ is just $\rho(\tau)$ measured in atomic units, cf. (1.5), and the same holds for $H^{RW}(t)$ and $H(\tau)$, cf. Section 3. The quantities in (1.4) as well as $H(\tau)$ and $\rho(\tau)$ must always be accompanied by a measurement prescription, a set of units (nuclear, atomic, or gravitational, cf. Sections 2 and 6). The scale factors of metric and permeability tensor define the time variation of all fundamental constants without reference to a specific set of measuring rods. This is not trivial, as in a theory of varying constants conversion factors are time dependent.

Equation (1.6) demonstrates, that an expanding space-time is equivalent to a static space-time in which the measuring rods are contracting. The cosmological redshift can be described either by an expanding galaxy background, or by a time variation of the constants of nature, which is such, that all local physical systems contract at the same rate. We may describe the cosmological redshift either by the assumption that the distance between the galaxies increases and the size of the atoms stays constant ($h(\tau) \equiv 1$), or by the assumption that the atomic radii contract and the distance between the galaxies stays constant, ($a(\tau) \equiv 1$). It is here of course understood, that the radii of all local systems, nuclear, atomic, and gravitational, including galaxy diameters, contract at the same rate, which, as we will argue, does not happen. So, the speed of light depends on the rods, by which we measure it, and there is in fact no rod that makes it constant throughout the cosmic evolution. The redshift is not only determined by the time variation of the photon frequency, but also by the cosmic scaling of atomic energy levels, which serve as measuring rods.

In Section 2 the scaling law for the gravitational constant is derived. In Tomaschitz (1998b) a theory of gravity was developed which is based on a scalar gravitational field coupled to the permeability tensor like the electromagnetic field. In this context we study the Kepler problem and derive scaling laws for planetary orbits. If we assume $H_0 = 100h_0$ km s^{-1} Mpc^{-1}, $h_0 \approx 0.7$ (HST Key Project, cf. Mould *et al.*, 2000), we find

$$\frac{\hbar^2 H_0}{kcm_e m_p m_n} \approx 0.49, \tag{1.7}$$

and there is the coincidence $\hbar^2 H_0/(kcm_\pi^3) = 1/(4\pi)$ for $h_0 \approx 0.6802$ (which happens to be the supernovae estimate of the Key Project Group) and a pion mass of 139.567 MeV/c^2. We keep this ratio constant in the cosmic evolution, which actually means a variation of the large ratio $e^2/(4\pi km_e m_p) \approx 2.27 \times 10^{39}$, and this is the point where the deviation from standard RW cosmology occurs. The time variation of this ratio is such that it converges to zero for $\tau \to 0$. Hence, in this limit, the atomic Coulomb potential will be ultimately overpowered by the Newton potential. Otherwise, in an epoch where the gravitational interaction can be neglected in microscopic interactions, weak and strong interactions define units that scale at the same rate like the atomic rods (1.5), cf. Section 6. As $\hat{c}(\tau)$ scales like $\mathbf{v}_n$ in (1.5), the speed of light is really constant in these units, but not so in gravitational units, see after (2.14).

In Section 3, we sketch the luminosity-distance in a permeable space-time, and relate the Hubble constant and the deceleration parameter to the scale factors of the metric and the permeability tensor. We then derive a relation between the logarithmic time derivative of the gravitational constant, the Hubble constant, and the deceleration parameter, based on the constancy of the ratio (1.7),

$$\dot{k}_0/k_0 = -H_0(1 + q_0), \tag{1.8}$$

in atomic units. Observational bounds on the variation of this ratio, steming from a variety of sources listed at the end of Section 3, give a much tighter bound on q_0 than obtained from the luminosity distance or angular diameters.

In Section 4, we discuss the redshift scaling of angular diameters and surface brightness, as well as of the linear sizes of radio sources, and we point out the implications of the varying gravitational constant on cosmological age dating. The variation of k scales linear diameters in a way that angular diameters decrease for high z. We also indicate the reconstruction of the scale factors of the metric and the permeability tensor from the z-dependence of the angular diameters and the surface brightness.

Source counts are studied in Section 5, based on a scaling law for the number density derived from the constancy of the moderate ratio Ω_m. We estimate, from the maximum in the quasar distribution, the present curvature radius of the hyperbolic 3-space, $R \approx 6.3c/H_0$. In the Conclusion, Section 6, the scaling laws for the Fermi constant and the strong interaction are derived, and we discuss the scaling laws for black-body radiation in a dispersionless, permeable space-time.

2. The Scaling Law for the Gravitational Constant

To derive the scaling law for k, it is sufficient to consider a Newtonian potential capable of producing Kepler ellipses; a gravitational theory that includes the permeability tensor as a dynamical variable and produces the right perihelion shifts can be found in Tomaschitz (1998b). In the Newtonian limit, the action of the gravitational potential reads as

$$S_\varphi = \int L_\varphi \sqrt{-g}\,dx, \quad L_\varphi = -\frac{c^3}{8\pi\alpha^2 k} g^{P-1\mu\nu} \varphi_{,\mu}\varphi_{,\nu}; \tag{2.1}$$

the tensors $g_{\mu\nu}$ and $g^P_{\mu\nu}$ are defined at the beginning of the Introduction. The gravitational potential is coupled to the permeability tensor quite similarly as the electromagnetic field, cf. (1.1), and the Lagrangian and the action of a particle moving in this potential reads

$$L_p(s) = -mcG(\varphi)\sqrt{-g^P_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}, \quad L_p(x) = \int L_p(s)\frac{\delta(x - x(s))}{\sqrt{-g}} ds,$$

$$G(\varphi) := 1 - \tilde{\alpha}^{-1}\varphi + O(\varphi^2), \quad S_p = \int L_p(s)ds = \int L_p(x)\sqrt{-g}\,dx. \tag{2.2}$$

The numerical constant $\tilde{\alpha}$ in $G(\varphi)$ gets important if we consider perihelion shifts, but in the Newtonian limits studied here it does not really enter, as it can be scaled into φ. (In this section we denote, as usual, derivatives with respect to s by a dot, and τ-derivatives by a prime.)

The source counts discussed in Section 5 provide evidence for a negatively curved 3-space. We choose as coordinate representation of the hyperbolic 3-space the Poincaré ball B^3 with metric $d\sigma^2 = 4(1 - |\mathbf{x}|^2/R^2)^{-2} d\mathbf{x}^2$, $|\mathbf{x}| < R$ (Cartesian coordinates), so that $x^\mu = (\tau, \mathbf{x})$ and $dx = d\mathbf{x}d\tau$ in (2.2). This line element is isometric to that defined after (1.5).

We consider a static point source of mass m_s, located at $\mathbf{x} = 0$, so that $ds = ch(\tau)d\tau$, in (2.2), and therefore

$$L_p(\tau, \mathbf{x}) = -m_s c^2 h(\tau) G(\varphi) \frac{\delta(\mathbf{x})}{\sqrt{-g}}, \quad \frac{\delta L_p(\tau, \mathbf{x})}{\delta\varphi} = \frac{m_s c h(\tau)}{\tilde{\alpha}} \frac{\delta(\mathbf{x})}{\sqrt{\gamma}}, \tag{2.3}$$

where γ is the determinant of the 3-space metric $a^2(\tau)d\sigma^2$. We obtain from (2.1)

$$\left(\frac{\delta L_\varphi}{\delta \varphi_{,\mu}}\right)_{;\mu} = -\frac{c^3}{4\pi\tilde{\alpha}^2 k}\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}g^{P-1\mu\nu}\varphi_{,\nu})}{\partial x^\mu}. \tag{2.4}$$

The Lagrange equation for the potential of a static point source m_s is derived from the combined action $S_\varphi + S_p$,

$$-\frac{1}{c^2a^3}\frac{\partial}{\partial\tau}\left(\frac{a^3}{h^2}\frac{\partial\varphi}{\partial\tau}\right) + \frac{1}{b^2}\Delta_B\varphi = -\frac{4\pi\tilde{\alpha}km_s h}{c^2a^3}\frac{\delta(\mathbf{x})}{\sqrt{\gamma^B}}, \tag{2.5}$$

which immediately follows from (2.3) and (2.4). Δ_B denotes the Laplace-Beltrami operator and γ^B the determinant of the B^3-metric $d\sigma^2$ as defined after (2.2). Since the time variation is adiabatic, we drop the term with the time derivatives in (2.5). The Poisson equation in B^3, $\Delta_B\tilde{\varphi} = -4\pi\delta(\mathbf{x})/\sqrt{\gamma^B}$, is solved by

$$\tilde{\varphi} = \frac{1}{2R}\left(\frac{R}{r} + \frac{r}{R} - 2\right) = \frac{1}{R}\left(\frac{Ra(\tau)}{d(\tau,\mathbf{x})} + O\left(\frac{d}{Ra}\right)\right), \tag{2.6}$$

where $d(\tau,\mathbf{x})$ is the spatial distance from the source located at $\mathbf{x} = 0$, cf. Tomaschitz (1998c). As $Ra(\tau)$ is the curvature radius of the 3-space, we may take the asymptotic limit in (2.6) for granted. Hence, the Newtonian potential of a static point source m_s reads as

$$\varphi = \frac{\tilde{\alpha}km_s}{c^2}\frac{b^2h}{a^2}\frac{1}{d(\tau,\mathbf{x})}. \tag{2.7}$$

Next we consider a particle in this potential. Introducing cosmic time as curve parameter, we have from (2.2)

$$L_p(s)ds = -mc^2h(1 - \tilde{\alpha}^{-1}\varphi + \ldots)\sqrt{1 - b^2h^{-2}c^{-2}\gamma^B_{ij}x'^i x'^j}d\tau, \tag{2.8}$$

where γ^B_{ij} denotes the B^3-metric. The velocity reads as $|\mathbf{v}|^2 = a^2\gamma^B_{ij}x'^i x'^j$. In the Newtonian limit, we obtain from (2.8) the Lagrangian

$$L = -mc^2h + \frac{1}{2}\frac{mb^2}{a^2h}|\mathbf{v}|^2 + \frac{1}{\tilde{\alpha}}mc^2h\varphi, \tag{2.9}$$

with φ as in (2.7). We may write this as

$$L = -\hat{m}\hat{c}^2 + \frac{1}{2}\hat{m}|\mathbf{v}|^2 + \frac{\hat{k}\hat{m}\hat{m}_s}{d(\tau,\mathbf{x})}, \tag{2.10}$$

with $\hat{m}$ and $\hat{c}$ defined in (1.4) and

$$\frac{\hat{k}(\tau)}{\hat{c}^2(\tau)} = \frac{k}{c^2}h^2(\tau). \tag{2.11}$$

If we express k in terms of the Hubble constant via (1.7), we obtain the scaling law for the gravitational constant,

$$\hat{k}(\tau) = k_0\kappa(\tau)h^{14/3}(\tau), \quad \kappa(\tau) := \frac{1}{h(\tau)}\frac{H(\tau)}{H_0}, \tag{2.12}$$

The identification of $H(\tau) := \dot{b}(\tau)/b(\tau)$ as Hubble parameter in a permeable space-time was already derived in Tomaschitz (1998c), see also Section 3. k_0 is the present-day value of the gravitational constant. The exponent 14/3 can also be guessed on dimensional grounds. In atomic units as defined in (1.5), we find the time variation of the gravitational constant as $k(t) = k_0\kappa(\tau(t))$.

We identify $d(\tau, \mathbf{x})$ in (2.10) with the Euclidean radial coordinate r and $|\mathbf{v}|$ with the Euclidean velocity. In polar coordinates, we have $|\mathbf{v}|^2 = r'^2 + r^2\theta'^2$. The angular momentum, $M = \hat{m}r^2\theta'$, is still conserved, but not any more the energy,

$$E(\tau) = \hat{m}\hat{c}^2 + \frac{1}{2}\hat{m}r'^2 + \frac{M^2}{2\hat{m}r^2} - \frac{\hat{k}\hat{m}\hat{m}_s}{r}. \tag{2.13}$$

The constants in (2.13) vary adiabatically on the time scale of a planetary period, of course. $E(\tau)$ is minimized by a circular orbit, whose radius, orbital velocity, and period scale as

$$r = \frac{M^2}{\hat{k}\hat{m}^2\hat{m}_s} \propto \kappa^{-1}h^{1/3}, \quad |\mathbf{v}| = r\theta' \propto \kappa h^{4/3}, \quad T \propto \kappa^{-2}h^{-1}. \tag{2.14}$$

Remarks: (1) Relation (2.12) actually constitutes the fundamental departure from standard RW cosmology. In fact, if we require the ratio $e^2/(km^2)$ to be constant instead of (1.7), this would mean to put $\kappa(\tau) \equiv 1$, independent of the Hubble constant. Then gravitational measuring rods scale exactly at the same rate as atomic ones, and we recover standard RW cosmology in the formalism of varying rods as discussed in Section 1. (2) The scaling stated in (2.14) for the orbital radius holds for the size of any gravitating system kept together by Newtonian potentials, in particular for galaxy diameters, which readily follows from the virial theorem (Teller, 1948). The scaling (2.14) is of course adiabatic, the orbit is not really closed. If the speed of light is measured in gravitational units (2.14), we find $\hat{c}(\tau) \propto \kappa^{-1}|\mathbf{v}|$, cf. (1.4), and thus $c(\tau) = c\kappa^{-1}$. In atomic units (1.5), we have of course a constant speed of light, $c(\tau) = c$, and the planetary orbital velocity (2.14) scales as $|\mathbf{v}| \propto \kappa$. The scale factors are chosen in a way, that atomic and gravitational units coincide at the present epoch, cf. (1.2).

As pointed out in (1.5), atomic periods scale as $T \propto h^{-1}$. However, this only holds if the Newtonian potential can be neglected. If the gravitational constant diverges for $\tau \to 0$, then the Newtonian potential will overpower the Coulomb potential at some point, and atoms will be kept together, if at all, by gravitational forces in this limit. The atomic period evidently scales as $T_{at.} \propto (-ee'/(4\pi) + \kappa(\tau)kmm')^{-2}h^{-1}(\tau)$, with present-day constants, and we have in the indicated limit $T_{at.} \sim T_{grav.} \propto \kappa^{-2}h^{-1}$, cf. (2.14). Time may be defined by counting either gravitational or atomic periods; in any case it is related to the cosmic time parameter via $dt \propto T^{-1}(\tau)d\tau$. If we count time in siderial years, we have to put

$$dt = \kappa^2(\tau)h(\tau)d\tau = (H(\tau)/H_0)^2 h^{-1}(\tau)d\tau, \tag{2.15}$$

and if time means the counting of atomic periods as defined by a hyperfine transition frequency, we have

$$dt = \frac{\left(1+\kappa(\tau)\frac{kmm'}{-ee'/(4\pi)}\right)^2}{\left(1+\frac{kmm'}{-ee'/(4\pi)}\right)^2} h(\tau)d\tau. \tag{2.16}$$

At the present epoch, dt and $d\tau$ coincide in either case, and the fundamental constants in (2.15) and (2.16) are present-day values. In Section 4, we will demonstrate that for most part of the cosmic evolution atomic time relates to cosmic time as $dt \approx h(\tau)d\tau$, i.e., the effect of the Newton potential can be neglected. However, in the limit $\tau \to 0$, atomic time coincides with gravitational time, provided one finds a suitable periodic system to count time in this limit.

Equations (1.4) and (2.12) constitute the scaling laws for c, m, e, and k; $\hbar$ and the small ratios α and (1.7) do not scale. In the next three sections we discuss the observational consequences of this time scaling, and then turn in Section 6 to the nuclear constants.

3. Hubble Constant and Deceleration Parameter in a Permeable Space-Time

The luminosity-distance relation (Weinberg, 1972; Sandage, 1988) reads $L_{app} = L/(4\pi d_L^2)$, with the luminosity distance

$$d_L = Ra(\tau_0)b(\tau_0)b^{-1}(\tau_{em}) \sinh[\Lambda \int_{\tau_{em}}^{\tau_0} R_P^{-1}(\tau)d\tau]. \tag{3.1}$$

$R_P(\tau) := b(\tau)h^{-1}(\tau)$, and $\Lambda := c/R$. The 3-space is negatively curved (see after (2.2) and Section 1) with curvature radius $Ra(\tau)$, therefore the 'sinh' in (3.1). R is the curvature radius of the 3-space at the present epoch, cf. (1.2). We do not give a derivation of (3.1) here, as d_L immediately follows from Equation (3.14) in Tomaschitz (1998c), if we insert there $E_n \propto h(\tau)$, $T_n \propto h^{-1}(\tau)$, and $E^{rad} \propto \omega \propto R_P^{-1}(\tau)$, cf. (1.5) and (1.2). The metric distance between source and observer [with respect to the line element $a(\tau)d\sigma$ on the 3-space] reads

$$d_M(\tau) = ca(\tau) \int_{\tau_{em}}^{\tau_0} R_P^{-1}(\tau)d\tau. \tag{3.2}$$

The redshift is determined by the scale factor $b(\tau)$ of the permeability tensor as

$$1 + z = b(\tau_0)/b(\tau_{em}), \tag{3.3}$$

(Tomaschitz, 1998c). [In deriving (3.3), the photon energy has to be normalized by the atomic energy levels, $E_n \propto h(\tau)$, cf. (1.5), we assume that the emission takes place at a time at which the gravitational potential in atoms can still be neglected, cf. the discussions following (2.14) and (4.12).]

If τ_{em} is close to τ_0, we may substitute into (3.1)–(3.3) the series expansions of $h(\tau)$ and $b(\tau)$ in powers of $\tau_0 - \tau$. To this end, we define

$$H(\tau) := \dot{b}/b, \quad p(\tau) := -\ddot{b}b/\dot{b}^2, \quad q(\tau) := p(\tau) + \frac{\dot{h}}{h}\frac{b}{\dot{b}}; \tag{3.4}$$

H_0, p_0, and q_0 denote the respective values at the present epoch τ_0. We find

$$H_0(\tau_0 - \tau_{em}) = z - (1 + p_0/2)z^2 + O(z^3), \tag{3.5}$$

$$D(z) := H_0 \int_{\tau_{em}}^{\tau_0} h(\tau) b^{-1}(\tau) d\tau$$

$$= \frac{h_0}{b_0} H_0(\tau_0 - \tau_{em}) \left(1 + \frac{1}{2}\left(1 - \frac{1}{H_0}\frac{\dot{h}_0}{h_0}\right) H_0(\tau_0 - \tau_{em}) + \ldots\right), \tag{3.6}$$

$$d_L = R(1+z)\sinh(\Lambda H_0^{-1} D(z)) = \frac{cz}{H_0}\left(1 + \frac{z}{2}(1 - q_0) + \ldots\right). \tag{3.7}$$

In this way H_0 is identified as Hubble constant and q_0 as deceleration parameter. d_L relates to the metric distance at absorption and emission time as

$$\frac{d_L}{d_M(\tau_0)} = (1+z)\Delta_H(z), \quad \Delta_H(z) := \frac{\sinh(\Lambda H_0^{-1} D(z))}{\Lambda H_0^{-1} D(z)}, \tag{3.8}$$

$$\frac{d_M(\tau_{em})}{d_M(\tau_0)} = \frac{a(\tau_{em})}{a(\tau_0)}, \quad d_M(\tau_0) = \frac{cz}{H_0}\left(1 - \frac{z}{2}(1 + q_0) + \ldots\right). \tag{3.9}$$

R does not enter in the above expansions in the indicated order. The luminosity distance for a flat 3-space is obtained by performing the limit $R \to \infty$ in (3.7), which means to put $\Delta_H(z) \equiv 1$. (In the case of a positively curved 3-space, we have to replace $\sinh x$ by $\sin x$ in Δ_H.) If $D(z)$ diverges in the high-z limit, i.e., if no horizon appears in the look-back time, then the luminosity distance increases exponentially in a hyperbolic 3-space. When this exponential increase sets in, or at which $z\,\Delta_H(z)$ starts to differ from 1 in a noticeable way, this evidently depends on the present curvature radius. As the gravitational constant is varying, the Einstein equations are not applicable. Accordingly, R cannot be obtained from H_0 and q_0, but has to be determined independently, see after (5.9).

Returning to (2.12), we find

$$\frac{\dot{H}}{H} = -H(\tau)(1 + p(\tau)), \quad \frac{\dot{\kappa}}{\kappa} = \frac{\dot{H}}{H} - \frac{\dot{h}}{h} = -H(\tau)(1 + q(\tau)). \tag{3.10}$$

At the present epoch, we may take the time derivatives in (3.10) with respect to atomic time, because $dt \approx d\tau$, cf. (2.16) and (1.2). In atomic units, we have $k(t) \propto \kappa(\tau(t))$, as pointed out after (2.12), and hence $d \log k(t_0)/dt = -H_0(1 + q_0)$, as announced in Equation (1.8). This gives a very tight bound on q_0; with H_0 as stated after (1.7), and a very conservative $|\dot{k}_0/k_0| < 10^{-11}$ yr^{-1}, we safely obtain $q_0 = -1 + \varepsilon$, $|\varepsilon| < 0.15$. Current bounds on $|\dot{k}_0/k_0|$ lie in the range $10^{-11} - 10^{-12}$ yr^{-1}, inferred from radar tracking of planets (Shapiro, 1990), binary pulsars and neutron stars (Kaspi *et al.*, 1994; Thorsett, 1996), and lunar laser ranging (Dickey *et al.*, 1994; Williams *et al.*, 1996). More phenomenological estimates (roughly in the same range) are obtained from planetary palaeoradii (McElhinny *et al.*, 1978), luminosity functions of white dwarfs (García-Berro *et al.*, 1995), main-sequence fitting of globular clusters (Degl'Innocenti *et al.*, 1996), and from helioseismology (Guenther *et al.*, 1998). Bounds from primordial nucleosynthesis, quoted in García-Berro *et al.* (1995) and Guenther *et al.* (1998), are on the lower end, 10^{-12} yr^{-1}, but rely on the Einstein equations or their generalizations. A time variation of k in the upper half of this range could also resolve the 'early faint sun paradox', without invoking an enhanced greenhouse effect and/or a lower albedo to reconcile the Earth's high surface temperature with the weak early solar luminosity (Newman and Rood, 1977; Kasting and Grinspoon, 1991; Sagan and Chyba, 1997).

This laboratory estimate on q_0 is of course completely independent of the cosmological attempts to extract q_0 from the luminosity distance, the angular diameters, and the surface brightness, cf. Section 4. Riess *et al.* (1998) find $q_0 = -1 \pm 0.4$; however, there is no consensus even on the sign of this parameter.

4. Angular Diameters and Surface Brightness: How Rigid are Galactic Measuring Rods?

We denote by $y(\tau_{em})$ the intrinsic diameter of a galaxy (measured in its locally geodesic rest frame) and by $d_M(\tau_{em})$ the metric distance, between galaxy and observer, cf. (3.2), both at emission time. The angular diameter, i.e., the angle measured by the observer between the rays arriving from the opposite ends of the diameter, reads $\theta = y(\tau_{em})/d_M(\tau_{em})$, to be expressed as a function of the redshift and the arrival time τ_0 of the photons. The galaxy diameter $y(\tau)$ scales like the planetary radius in (2.14) (Teller, 1948),

$$y(\tau_{em}) = y_0 h^{1/3}(\tau_{em}) \kappa^{-1}(\tau_{em}). \tag{4.1}$$

In our first example, we specify the scale factors in the line elements ds^2 and ds_P^2 of metric and permeability tensor as power laws,

$$a(\tau) = (\tau/\tau_0)^{\alpha}, \quad b(\tau) = (\tau/\tau_0)^{\beta}, \quad h(\tau) = (\tau/\tau_0)^{\gamma}, \tag{4.2}$$

so that $\gamma = 3(\alpha - \beta)$ and $H(\tau) = \beta/\tau$, cf. (1.2) and (3.4). By means of (3.2)–(3.4), (3.9) and (2.12), we readily calculate

$$\tau_{em}/\tau_0 = (1+z)^{-1/\beta}, \quad q_0 = (\gamma+1)/\beta - 1,$$

$$d_M(\tau_{em}) = cH_0^{-1}(1+z)^{-\alpha/\beta} D(z),$$

$$D(z) = \frac{(1+z)^{-q_0} - 1}{-q_0}, \quad \kappa(\tau_{em}) = (\tau_{em}/\tau_0)^{-(\gamma+1)} = (1+z)^{(\gamma+1)/\beta}, \qquad (4.3)$$

($D(z)$ is defined in (3.6)), so that

$$y(\tau_{em}) = y(\tau_0)(\tau_{em}/\tau_0)^{4(\alpha-\beta)+1} = y_0(1+z)^{-(4(\alpha-\beta)+1)/\beta}. \qquad (4.4)$$

For comparison, we note the scaling of the diameter for $\kappa(\tau) \equiv 1$, $y_{\dot{\kappa}=0} = y_0(1+z)^{1-\alpha/\beta}$, cf. the Remarks following (2.14). The redshift dependence of θ follows from (4.3) and (4.4),

$$\theta(z) = \frac{H_0 y_0}{c} \frac{(1+z)^{-q_0}}{D(z)}. \qquad (4.5)$$

Hence, for $z \to \infty$,

$$\theta(q_0 < 0) \sim 1, \quad \theta(q_0 = 0) \sim 1/\log z, \quad \theta(q_0 > 0) \sim z^{-q_0}. \qquad (4.6)$$

For comparison, if $\kappa(\tau) \equiv 1$ [which is equivalent to standard RW cosmology with expansion factor $a_{RW}(t) = t^{1/(1+q_0)}$, cf. (1.6)], we find the RW results

$$\theta_{\dot{\kappa}=0}(q_0 < 0) \sim z^{1+q_0}, \quad \theta_{\dot{\kappa}=0}(q_0 = 0) \sim z/\log z, \quad \theta_{\dot{\kappa}=0}(q_0 > 0) \sim z. \qquad (4.7)$$

In the following we put $q_0 = -1 + \varepsilon$, cf. the end of Section 3. To obtain redshifts, we evidently have to require $\varepsilon > 0$. The case $\varepsilon = 0$, i.e., $\gamma = -1$, is also acceptable in this respect, leading to a steady state cosmology. Otherwise, the choice of α and β is physically undistinguishable, as long as it leads to the same q_0; it just determines how much of the redshift is due to the expanding galaxy grid, also see the discussion following (4.28). Convenient choices are $\alpha = 0$, leading to a static galaxy background, in which the redshift is entirely an effect of the contracting measuring rods, or $\alpha = 1$, which results in an expanding space-time isometric to the forward light cone (Milne universe, cf. Robertson and Noonan, 1968). If $\alpha = \beta$, the measuring rods stay invariant, and the redshift is caused by the space expansion only.

As long as $\kappa(\tau) \equiv 1$, the permeable space-time is always equivalent to a RW cosmology, cf. Section 1. If, however, the gravitational constant varies in a way that the small ratio (1.7) stays constant instead of $e^2/(km^2)$, which requires (2.12), then this has a substantial impact on the angular diameter. The RW angular diameters (4.7) increase for large redshifts, with the exception of the steady state case $\varepsilon = 0$, and this increase is not observed (Sandage, 1988), whereas the diameter (4.6) approaches a finite limit value in the relevant ε-range, $|\varepsilon| < 0.15$. In our second example, the varying gravitational constant will even lead to a power law decrease of θ for high z.

Unlike the luminosity distance, the surface brightness crucially depends on the time variation of the gravitational constant. The angular diameter corresponds to a solid angle of $\pi\theta^2/4$, since θ is the opening angle of a cone with apex at the observer and the galaxy diameter as base line. The surface brightness is defined as the energy flux arriving at the observer per unit solid angle,

$$SB = \frac{4L_{app}}{\pi\theta^2} = \frac{L}{\pi^2}\frac{1}{(\theta d_L)^2}. \tag{4.8}$$

The luminosity-distance (3.1) can readily be assembled from (4.3) and (3.8),

$$d_L/d_M(\tau_{em}) = (1+z)^{1+\alpha/\beta}\Delta_H(z). \tag{4.9}$$

If we assume $\Delta_H(z) \approx 1$, i.e., if we neglect the 3-space curvature, we find

$$SB(z) = \text{const. } y^{-2}(z)(1+z)^{-2(1+\alpha/\beta)}, \tag{4.10}$$

with y as given in (4.4). Clearly, if we insert $y_{\dot{\kappa}=0}$ (defined after (4.4)) into (4.10), we obtain the model-independent RW result, $SB \propto (1+z)^{-4}$ (Sandage and Perelmuter, 1991; Moles *et al.*, 1998; Petrosian, 1998).

Atomic time relates to cosmic time as

$$t_{at.}(\tau_0, \tau) = \int_\tau^{\tau_0} h(\tau)d\tau = \frac{\tau_0}{\varepsilon\beta}(1 - (\tau/\tau_0)^{\varepsilon\beta}), \tag{4.11}$$

cf. (2.16), and gravitational time relates to τ as

$$t_{gr.}(\tau_0, \tau) = \int_\tau^{\tau_0} \kappa^2(\tau)h(\tau)d\tau = \frac{\tau_0}{\varepsilon\beta}((\tau/\tau_0)^{-\varepsilon\beta} - 1), \tag{4.12}$$

with $\kappa(\tau) = (\tau/\tau_0)^{-\varepsilon\beta}$, cf. (2.15). The definition of atomic time (4.11) is of course only valid as long as we can neglect the mass dependent ratio in (2.16). In the following, we assume $h_0 \approx 0.7$, cf. (1.7), so that $1/H_0 \approx 14$ Gyr. We put $\alpha = \beta = 1/\varepsilon$; the present epoch then reads as $\tau_0 \approx 14\varepsilon^{-1}$ Gyr, and atomic time coincides with the cosmic time parameter in the approximation (4.11). (As mentioned, $\varepsilon = 0$ corresponds to the steady state case, with an infinite age and constant H_0 and k. An example in which the cosmic age is quite moderate, even if ε is very small, will be discussed below.) The meteoritic age of the solar system is $\Delta t_{at.} = \tau_0 - \tau_s \approx 4.57$ Gyr (Bahcall *et al.*, 1995), so that $\tau_s/\tau_0 \approx 1 - (4.57/14)\varepsilon$. (At the beginning of Section 6, it is pointed out that the radiometric dating methods are not affected by the variation of the constants.) As $\varepsilon < 0.15$, $\kappa(\tau_s)$ is moderate, and this justifies the neglect of the gravitational potential in the definition of atomic time. The same holds, by the way, for the age of the Galaxy, which is unlikely to exceed 20 Gyr (VandenBerg *et al.*, 1996; Cowan *et al.*, 1997, 1999; Lineweaver, 1999). What remains is to convert the atomic time of 4.57 Gyr into gravitational time; this is readily done by means of (4.12), $\Delta t_{gr.} \approx 4.57(1 - (4.57/14)\varepsilon)^{-1}$ Gyr. For $\varepsilon < 0.15$, there is no substantial difference between the atomic and gravitational age of the solar system. Also note that the gravitational age of the universe is infinite, as (4.12) diverges for $\tau \to 0$. In this limit the approximation (4.11) is not any more valid, and one has to use the exact expression (2.16) as integrand in (4.11). Accordingly, also the atomic age of the universe is infinite, as atomic clocks turn gravitational for small τ.

Our second example is defined by the scale factors

$$a(\tau) = \frac{\sinh^{\mu}\left(\frac{\alpha}{\lambda}\frac{\tau}{\tau_0}\right)}{\sinh^{\mu}(\alpha/\lambda)}, \quad b(\tau) = \frac{\sinh^{\lambda}\left(\frac{\alpha}{\lambda}\frac{\tau}{\tau_0}\right)}{\sinh^{\lambda}(\alpha/\lambda)}, \quad h(\tau) = \frac{\sinh^{3(\mu-\lambda)}\left(\frac{\alpha}{\lambda}\frac{\tau}{\tau_0}\right)}{\sinh^{3(\mu-\lambda)}(\alpha/\lambda)}, \tag{4.13}$$

$\alpha, \lambda > 0$. This means a power law scaling for $\tau \to 0$ as in the first example, so that the universe has a finite age τ_0, and the exponential scaling for $\tau \to \infty$ is suggested by the fact that q_0 is close to -1, see after (4.6). We find, cf. (3.4),

$$H(\tau) = \frac{\alpha}{\tau_0} \coth\left(\frac{\alpha}{\lambda}\frac{\tau}{\tau_0}\right), \quad 1 + p(\tau) = \frac{1}{\lambda}\cosh^{-2}\left(\frac{\alpha}{\lambda}\frac{\tau}{\tau_0}\right), \tag{4.14}$$

which requires $p_0 > -1$. p_0 is now related to the deceleration parameter by $p_0 = 3(\lambda - \mu)/\lambda + q_0$. We find

$$\tau_0 = \frac{\alpha(p_0, \lambda)}{H_0\sqrt{1-(1+p_0)\lambda}}, \quad \alpha(p_0, \lambda) = \frac{1}{2}\lambda \log \frac{1+\sqrt{1-(1+p_0)\lambda}}{1-\sqrt{1-(1+p_0)\lambda}}, \tag{4.15}$$

which requires $0 < \lambda < (1+p_0)^{-1}$. We readily calculate

$$\frac{\dot{H}_0}{H_0} = -\frac{\alpha}{\lambda\tau_0}\frac{1}{\sinh(\alpha/\lambda)\cosh(\alpha/\lambda)}, \tag{4.16}$$

$$\sinh\left(\frac{\alpha}{\lambda}\frac{\tau_{em}}{\tau_0}\right) = \frac{\sinh(\alpha/\lambda)}{(1+z)^{1/\lambda}}, \quad \sinh(\alpha/\lambda) = \sqrt{\frac{1-(1+p_0)\lambda}{(1+p_0)\lambda}}, \tag{4.17}$$

$$\tau_{em}(z \to \infty) \sim \frac{1}{H_0}\sqrt{\frac{\lambda}{(1+p_0)}}z^{-1/\lambda}, \quad \frac{d_M(\tau_{em})}{d_M(\tau_0)} = (1+z)^{-\mu/\lambda}; \tag{4.18}$$

the last relation follows from (3.9). To obtain the large-z asymptotics of the luminosity-distance, we note, cf. (3.6),

$$D(z) = H_0\frac{\lambda\tau_0}{\alpha}\sinh^{-3\mu+4\lambda}(\alpha/\lambda)\int_{\frac{\alpha}{\lambda}\frac{\tau_{em}}{\tau_0}}^{\alpha/\lambda} \sinh^{3\mu-4\lambda}(\tau)d\tau =$$

$$\lambda\int_{(1+z)^{-1/\lambda}}^{1} \frac{y^{3\mu-4\lambda}dy}{\sqrt{(1+p_0)\lambda + [1-(1+p_0)\lambda]y^2}}. \tag{4.19}$$

$D(z)$ can readily be expressed in terms of hypergeometric functions, but for the following asymptotic results this is not needed. If $1 + 3\mu - 4\lambda > 0$, $D(z)$ admits a finite limit value for $z \to \infty$. Otherwise, if $1 + 3\mu - 4\lambda < 0$, it diverges,

$$D(z \to \infty) \sim z^{(4\lambda-3\mu-1)/\lambda}. \tag{4.20}$$

We so find the asymptotic behavior of the luminosity-distance as

$$d_L \sim z, \ \ d_L \sim z \log z \, \Delta_H(z), \ \ d_L \sim z^{(5\lambda-3\mu-1)/\lambda} \Delta_H(z), \tag{4.21}$$

for $1 + 3\mu - 4\lambda > (=, <)0$, respectively. In the first asymptotic relation in (4.21) we need not indicate Δ_H, as it approaches a finite limit value due to the horizon. Analogously, the asymptotics of the metric distance at emission time reads as

$$d_M(\tau_{em}) \sim z^{-\mu/\lambda}, \ \ d_M(\tau_{em}) \sim z^{-\mu/\lambda} \log z, \ \ d_M(\tau_{em}) \sim z^{-(4(\mu-\lambda)+1)/\lambda}. \tag{4.22}$$

(In (4.20)–(4.22), there are of course constant factors in front of the z-terms, which we did not indicate.)

As for the source diameter, we find from (4.1), (2.12), (4.14) and (4.17),

$$y(\tau_{em}) = y_0 \frac{(1+z)^{-4(\mu-\lambda)/\lambda}}{\sqrt{1 + \lambda(1+q_0)((1+z)^{2/\lambda} - 1)}}, \tag{4.23}$$

so that $y(z \to \infty) \sim z^{-(1+4(\mu-\lambda))/\lambda}$, and $y(z \to 0) = y_0(1 - (q_0 + \mu/\lambda)z + \ldots)$. For comparison with the standard theory, if $\kappa \equiv 1$ in (4.1), we have $y_{\dot{\kappa}=0}(\tau_{em}) = y_0(1+z)^{-(\mu-\lambda)/\lambda}$. The large-$z$ asymptotics of the angular diameter can now be readily compiled; we obtain, for $1 + 3\mu - 4\lambda > (=, <)0$,

$$\theta \sim z^{-(1+3\mu-4\lambda)/\lambda}, \ \ \theta \sim 1/\log z, \ \ \theta \sim 1, \tag{4.24}$$

respectively. If $\kappa \equiv 1$, we find

$$\theta_{\dot{\kappa}=0} \sim z, \ \ \theta_{\dot{\kappa}=0} \sim z/\log z, \ \ \theta_{\dot{\kappa}=0} \sim z^{(3(\mu-\lambda)+1)/\lambda}, \tag{4.25}$$

for the respective parameter ranges. In the limit $z \to 0$, we have in any case $\theta =$ const. $z^{-1}(1 + z(1-q_0)/2 + \ldots)$, to be compared to the standard RW result $\theta_{\dot{\kappa}=0} =$ const. $z^{-1}(1 + z(q_0+3)/2 + \ldots)$. The observational state of the art concerning angular sizes of radio sources is discussed in Dabrowski *et al.* (1995); it does not seem possible as yet to extract q_0 from the small redshift limit.

As for the surface brightness (4.8), we obtain, from (3.8) and (4.18),

$$SB(z) = \text{const. } y^{-2}(z)(1+z)^{-2(1+\mu/\lambda)}, \tag{4.26}$$

with (4.23) substituted. (We again assume $\Delta_H(z) \approx 1$, neglecting the 3-space curvature.) Hence, $SB(z \to \infty) \sim z^{2(1+3\mu-5\lambda)/\lambda}$, and $SB(z \to 0) =$ const. $(1 + 2(q_0 - 1)z + \ldots)$. The small redshift limit is evidently close to the standard RW result (with $y_{\dot{\kappa}=0}$ in (4.26)), since $q_0 = -1 + \varepsilon$, $|\varepsilon| < 0.15$.

We may write in (4.15) $(1 + p_0)\lambda = 3(\lambda - \mu) + \varepsilon\lambda$. The exponents λ and μ in (4.13) must be such, that

$$0 < \varepsilon\lambda + 3(\lambda - \mu) < 1, \ \ 0 \le 1 + 3\mu - 4\lambda. \tag{4.27}$$

The first of these inequalities follows from the inequalities stated after (4.14) and (4.15), and the second follows from (4.24), as the angular diameter should decay for high z. Moreover, α in (4.15) should not be too small, otherwise we cannot accommodate the age of the Galaxy. If we take all that into account, we conclude $\lambda \approx \mu$, and λ and μ cannot be much larger than one, since ε is small. If in addition the surface brightness is not to diverge for high z, the range of these exponents cannot be far outside the interval [1/2,1].

Firstly, if $\lambda = \mu = 1$, we obtain a de Sitter cosmology with a varying gravitational constant, where the angular diameter approaches zero logarithmically, cf. (4.24). [In standard de Sitter cosmology, $\theta_{\dot{\kappa}=0}$ is increasing, cf. (4.25).] The high-z asymptotics of the surface brightness reads as $SB \sim z^{-2}$. This is more or less the steepest descent that can be achieved with the scale factors (4.13), if the angular diameter is not to diverge, and as long as $\Delta_H(z) \approx 1$, see after (5.9).

Secondly, if $\theta \sim z^{-1}$, we have to require $1 + 3\mu - 5\lambda = 0$, cf. (4.24); we then obtain $SB \sim 1$, and the condition $\lambda \approx \mu$ suggests at first $\lambda = \mu = 1/2$. We readily calculate, for $\lambda = \mu$, $\alpha = (\mu/2)\log(4/(\varepsilon\mu)) + O(\varepsilon)$, cf. (4.15). We find $\alpha \approx 1$, based on $\mu = 1/2$ and the bound $\varepsilon = 0.15$, and thus $\tau_0 \approx 14$ Gyr; a smaller ε would clearly increase this lower bound on the age of the universe. However, a tiny ε need not lead to an excessive age, if we put $\mu = 1/2 - |\varepsilon|^\eta$, $\lambda = 1/2 - (3/5)|\varepsilon|^\eta$, with $0 < \eta < 1$. Then we still have $\theta \sim z^{-1}$ and $SB \sim 1$, but for α we now obtain $\alpha \approx (1/4)\log(10/(3|\varepsilon|^\eta))$. We can in fact prescribe a moderate τ_0 commensurate with the galactic age, and choose η accordingly.

Finally, the asymptotics $\theta \sim z^{-1/2}$ and $SB \sim z^{-1}$ follows from $\lambda = \mu = 2/3$, which results, for $\varepsilon = 0.15$, in $\alpha \approx 1.2$ and thus in a lower bound of $\tau_0 \approx 17$ Gyr for the age of the universe. As pointed out, a smaller ε results in a larger age, which can be reduced by choosing $\lambda \approx \mu = 2/3 + O(|\varepsilon|^\eta)$, $0 < \eta < 1$; the indicated exponents of θ and SB can only mildly be affected by the $O(|\varepsilon|^\eta)$ terms.

Observational attempts to infer the scaling exponents of angular diameter and surface brightness are not yet conclusive (Dabrowski *et al.*, 1995; Moles *et al.*, 1998); we discuss two results concerning the linear sizes of radio sources. For $\lambda \approx \mu$ and high z, we obtain the diameter scaling $y \approx z^{-1/\lambda}$, as pointed out after (4.23). Neeser *et al.* (1995) find, in the framework of the standard model, from a sample mainly consisting of radio galaxies, a scaling exponent $-1/\lambda = -1.2 \pm 0.5$ for $q_0 = 0$, and $-1/\lambda = -1.7 \pm 0.4$ for $q_0 = 1/2$. On the other hand, it follows from the Remark after (4.6), that $q_0 = 0$ corresponds to $\lambda = \mu = 1$, and $q_0 = 1/2$ to $\lambda = \mu = 2/3$ for $\tau \to 0$ in (4.13). Accordingly, there is agreement in both cases well within the error bounds. As for quasars, Barthel and Miley (1988) find for the redshift scaling of the linear size the bounds $3/2 < 1/\lambda < 2$. These results would suggest $\lambda \approx \mu \approx 2/3$, or at least $1/2 < \lambda \approx \mu < 1$ as restrained after (4.27). However, there also exist several other, rather diverging estimates on the scaling exponents of the linear and angular sizes of radio galaxies and quasars (Dabrowski *et al.*, 1995; Neeser *et al.*, 1995, and references therein).

The considerations concerning atomic and gravitational time are quite analogous to those following (4.11) and (4.12), and will not be repeated here. For a cosmic age exceeding 14 Gyr, there is no noticeable difference between the atomic and gravitational age of the solar system.

As the gravitational constant varies, the Einstein equations are not applicable, but there is the possibility to reconstruct the scale factors, if we know the angular diameter or the surface brightness as a function of the redshift. In the following we write τ for τ_{em}. We insert (3.3) into (3.1), regard τ as a function of z, differentiate, make use of (1.2), and obtain

$$\frac{ch(\tau)}{H(\tau)} = \delta_R(z)\frac{d}{dz}\frac{d_L(z)}{1+z}, \quad \delta_R(z) := \left(1 + \frac{d_L^2(z)}{R^2(1+z)^2}\right)^{-1/2},$$

$$-\frac{\delta_R(z)}{1+z}\frac{d}{dz}\frac{d_L(z)}{1+z}dz = ch(\tau)d\tau. \tag{4.28}$$

If $d_L(z)$ is known, and $h(\tau)$ is arbitrarily prescribed, then we can solve the second equation in (4.28) for $z(\tau)$, and so we find $b(\tau) = b(\tau_0)(1 + z(\tau))^{-1}$, cf. (3.3), and the expansion factor $a(\tau)$ via (1.2). $h(\tau)$ determines the fraction of the redshift caused by the space expansion; if we choose $h(\tau) \equiv 1$, then the redshift is entirely due to the space expansion, and if we solve (4.28) with $h(\tau) = (1 + z)^3$, then it is a consequence of the contracting measuring rods in a static space time, $a(\tau) \equiv b(\tau_0) = 1$, cf. Section 1. The integration constant in $z(\tau)$ is fixed by the integration ranges $\int_0^z$ and $\int_{\tau_0}^{\tau}$, cf. Weinberg (1972).

From (4.1), (3.8), (1.2), and (4.28), we readily derive

$$\theta(z) = \frac{y_0 H_0}{c}\frac{\delta_R(z)(1+z)}{R \operatorname{arcsinh}\left(\frac{d_L(z)}{R(1+z)}\right)}\frac{d}{dz}\frac{d_L(z)}{1+z},$$

$$SB(z) = \frac{L}{\pi^2}\frac{c^2}{y_0^2 H_0^2}\frac{R^2 \operatorname{arcsinh}^2\left(\frac{d_L(z)}{R(1+z)}\right)}{\delta_R^2(z)(1+z)^2 d_L^2(z)\left(\frac{d}{dz}\frac{d_L(z)}{1+z}\right)^2}. \tag{4.29}$$

(In the case of positive 3-space curvature, we replace in δ_R the plus by a minus sign, and arcsinh by arcsin.) Both equations can easily be integrated, in particular for $R = \infty$, if the left side is known. Returning to the second example discussed after (4.27), we assume $\theta = \tilde{a}_\theta z^{-1} + \tilde{b}_\theta z^{-2} + O(z^{-3})$, so that we obtain $d_L(z) = \tilde{a}_L z + \tilde{b}_L + O(z^{-1})$ from (4.29) (with $R = \infty$). If we choose $h(\tau) \sim \tilde{h}\tau^{2\gamma}$ in (4.28), we find $z^{-3}dz \sim$ const. $\tau^{2\gamma}d\tau$, and thus $b(\tau \to 0) \sim \tilde{b}\tau^{1/2+\gamma}$ and $a(\tau) \sim \tilde{a}\tau^{1/2+5\gamma/3}$. With $\gamma = -(3/5)|\varepsilon|^\eta$, we recover the exponents $\lambda = 1/2 - (3/5)|\varepsilon|^\eta$ and $\mu = 1/2 - |\varepsilon|^\eta$ of the scale factors, as discussed above. Finally, we may eliminate d_L and d_L' in one of the equations (4.29), by means of

$$SB(z)\theta^2(z)d_L^2(z) = L/\pi^2, \quad -\frac{d_L'}{d_L} = \frac{1}{2}\frac{SB'}{SB} + \frac{\theta'}{\theta}. \tag{4.30}$$

If SB and θ are observationally determined, this allows a consistency check, as well as a possibility to infer R, the curvature radius of the 3-space at the present epoch; in the next section, R will be determined by a very different method.

5. Implications of the Constancy of Ω_m on Source Counts and the Curvature Radius

A moderate dimensionless ratio can be composed of k, H_0, and the present-day mass density of the universe ρ_m,

$$\Omega_m := \frac{8\pi}{3}\frac{k\rho_m}{H_0^2} \approx 0.3 \pm 0.2, \tag{5.1}$$

(Riess *et al.*, 1998; Lineweaver, 1999; Perlmutter *et al.*, 1999), whose constancy requires for the mass and number densities the scaling laws

$$\rho_m \propto H(\tau)h^{-11/3}(\tau), \quad \rho_N \propto H(\tau)h^{-2}(\tau), \tag{5.2}$$

respectively. (The factor $8\pi/3$ in (5.1) is just a convention in connection with the Einstein equations, and irrelevant in this context.) In deriving (5.2), we used the scaling laws (2.12), and (1.4).

In the following we study source counts based on the scaling law (5.2) for the number density, which is assumed to hold universally, for optical and radio sources alike. Conservation of the source number would require, contrary to (5.2), $\rho_N(\tau) \propto b^{-3}(\tau)h^{-1}(\tau)$, as pointed out after (1.6). Densities leading to a non-conserved source number were already discussed in Tomaschitz (1998c) in the context of a flat 3-space. There is a peak in the quasar distribution at $z_{\max} \approx 2.3$ (Hartwick and Schade, 1990; Schmidt *et al.*, 1995; Maloney and Petrosian, 1999), and the scaling exponent of the density ρ_N was determined in a way that this peak appears in the number density dN/dz. This required a density slightly different from the steady state case, mentioned after (1.6), $\rho_N \propto \tau^{2\lambda-3}$, $\lambda \approx 1.9$, for $R = \infty$ and exponents $\gamma = -1$, $\alpha = 1$, and $\beta = 4/3$ in (4.2). In this paper we use the scaling law (5.2) for the quasar density ρ_N, and determine the present-day curvature radius R of the 3-space in a way that the observed peak in the density dN/dz is reproduced. We start with

$$dN = 32\pi a^3(\tau)\rho_N(\tau)\frac{r^2 dr}{(1 - r^2/R^2)^3} = \hat{\rho}_N(\tau)\,\text{area}\,(r_H)dr_H,$$

$$\hat{\rho}_N(\tau) := a^3(\tau)\rho_N(\tau), \quad r_H := \int_{\tau_{em}(z)}^{\tau_0} R_P^{-1}(\tau)d\tau,$$

$$\text{area}\,(r_H) := 4\pi R^2 \sinh^2(r_H/R) = 4\pi\frac{d_L^2(z)}{(1+z)^2}, \tag{5.3}$$

cf. Tomaschitz (1998). Using (4.28), we find

$$\frac{dr_H}{dz} = -c\frac{h}{b}\frac{d\tau}{dz} = \delta_R(z)\frac{d}{dz}\frac{d_L(z)}{1+z}, \tag{5.4}$$

$$dN(z) = \frac{4\pi}{3}\hat{\rho}(\tau_{em}(z))\delta_R(z)\frac{d}{dz}\frac{d_L^3(z)}{(1+z)^3}dz. \tag{5.5}$$

A conserved quasar number means $\rho_N \propto b^{-3}h^{-1}$, as discussed after (1.6), and thus $\hat{\rho}_N(\tau) = \rho_N(\tau_0)$, cf. (1.2). With this density we obtain, via (3.7),

$$dN(z)/dz = 4\pi R^2 c H_0^{-1}\rho(\tau_0)\sinh^2(\Lambda H_0^{-1}D(z))dD(z)/dz. \tag{5.6}$$

This is just the standard RW result, but there is no peak in this distribution, with $D(z)$ as in (4.3) or in (4.19). However, this changes if we assume the quasar density to scale according to (5.2), $\rho_N(\tau) = \rho_N(\tau_0)H_0^{-1}H(\tau)h^{-2}(\tau)$. We find, with (4.28) and (5.5),

$$\hat{\rho}_N(\tau_{em}(z)) = \rho_N(\tau_0)\frac{c}{H_0}\frac{1}{\delta_R(z)(1+z)^3}\left(\frac{d}{dz}\frac{d_L(z)}{1+z}\right)^{-1}, \tag{5.7}$$

$$dN/dz = 4\pi\rho_N(\tau_0)\frac{c}{H_0}\frac{d_L^2(z)}{(1+z)^5} = 4\pi R^2\rho_N(\tau_0)\frac{c}{H_0}\frac{\sinh^2(\Lambda H_0^{-1}D(z))}{(1+z)^3}. \tag{5.8}$$

A peak in (5.8) at $z_{\max}$ requires $N''(z_{\max}) = 0$, or

$$\tanh(\Lambda H_0^{-1}D(z_{\max})) = (2/3)\Lambda H_0^{-1}(1+z_{\max})dD(z_{\max})/dz. \tag{5.9}$$

What remains is to solve (5.9) for Λ ($\Lambda := c/R$, cf. (3.1)), with $z_{\max} \approx 2.3$. If $\varepsilon \to 0$, we may approximate $D(z) \approx z$ in (4.3), and the same approximation also holds in this limit for $D(z)$ in (4.19), as long as z is not very large. (This follows from $q_0 = -1 + \varepsilon$ and $\lambda - \mu = O(|\varepsilon|^\eta)$, cf. the discussion after (4.27).) Hence, the solution of (5.9) generating a maximum in the number density (5.8) at $z_{\max}$ reads in either case $\Lambda H_0^{-1} \approx 0.16$, which suggests a present-day curvature radius of $R \approx 6.3c/H_0$, if $\varepsilon \ll 0.15$. On this basis we can estimate the effect of the space curvature on the luminosity distance, $\Delta_H(z) < 1.1$, for $z < 5$, cf. the discussion following (3.9).

With $\Lambda H_0^{-1} \approx 0.16$, we calculate the second solution of (5.9), $z_{\min} \approx 5.9$, which corresponds to a minimum of (5.8), before the exponential increase. The extrema of (5.8) read as $N'(z_{\max})/(4\pi R^2\rho_N c H_0^{-1}) \approx 4.1\times 10^{-3}$ and $N'(z_{\min})/(4\pi R^2\rho_N c H_0^{-1}) \approx 3.6\times 10^{-3}$. The decrease of $N'(z)$ in the range $[z_{\max}, z_{\min}]$ is thus mild, and one may wonder if this is the reason why the fall-off in the quasar density is not found in all surveys, and that rather large error bounds are still occasionally cited for $z_{\max}$, cf. the literature quoted above, and references therein.

Remark: The numerical values given here for ΛH_0^{-1} and $z_{\min}$ hold for $\varepsilon \to 0$. If $\varepsilon \approx 0.15$, which is rather unlikely, cf. the end of Section 3, one has to use the exact expression for $D(z)$ in (5.9) to obtain reasonably accurate results, but even then there is no qualitative change. It is quite possible that the bound on the logarithmic time derivative of the gravitational constant (and hence on ε) cited after (3.10) can be improved by several orders of magnitude, so that the time variation of k has a noticeable effect only at a very early epoch; binaries are promising candidates for this endeavor (Kaspi *et al.*, 1994; Thorsett, 1996). But however small the variation of k may be today, the gravitational attraction overpowers all other interactions in the limit $\tau \to 0$, cf. the discussion following (2.14).

6. Conclusion

Three independent dimensionless ratios can be composed from the six laboratory constants $m, c, \hbar, e, g_F, g_s$. They can be chosen to be moderate numbers,

$$e^2/(4\pi\hbar c) \approx 1/137, \quad g_s^2/(4\pi\hbar c) := \alpha_s(m_Z) \approx 0.12, \quad g_F m_W^2 c/\hbar^3 = 0.075, \tag{6.1}$$

cf. Caso *et al.* (1998). [The definition of the strong coupling constant g_s is somewhat arbitrary here; if one defines it phenomenologically by means of a Yukawa potential, and then infers its numerical value from the deuteron binding energy (Davies, 1972), one arrives at the same conclusion, namely, that $g_s^2/(\hbar c)$ is a moderate number.] As small dimensionless ratios should not scale in cosmic time, we readily obtain with (1.4) the scaling laws for the strong interaction and the Fermi constant,

$$g_s \propto h^{2/3}(\tau), \quad g_F \propto h^2(\tau). \tag{6.2}$$

By virtue of the scaling laws (1.4), the constancy of the Planck constant, and (6.2), it is easy to see from the Gamow formula for α-decay and Fermi's formula for the ft-value, or simply on dimensional grounds, that the decay constants scale as $\lambda(\tau) \propto h(\tau)$, for α and β-decay alike, as long as electromagnetic, strong, and weak interactions are not overpowered by the diverging gravitational constant, cf. the discussion following (4.12). This means $\lambda(\tau)d\tau \approx \lambda_0 dt$, cf. (2.16), if dt measures atomic time, and there is overwhelming evidence for this relation to hold within the last 4.5 Gyr, from radiometric age dating of rocks and meteorites (Dyson, 1972; Lindner *et al.*, 1986). As long as $km_e m_p/e^2 \ll 1$, atomic and nuclear measuring rods are determined by the six mentioned laboratory constants, and they are equivalent, admitting time-independent conversion factors. Atomic and nuclear clock rates are then related to cosmic time via $dt \approx h(\tau)d\tau$.

The gravitational constant is not included in the mentioned set of laboratory constants, as no small ratio can be composed with it. However, it matches very well with the Hubble constant, resulting in the small dimensionless ratio (1.7), whose constancy requires the scaling law (2.12) for the gravitational constant. Gravitational time is related to cosmic time via (2.15). Atomic and gravitational time just mean different sets of clocks, whose rates are connected by conversion factors depending on the cosmic time parameter. It does not really matter which clocks we use, because the measurements can be converted, as exemplified after (4.12). All clocks turn gravitational for $\tau \to 0$.

Let us next consider the scaling laws for the cosmic background radiation. In the Planck distribution, the time dependence of the frequency (outlined after (1.2)) is absorbed in the $k_B T$ factor so that $h\nu/(k_B T) \propto 1$, the temperature being redshifted according to (3.3). This requires

$$k_B \propto h(\tau), \quad T \propto b^{-1}(\tau). \tag{6.3}$$

[No confusion should arise here between the unreduced Planck constant h, and the scale factor $h(\tau)$.] The total energy density and the number density of the photon background scale as

$$\rho_E \propto (k_B T)^4/(\hbar c)^3 \propto b^{-4}(\tau), \quad \rho_N \propto (k_B T)^{-1}\rho_E \propto h^{-1}(\tau) b^{-3}(\tau), \tag{6.4}$$

respectively; ρ_N scales like the galaxy density discussed after (1.6), leading to a conserved photon number. The Planckian shape of the microwave spectrum is perfectly conserved, as the time variation of the fundamental constants (with the exception of the gravitational constant, which does not enter here) can be absorbed in the expansion factor, cf. (1.6). [Theories of varying fundamental constants as well as 'tired-light' theories are frequently marred by a distorted Planck spectrum (Steigman, 1978).] The scaling laws for pressure and specific entropy/heat capacities are likewise readily read off from the standard formulas for black-body radiation, $P \propto b^{-4}(\tau)$, and $s \propto c_{V,P} \propto b^{-3}(\tau)$, respectively, and the spectral energy and number densities scale as

$$\rho_E(\nu) \propto h^{-1}(\tau) b^{-3}(\tau), \quad \rho_N(\nu) = \rho_N(\nu)/(h\nu) \propto h^{-2}(\tau) b^{-2}(\tau). \tag{6.5}$$

The cosmic time scaling of a possible cosmic tachyon background radiation, a Bose gas composed of the quantized eigenmodes of a conformally coupled Proca field with negative mass square, as well as the cosmic time scaling of the mass of superluminal particles is discussed in Tomaschitz (1999a-c).

Locally, for a massive Fermi gas, we have $T \propto 1$ and $mc^2/(k_B T) \propto 1$, hence

$$\rho_E \propto m^4 c^5/\hbar^3 \propto 1, \quad \rho_M \propto \rho_E c^{-2} \propto h^{-8/3}, \quad \rho_N \propto \rho_E/(mc^2) \propto h^{-1}, \tag{6.6}$$

for energy, mass and particle density, respectively. We find $P \propto s \propto c_{V,P} \propto 1$, and for the spectral energy density $\rho_E(\nu) \propto h^{-1}$; explicit formulas for the thermodynamic variables are given in Chandrasekhar (1967). The densities (6.6) dimensionally scale like the atomic measuring rods. Unlike the source density ρ_N in (5.2), they lead to the usual conservation laws; there is no mass creation in galaxies, no accretion of stellar matter from within.

The cosmology studied in this paper is based on two symmetric tensor fields, a space-time metric and a symmetric permeability tensor representing the world ether. This tensor is assumed as homogeneous and isotropic; it is determined by two scale factors $h(\tau)$ and $b(\tau)$, both functions of cosmic time like the expansion factor $a(\tau)$ in the RW metric, cf. Section 1. Electromagnetic fields are coupled to the permeability tensor like in a dielectric medium, cf. (1.1). Classical mechanics in the ether is defined by replacing in the Hamilton-Jacobi equation the space-time metric by the permeability tensor. The relation (1.2) among the scale factors is necessary to make the ether dispersion free, a prerequisite for black-body radiation. If we furthermore require the large ratio $e^2/(km^2)$ to be constant in cosmic time [instead of the moderate ratio $\hbar^2 H_0/(kcm^3)$, which we assumed constant in this paper], then we recover traditional RW cosmology, as pointed out in Section 1. This clearly raises questions on the reality of the space expansion, cf. Sandage (1988), as it is undistinguishable from a static space-time in which the fundamental constants vary in a way that all measuring rods contract at the same rate. The curvature sign of the 3-space can be tested by source counts, or by the redshift dependence of angular diameters and surface brightness, or by the qualitative dynamics of freely moving objects (Tomaschitz, 1997), or by measuring the angle deficiency/excess of triangles, or by the angular anisotropy of the microwave background, but not so the space expansion.

Certain dimensionless ratios of moderate magnitude are kept constant like in Dirac's large number hypothesis (Dirac, 1937, 1974; Dyson, 1972); there are experimental bounds, cited after (1.4), which make a time variation of the ratios (6.1) rather unlikely within the age of the Earth. However, we do not adopt the view that all large dimensionless ratios, composed of the laboratory constants (including k), H_0, the curvature radius of the 3-space, and the cosmological particle densities, have been small in the past. There is nothing strange on large numbers in an open universe, and a time variation does not really explain them; one would rather expect a physical explanation of $e^2/(km_p m_e)$ to be tantamount to a genuinely unified theory of electricity and gravitation.

In this paper we presented a cosmology in which the moderate ratios $\hbar^2 H_0/(kcm^3)$ and Ω_m stay constant in the cosmic evolution. Several observable consequences rather different from standard RW cosmology were discussed in detail: a very stringent laboratory estimate of the deceleration parameter, $q_0 = -1 + \varepsilon$, $|\varepsilon| < 0.15$, cf. the end of Section 3, the power law decay of angular diameters for high redshifts, the possibility of a cosmic age consistent with even the highest estimates of the galactic age, cf. Section 4, and the peak in the number density (5.8) of the quasar distribution.

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