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# CONFORMAL TACHYONS

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We study tachyons conformally coupled to the background geometry of a Milne universe. The causality of superluminal signal transfer is scrutinized in this context. The cosmic time of the comoving frame determines a distinguished time order for events connected by superluminal signals. An observer can relate his rest frame to the galaxy frame, and compare so the time order of events in his proper time to the cosmic time order. All observers can in this way arrive at identical conclusions on the causality of events connected by superluminal signals. An unambiguous energy concept for tachyonic rays is defined by means of the cosmic time of the comoving reference frame, without resorting to an antiparticle interpretation. On that basis we give an explicit proof that no signals can be sent into the past of observers. Causality violating signals are energetically forbidden, as they would have negative energy in the rest frame of the emitting observer. If an observer emits a superluminal signal, the tachyonic response of a second observer cannot reach him prior to the emission, i.e. no predetermination can occur.

## 1. Introduction

Superluminal particles (tachyons) are a possibility suggested by a straightforward modification of the formalism of classical relativistic mechanics, they are a natural extension of the classical particle concept. However, relativistic theories of superluminal motion[1–7] are marred by causality violation as Lorentz boosts may change the time order of events connected by superluminal signals: If a uniformly moving observer $O_1$ sees a tachyon $T$ moving from space point $A$ to space point $B$, then a second observer $O_2$ related to the first by a Lorentz boost may well see it heading from $B$ to $A$. [To see a tachyon moving from $A$ to $B$ just means to observe the change effected by the tachyon at $A$ (emission) prior to the change effected at $B$ (absorption). By definition, emission always happens prior to absorption, without reference to energy transfer, at this point.] Observer $O_1$ concludes that the change at $A$ (effected by the emission of $T$) causes the change at $B$ (effected by the absorption of $T$). Observer $O_2$, however, concludes that the change at $B$ (emission) causes the change at $A$ (absorption). Both observers base their conclusion on the assumption that the cause precedes the effect. For observer $O_1$, the cause is the change that takes place at space point $A$ by the emission of the tachyon, and the effect is the change taking place at $B$ by its absorption. The same holds for observer $O_2$, but with $A$ and $B$ interchanged. What appears as emission to observer $O_1$ is absorption for observer $O_2$, and vice versa, as the time order (proper time) in the two rest frames is different. According to the relativity principle, the conclusions of both observers concerning cause and effect must be regarded as equally real, as physically equivalent. This leads to a violation of the traditional causality principle, which may be stated as follows:[8] (i) Every effect has a cause. (ii) The cause precedes the effect. (iii) The distinction of cause and effect is unambiguous. The third condition simply means that all observers come to the same conclusion on what is cause and effect. The conclusions of observers $O_1$ and $O_2$ are evidently different.

**Remarks.** (1) In Refs. 3 and 5 cause and effect are defined by energy loss and energy gain, respectively, which is a relativistically invariant characterization if properly done, but it conflicts with condition (ii) of the causality principle. (2) Emission and absorption are defined in a frame dependent, geometric way: We say that in the rest frame of a given observer the tachyon is emitted at space point $A$ and absorbed at $B$, if $A$ is the initial and $B$ the terminal point of its trajectory, parametrized by the observer's proper time.

In Robertson–Walker cosmology, there exists a coordinate frame in which all galaxies and galactic observers have constant space coordinates, despite their mutual recession. A universal cosmic time is defined by this comoving frame, and thus a distinguished time order. Every observer can compare the time order of events in his proper time to the universal cosmic time order, and all observers arrive in this way at the same conclusion on the causal connection of events related by superluminal signals, even though the cosmic time order may be inverted in their proper time.

In Minkowski space, there seems to be at first sight a very straightforward generalization of the energy concept for subluminal particles to tachyons. But it turns out that the sign of the energy of tachyons is not preserved under Lorentz boosts. There has been a rescue attempt[3,6] to reinterpret tachyons of negative energy as antiparticles with positive energy, similar to the negative energy solutions of the Dirac equation, and to define so a positive energy in an invariant way. However, this does not solve the causality problem.[8] In the theory advanced in this paper, the energy of tachyons is defined by means of the universal cosmic time of the comoving frame without using the quantum mechanical antiparticle concept.

A conformal classical field theory of tachyons (Proca equation with negative mass square), the spectral energy density of a tachyon background radiation, and the interaction of tachyons with matter are studied in Refs. 9–11. In this paper, we focus on the classical mechanics of conformally coupled tachyons, obtained from this field theory in the semiclassical limit.

We consider as background geometry the Milne universe,[12] a flat Robertson–Walker cosmology with a linear expansion factor and an open, negatively curved three-space. In this cosmology the rest frames of uniformly moving observers can be synchronized by Lorentz boosts without resorting to locally geodesic neighborhoods, but otherwise the reasoning in this paper also applies to any other Robertson–Walker cosmology.

In Sec. 2, we discuss the world-lines of conformal tachyons, both in comoving and globally geodesic frames, and define an unambiguous energy concept for tachyons based on the comoving reference frame. We study tachyonic signal exchange effected by two galactic, i.e. comoving, observers, and show that no causality violation or predetermination can occur. In Sec. 3, we investigate superluminal signal exchange between nongalactic, uniformly moving observers.

The causality proof is given as follows. A geodesic observer $O_A$ emits a tachyon $T_A$, which is absorbed by a second uniformly moving observer $O_B$. As soon as the absorption takes place, observer $O_B$ emits as his response a tachyon $T_B$, which is in turn absorbed by observer $O_A$. In Secs. 2 and 3 it is demonstrated that in the geodesic rest frame of observer $O_A$ the response $T_B$ does not arrive prior to the emission of tachyon $T_A$; we show that in the rest frame of observer $O_A$ the emission of tachyon $T_A$ is not predetermined by the response $T_B$ to it. The proof makes use of the tachyonic energy concept developed in Sec. 2. An observer can only emit tachyons of positive energy, and hence the geometric possibility of sending signals into the past of observers, as pointed out at the beginning of this section, is energetically excluded. In Sec. 4 we present our conclusions. In the appendix, we give the causality proof for a static Minkowski universe.

## 2. The Energy Concept for Conformal Tachyons

At first we consider tachyonic rays in RW coordinates (comoving frame). The line element of the Milne universe reads as

$$d\lambda^2 = -d\tau^2 + \left(\frac{\tau}{u}\right)^2 (du^2 + |dz|^2)\,; \tag{2.1}$$

we use as coordinate representation of the three-space the Poincaré half-space $H^3$, with Cartesian coordinates $u$, $z$; $u > 0$ cf. Ref. 13. The manifold defined in this way is isometric to the forward light cone.[14] In the following we consider geodesic motion along the $u$-semiaxis, and put $z = dz = 0$, as $H^3$ is homogeneous. The isometry which maps the $(\tau, u)$-plane onto the $(t, x)$-plane $(t^2 > x^2, t > 0)$ of the light cone reads as

$$\begin{aligned} t &= \frac{\tau}{2}(u + u^{-1})\,, \qquad & x &= \frac{\tau}{2}(u - u^{-1})\,, \\ \tau &= \sqrt{t^2 - x^2}\,, \qquad & u &= \sqrt{(t+x)(t-x)^{-1}}\,. \end{aligned} \tag{2.2}$$

Along the $u$-semiaxis, the geodesic world-lines of a particle with negative mass-square are determined by

$$-\dot{\tau}^2(\lambda) + \left(\frac{\tau \dot{u}(\lambda)}{u}\right)^2 = \mu^2\,, \qquad \frac{\tau^2 \dot{u}}{u} = s \tag{2.3}$$

(derived from $L = -\mu\sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}$), $s$ is a real integration parameter. The tachyon mass $\mu$ varies in cosmic time inversely proportional to the expansion factor, $\mu = m/\tau$, $m > 0$ in our notation. There is some formal analogy to the cosmic time variation of mass suggested in Dirac's large numbers hypothesis.[15–17] However, the tachyon mass is not a rest mass and should not be taken too literally. The concept of tachyonic charge, and atomic emission and absorption processes effected by tachyon radiation are discussed in Ref. 9. The conformal time scaling of the tachyon mass is necessary to achieve the conformal coupling of the wave equation for tachyons, which results in the tachyonic world-lines

$$u(\tau) = \kappa \tau^{\delta(s)}\,, \qquad \delta(s) := s(s^2 - m^2)^{-1/2}\,, \tag{2.4}$$

via the semiclassical limit.[10] These superluminal rays solve (2.3). Due to the conformal coupling, tachyonic world-lines are a straightforward extension of the ray concept of geometric optics; light rays are obtained by putting $\delta = \pm 1$ in (2.4).

In the comoving frame, energy and momentum of conformal tachyons moving along the rays (2.4) are defined in analogy to subluminal particles, by

$$E = \frac{s}{\delta \tau} = \frac{m}{\sqrt{v_{\mathrm{co}}^2 - 1}}\,, \qquad p = \frac{s u(\tau)}{\tau^2}\,, \qquad |p| = \frac{|s|}{\tau} = \frac{m v_{\mathrm{co}}}{\sqrt{v_{\mathrm{co}}^2 - 1}}\,, \tag{2.5}$$

$v_{\mathrm{co}} = |\delta|$. (The sign of $\delta$ evidently determines whether the tachyon moves the $u$-semiaxis up or downwards.) In the forward light cone, we obtain the trajectories (2.4) as

$$t(\tau) = \frac{\tau}{2}\big(u(\tau) + u^{-1}(\tau)\big)\,, \qquad x(\tau) = \frac{\tau}{2}\big(u(\tau) - u^{-1}(\tau)\big)\,, \tag{2.6}$$

parametrized by cosmic time. By transforming $(E, p)$ in (2.5) like a contravariant two-vector, we define in the forward light cone

$$E = \frac{s}{\tau^2}\left(\frac{t}{\delta} + x\right) = \frac{\mu\, \mathrm{sign}(t + \delta x)}{\sqrt{v^2 - 1}}\,, \tag{2.7}$$

$$p = \frac{s}{\tau^2}\left(t + \frac{x}{\delta}\right) = \frac{\mu v\, \mathrm{sign}(t + \delta x)}{\sqrt{v^2 - 1}}\,, \tag{2.8}$$

with mass and velocity

$$\mu = \frac{m}{\sqrt{t^2 - x^2}}\,, \qquad v = \frac{p}{E} = \frac{x/t + \delta}{1 + \delta x/t}\,. \tag{2.9}$$

In deriving (2.7)–(2.9), we made use of

$$\frac{dt}{d\tau} = \tau^{-1}(t + \delta x)\,, \qquad \frac{dx}{d\tau} = \tau^{-1}(\delta t + x)\,, \tag{2.10}$$

along the rays.

**Remarks.** (1) Negative energy indicates a time inversion, $dt/d\tau < 0$, cf. (2.7) and (2.10). (2) Equations (2.7) and (2.8) are not a covariant definition of energy and momentum, as they are based on the comoving reference frame. The same holds for the time variation of the tachyon mass, likewise defined by means of the cosmic time of the comoving frame.

The time coordinate $t(\tau)$ in (2.6) admits a minimum at $\tau_\infty$,

$$\tau_\infty = \kappa^{-1/\delta}\left(\frac{\delta - 1}{\delta + 1}\right)^{1/(2\delta)}\,, \qquad u_\infty = \sqrt{\frac{\delta - 1}{\delta + 1}}\,, \tag{2.11}$$

$$t_\infty = \tau_\infty \frac{|\delta|}{\sqrt{\delta^2 - 1}}\,, \qquad x_\infty = \tau_\infty \frac{-\operatorname{sign}(\delta)}{\sqrt{\delta^2 - 1}}\,, \tag{2.12}$$

where $u_\infty = u(\tau_\infty)$, $t_\infty := t(\tau_\infty)$, and $x_\infty := x(\tau_\infty)$. $u_\infty$ does not depend on the integration constant $\kappa$, but solely on the tachyon velocity in the comoving frame. The velocity (2.9) in the geodesic frame diverges at $(t_\infty, x_\infty)$, since

$$\frac{x_\infty}{t_\infty} = -\frac{1}{\delta} = \tanh(\log u_\infty)\,, \qquad \sqrt{t_\infty^2 - x_\infty^2} = \tau_\infty\,. \tag{2.13}$$

This minimum of $t(\tau)$ is the reason for double images of tachyons in geodesic rest frames. In the $(t, x)$-frame, two tachyons appear in the interval $[t_\infty, \infty]$, whereas the tachyon is not visible at all in $[0, t_\infty]$. One of them carries positive and the other negative energy, because

$$E(\tau \to 0) \to -\infty\,, \qquad E(\tau \to \infty) \to \infty\,, \qquad E(\tau_\infty) = 0\,, \tag{2.14}$$

cf. (2.7), (2.6) and (2.4). These limits are independent of the sign of $\delta$ (i.e. of the orientation of the comoving velocity). Negative energy in individual rest frames can lead to double images[16,17] discussed after (2.21) and in Sec. 3.

The boost

$$t' = (1 - \alpha^2)^{-1/2}(t - \alpha x)\,, \qquad x' = (1 - \alpha^2)^{-1/2}(x - \alpha t)\,, \tag{2.15}$$

in the light cone corresponds via (2.2) to the transformation

$$\tau' = \tau\,, \qquad u' = \eta^{-1} u\,, \qquad \eta := (1 + \alpha)^{1/2}(1 - \alpha)^{-1/2}\,, \tag{2.16}$$

$|\alpha| < 1$, $\eta > 0$, in comoving coordinates (2.1). Galactic observers are characterized by constant space coordinates in the comoving frame, $u = \tilde{\kappa}$. In globally geodesic coordinates, the world-line of a galactic observer $\tilde{\kappa}$ reads $x = \alpha t$, $\alpha := (\tilde{\kappa}^2 - 1)(\tilde{\kappa}^2 + 1)^{-1}$, cf. (2.2); his time coordinate ranges in $[0, \infty]$. Galactic observers are related by Lorentz boosts (2.15). The forward light cone can be introduced as geodesic rest frame for every galactic observer $\tilde{\kappa}$, just by applying the Lorentz boost (2.15) with $\alpha$ as above, so that his world-line reads $x' = 0$ in his rest frame $t'^2 - x'^2 > 0$, $t' > 0$. In the comoving frame, this boost corresponds to a simple rescaling of the space coordinate, $u' = u/\tilde{\kappa}$, leaving cosmic time unchanged, cf. (2.16).

Next we study a superluminal signal exchange in the comoving galaxy frame. A tachyon $T_A$ is emitted at $(\tau_A, u_A = 1)$ and absorbed at $(\tau_B, u_B)$, $u_B > 1$, $\tau_B > \tau_A$, by two galactic observers $O_A$ and $O_B$ sitting at $u_A = 1$ and $u_B$, respectively. The trajectory of the tachyon is given in (2.4). We have to assume $\delta > 1$ in (2.4), so that the tachyon can reach $u_B$. The integration constant $\kappa$ in (2.4) and the arrival time read as

$$\kappa_A = \tau_A^{-\delta}\,, \qquad \tau_B = u_B^{1/\delta}\tau_A\,. \tag{2.17}$$

The geodesic rest frame $(t, x)$ of observer $O_A$, who emits $T_A$, is linked to the comoving frame by (2.2). In the $(t, x)$-frame, the initial and terminal points of the trajectory of $T_A$ are

$$\begin{pmatrix} t_A \\ x_A \end{pmatrix} = \begin{pmatrix} \tau_A \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} t_B \\ x_B \end{pmatrix} = \frac{\tau_B}{2}\,(u_B \pm u_B^{-1})\,. \tag{2.18}$$

As $u_B + u_B^{-1} > 2$, we find $t_B > t_A$. Moreover $u_\infty < 1$, cf. (2.11), so that no double image can emerge in the $(t, x)$-frame in the relevant time interval $[t_A, t_B]$, and the energy of tachyon $T_A$ is positive, cf. (2.7). (For a double image to occur, the tachyon must pass through $u_\infty$ in the comoving frame.) The world-line of observer $O_A$ is of course $x = 0$, and the world-line of observer $O_B$ reads as $x = \tilde{\alpha} t$ with $\tilde{\alpha} = (u_B^2 - 1)(u_B^2 + 1)^{-1}$.

To obtain the geodesic rest frame $(t', x')$ of observer $O_B$, we apply a Lorentz boost (2.15) with $\tilde{\alpha}$ as defined above or, equivalently, a coordinate change $u' = u/u_B$ in the comoving frame, followed by the transformation (2.2). In the $(t', x')$-frame, the world-line of observer $O_B$ is $x' = 0$, and the world-line of observer $O_A$ reads as $x' = -\tilde{\alpha} t'$. In the comoving $(\tau, u')$-frame, the world-line of the tachyon is given by (2.4) with $\kappa = \kappa_A/u_B$; it moves there from $u' = 1/u_B$ to $u' = 1$. Its initial and terminal points in the geodesic $(t', x')$-frame are

$$\begin{pmatrix} t'_A \\ x'_A \end{pmatrix} = \frac{\tau_A}{2}\,(u_B^{-1} \pm u_B)\,, \qquad \begin{pmatrix} t'_B \\ x'_B \end{pmatrix} = \begin{pmatrix} \tau_B \\ 0 \end{pmatrix}. \tag{2.19}$$

This is at first glance quite similar to (2.18); however, $t'_A < t'_B$ only holds if

$$\delta < \frac{\log u_B}{\log\left((u_B + u_B^{-1})/2\right)}\,, \tag{2.20}$$

cf. (2.19) and (2.17). If (2.20) is violated, a time inversion occurs, which can easily be understood as follows. Equations (2.11) hold with $\kappa = \kappa_A/u_B$, so that $u_\infty < u_B^{-1}$ is equivalent to

$$\delta < (u_B^2 + 1)(u_B^2 - 1)^{-1}\,. \tag{2.21}$$

Inequality (2.21) implies (2.20), but not vice versa. [If we define

$$f(u) := (u^2-1)\log u - (u^2+1)\log\left((u+u^{-1})/2\right),$$

we readily find $f'(u) > 0$ for $u > 1$, and $f(u) > 0$ follows by inspecting the limit $u \to 1$.] Accordingly, if (2.21) holds, then observer $O_B$ at $x' = 0$ will see a tachyon with positive energy emitted at $(t'_A, x'_A)$ and absorbed by him at a later instant $t'_B$, cf. (2.19). If, however, inequality (2.21) is violated by a high tachyonic velocity in the comoving frame ($\delta = v_{\rm co} > 0$), then $u_B^{-1} < u_\infty < 1$, and observer $O_B$ will see two tachyons emerging at $(t'_\infty, x'_\infty)$ [as defined in (2.12), with $\kappa = \kappa_A/u_B$] moving in opposite directions, one toward him with positive energy, and the second toward observer $O_A$ at $x'_A$ with negative energy. If inequality (2.20) is violated too, then the second tachyon arrives at $x'_A$ before the first reaches observer $O_B$.

At any rate, $E(t'_B, x'_B) > 0$, cf. (2.7) and (2.19); tachyon $T_A$ arrives with positive energy at $x' = 0$, where observer $O_B$ is located. The tachyon energy undergoes a sign change [so that $E(t'_A, x'_A) < 0$] if $(t'_\infty, x'_\infty)$ lies on the trajectory, which requires in the comoving $(\tau, u')$-frame $u_B^{-1} < u_\infty < 1$ to hold. ($u'_\infty = u_\infty$, independent of the rescaling of the $u$-coordinate.) But in this case there appear in actual fact two tachyons, one with positive and the other with negative energy.

Next we consider the response of the galactic observer $O_B$ at $u = u_B$, which emits at $(\tau_B, u_B)$ a tachyon $T_B$ defined by

$$u = \kappa_B \tau^{\hat\delta}, \qquad \kappa_B := u_B \tau_B^{-\hat\delta}, \tag{2.22}$$

compare (2.4). For tachyon $T_B$ to reach observer $O_A$ at $u = 1$, we need $\hat\delta < -1$; then its arrival time is $\tau_{A,\rm rec} = u_B^{-1/\hat\delta}\tau_B$.

In the geodesic rest frame $(t, x)$ of observer $O_A$, the absorption of $T_B$ takes place at

$$(t_{A,\rm rec}, x_{A,\rm rec}) = (u_B^{-1/\hat\delta}\tau_B, 0), \tag{2.23}$$

[correspondingly via (2.2) to $(\tau_{A,\rm rec}, u = 1)$ in the comoving frame] and the emission of $T_B$ happens at $(t_B, x_B)$ as given in (2.18). Analogous to (2.11), a double image of $T_B$ appears in the $(t, x)$-frame whenever

$$\hat u_\infty := \sqrt{\frac{|\hat\delta| + 1}{|\hat\delta| - 1}} \tag{2.24}$$

lies on the trajectory of $T_B$ connecting the two observers $O_{A,B}$ in the comoving $(\tau, u)$-frame. Tachyon $T_B$ reaches observer $O_A$ with positive energy, $E(t_{A,\rm rec}, 0) > 0$, but if $\hat u_\infty < u_B$, then observer $O_A$ sees two tachyons emerging at $(\hat t_\infty, \hat x_\infty)$ [defined as in (2.11) and (2.12), with $\kappa$ and $\delta$ replaced by $\kappa_B$ and $\hat\delta$, respectively], one moving toward him with positive energy, and the other toward $O_B$ with negative energy, cf. (2.14). A double image does not occur if $\hat u_\infty > u_B$ [equivalent to $|\hat\delta| < (u_B^2+1)(u_B^2-1)^{-1}$, cf. (2.21)]; then the energy of the tachyon is positive along the trajectory connecting the observers. In any case,

$$\tau_A = t_A < t_{A,\rm rec} = u_B^{-1/\hat\delta} u_B^{1/\delta}\tau_A, \tag{2.25}$$

with $u_B > 1$, $\delta > 1$, and $\hat{\delta} < -1$, cf. (2.23) and (2.17), so that the emission of tachyon $T_A$ at $(t_A, x = 0)$ happens prior to the absorption of $T_B$ at $(t_{A,\mathrm{rec}}, x = 0)$, and accordingly a predetermination of signal $T_A$ cannot be effected by the response $T_B$ of observer $O_B$.

In the geodesic rest frame $(t', x')$ of observer $O_B$, the coordinates of the absorption of tachyon $T_B$ (by observer $O_A$) read

$$\begin{pmatrix} t'_{A,\mathrm{rec}} \\ x'_{A,\mathrm{rec}} \end{pmatrix} = \frac{\tau_{A,\mathrm{rec}}}{2}\left(u_B^{-1} \pm u_B\right), \tag{2.26}$$

and the emission of $T_B$ by $O_B$ happens at $(t'_B, x'_B) = (\tau_B, 0)$ [upon arrival of tachyon $T_A$, cf. (2.19)]. Clearly, $t'_B < t'_{A,\mathrm{rec}}$, and tachyon $T_B$ has positive energy in this frame. No double images can appear, since $T_B$ moves in comoving coordinates $(\tau, u')$ from $u' = 1$ to $u' = u_B^{-1}$ and does not reach $\hat{u}_\infty$.

## 3. Superluminal Signal Exchange Between Uniformly Moving Observers

The geodesic world-lines of uniformly moving observers in the Milne universe (2.1) read in comoving coordinates

$$u(\tau) = \tilde{\kappa}\left(\frac{-\nu + \sqrt{\tau^2 + \nu^2}}{\nu + \sqrt{\tau^2 + \nu^2}}\right)^{1/2}, \tag{3.1}$$

with $\tilde{\kappa} > 0$, and $\nu$ is a real integration constant determining the speed of the observer (subluminal particle), $\mathbf{v}_{\mathrm{co}} = \tau u^{-1} du/d\tau = \nu(\nu^2 + \tau^2)^{-1/2}$. Equation (3.1) is readily obtained from (2.3) if we put there $\mu^2 = -1$, and we focus on geodesic motion along the $u$-semiaxis of $H^3$. The choice $\nu = 0$ evidently corresponds to galactic observers discussed in Sec. 2. The trajectories (3.1) are mapped into the forward light cone by (2.2),

$$x = \mathbf{v}t - \frac{2\tilde{\kappa}\nu}{\tilde{\kappa}^2 + 1}, \qquad \mathbf{v} := \frac{\tilde{\kappa}^2 - 1}{\tilde{\kappa}^2 + 1}, \tag{3.2}$$

with $t$ ranging in the interval $[|\nu|\tilde{\kappa}^{-\,\mathrm{sign}(\nu)}, \infty]$. This $(t, x)$-frame is the geodesic rest frame of a galactic observer $(\tilde{\kappa} = 1, \nu = 0)$, and relates to the geodesic rest frame $(t', x')$ of an observer $(\tilde{\kappa}, \nu)$ by a Lorentz boost (2.15) with $\alpha = \mathbf{v}$. The world-line of observer $(\tilde{\kappa}, \nu)$ reads in the $(t', x')$-frame $x' = -\nu$, with $t'$ ranging in $[|\nu|, \infty]$. The geodesic rest frame $(t', x')$ of observer $(\tilde{\kappa}, \nu)$ is therefore a truncated copy of the forward light cone, $t'^2 - x'^2 > 0$, $t' > |\nu|$. In this frame, the galaxies radially emanate from $x' = 0$, and because the observer is located at $x' = -\nu$, the galactic recession appears anisotropic to him, and so does the background radiation. In the following, we will frequently use the comoving frame $(\tau, u')$, $u' = u/\tilde{\kappa}$, which is connected to the $(t', x')$-frame via (2.2), cf. the discussion preceding (2.19).

As in Sec. 2, the first part of the signal exchange consists of a tachyon $T_A$ emitted at $(\tau_A, u_A = 1)$ and absorbed at $(\tau_B, u_B)$, $u_B > 1$, $\tau_B > \tau_A$; the trajectory of this tachyon is defined in (2.4) and (2.17). Emission and absorption are now effected by two observers $O_A$ and $O_B$, respectively, who move along world-lines as defined in (3.1). Their integration parameters $(\tilde{\kappa}_A, \nu_A)$ and $(\tilde{\kappa}_B, \nu_B)$, respectively, relate to the indicated emission and absorption events by

$$\tilde{\kappa}_A = \left( \frac{\nu_A + \sqrt{\tau_A^2 + \nu_A^2}}{-\nu_A + \sqrt{\tau_A^2 + \nu_A^2}} \right)^{1/2}, \qquad \tilde{\kappa}_B = u_B \left( \frac{\nu_B + \sqrt{\nu_B^2 + \tau_B^2}}{-\nu_B + \sqrt{\nu_B^2 + \tau_B^2}} \right)^{1/2}. \tag{3.3}$$

The integration constants $\nu_{A,B}$ may have either sign; if it is positive, the observer moves the $u$-semiaxis upwards. Evidently, $1/\tilde{\kappa}_A < 1$ if $\nu_A > 0$, and $1/\tilde{\kappa}_A > 1$ if $\nu_A < 0$. Likewise, for observer $O_B$, $u_B/\tilde{\kappa}_B < 1$ if $\nu_B > 0$, and $u_B/\tilde{\kappa}_B > 1$ if $\nu_B < 0$.

The geodesic rest frame of observer $O_A$ is defined by coordinates $(t'', x'')$ corresponding via (2.2) to the comoving frame $(\tau, u'')$, with $u'' = u/\tilde{\kappa}_A$. Likewise, the rest frame of $O_B$ is denoted by $(t', x')$ as in Sec. 2, corresponding to comoving coordinates $(\tau, u')$, $u' = u/\tilde{\kappa}_B$. In the $(t'', x'')$-frame, the world-line of observer $O_A$ reads as $x'' = -\nu_A$, and the world-line of observer $O_B$ is given by (3.2) with $(\tilde{\kappa} = \tilde{\kappa}_B/\tilde{\kappa}_A, \nu_B)$. In the $(t', x')$-frame, the world-line of $O_B$ reads $x' = -\nu_B$, and the trajectory of $O_A$ is defined by $(\tilde{\kappa} = \tilde{\kappa}_A/\tilde{\kappa}_B, \nu_A)$.

In the geodesic rest frame $(t'', x'')$ of observer $O_A$, emission and absorption of tachyon $T_A$ take place at

$$\begin{pmatrix} t''_A \\ x''_A \end{pmatrix} = \frac{\tau_A}{2} \left( \tilde{\kappa}_A^{-1} \pm \tilde{\kappa}_A \right) = \begin{pmatrix} \sqrt{\tau_A^2 + \nu_A^2} \\ -\nu_A \end{pmatrix}, \qquad \begin{pmatrix} t''_B \\ x''_B \end{pmatrix} = \frac{\tau_B}{2} \left( \frac{u_B}{\tilde{\kappa}_A} \pm \frac{\tilde{\kappa}_A}{u_B} \right), \tag{3.4}$$

which follows from $u'' = u/\tilde{\kappa}_A$ and (2.2). In the corresponding comoving $(\tau, u'')$-frame, tachyon $T_A$ moves from $u'' = 1/\tilde{\kappa}_A$ to $u'' = u_B/\tilde{\kappa}_A$, and therefore a double image of the tachyon appears in the $(t'', x'')$-frame, provided $\tilde{\kappa}_A^{-1} < u_\infty < u_B \tilde{\kappa}_A^{-1}$ holds, with $u_\infty$ as defined in (2.11), cf. the discussion following (2.13). [$u_\infty$ only depends on the velocity of the tachyon, and is not affected by a rescaling of the $u$-coordinate, unlike $\tau_\infty$ in (2.11).] In this case, $E(t''_A, x''_A) < 0$, and $E(t''_B, x''_B) > 0$, cf. (2.7), which follows from (2.14), compare the discussion after (2.21). The tachyon energy undergoes a sign change along the trajectory at $(t''_\infty, x''_\infty)$ [defined as in (2.11) and (2.12), with $\kappa = \kappa_A/\tilde{\kappa}_A$]. If $u_\infty > u_B/\tilde{\kappa}_A$, then the energy of the tachyon is negative along the track connecting the observers, and a time inversion occurs, $t''_A > t''_B$. If, finally,

$$u_\infty < \tilde{\kappa}_A^{-1}, \tag{3.5}$$

then the tachyonic energy (2.7) is positive along the world-line from $O_A$ to $O_B$, and the cosmic time order is preserved, $t''_A < t''_B$.

The energy of a tachyon is positive in the rest frame in which it is emitted, otherwise it would appear there prior to its emission. Accordingly, condition (3.5) is a necessary constraint on the velocity [i.e. on $\delta$ in (2.17)] of tachyon $T_A$; only galactic observers ($\nu_A = 0$) can emit tachyons of any velocity. The reference in (3.5) to the observer's velocity in the comoving frame [as determined by the integration constant $\nu_A$, see after (3.1)] once more underscores the nonrelativistic nature of superluminal signals; they are defined with respect to the galaxy background, the comoving reference frame. Condition (3.5) is made more explicit in the remark following (3.9).

Next we consider tachyon $T_A$ in the rest frame $(t', x')$ of observer $O_B$. The events $(\tau_\infty, u' = u_\infty)$ and $(t'_\infty, x'_\infty)$, indicating the splitting of the tachyon trajectory, are defined as in (2.11) and (2.12), with $\kappa = \kappa_A/\tilde{\kappa}_B$. If $\tilde{\kappa}_B^{-1} < u_\infty < u_B\tilde{\kappa}_B^{-1}$ holds, then $E(t'_A, x'_A) < 0$ and $E(t'_B, x'_B) > 0$, because $E$ in (2.7) is positive for $\tau > \tau_\infty$, and negative for $\tau < \tau_\infty$, cf. (2.14), and hence a double image appears in the $(t', x')$-frame. Otherwise, the tachyonic energy $E(t', x')$ is positive along the track connecting the observers if $u_\infty < \tilde{\kappa}_B^{-1}$, and negative if $u_\infty > u_B/\tilde{\kappa}_B$. Tachyon $T_A$ may well appear to observer $O_B$ with negative energy or as double image, as it is not emitted in his rest frame.

The second part of the signal exchange consists of a tachyon $T_B$ as defined in (2.22), emitted at $(\tau_B, u_B)$ by observer $O_B$ as his response to tachyon $T_A$. Tachyon $T_B$ is absorbed by observer $O_A$ at $(\tau_{A,\text{rec}}, u_{A,\text{rec}})$, where $\tau_{A,\text{rec}}$ is the solution of

$$\kappa_B \tau_{A,\text{rec}}^{\hat{\delta}} = \tilde{\kappa}_A \left( \frac{-\nu_A + \sqrt{\tau_{A,\text{rec}}^2 + \nu_A^2}}{\nu_A + \sqrt{\tau_{A,\text{rec}}^2 + \nu_A^2}} \right)^{1/2} = u_{A,\text{rec}}\,. \tag{3.6}$$

Because observer $O_A$ moves subluminally, he cannot arrive prior to tachyon $T_A$ at $u_B$, and therefore $\hat{\delta} < -1$ is necessary for tachyon $T_B$ to reach observer $O_A$, as already assumed after (2.22). It is also clear from the velocities that the tachyon hits the observer exactly once, the solution $\tau_{A,\text{rec}}$ is unique, and evidently $\tau_{A,\text{rec}} > \tau_B$. There are no restrictions on the subluminal velocities of the observers $O_{A,B}$, they may move up or down the $u$-semiaxis.

By making use of the comoving $(\tau, u'')$-frame defined after (3.3), and applying (2.2), we readily find for the absorption of $T_B$ by observer $O_A$ in his rest frame $(t'', x'')$ the coordinates

$$\begin{pmatrix} t''_{A,\text{rec}} \\ x''_{A,\text{rec}} \end{pmatrix} = \frac{\tau_{A,\text{rec}}}{2} \left( \frac{u_{A,\text{rec}}}{\tilde{\kappa}_A} \pm \frac{\tilde{\kappa}_A}{u_{A,\text{rec}}} \right) = \begin{pmatrix} \sqrt{\tau_{A,\text{rec}}^2 + \nu_A^2} \\ -\nu_A \end{pmatrix}. \tag{3.7}$$

The emission of $T_B$ coincides of course with the absorption of $T_A$ at $(t''_B, x''_B)$ as defined in (3.4). In the $(\tau, u'')$-frame, tachyon $T_B$ moves from $u'' = u_B/\tilde{\kappa}_A$ to $u'' = u_{A,\text{rec}}/\tilde{\kappa}_A$. (Since $\hat{\delta} < -1$, we have $u_{A,\text{rec}} < u_B$.) Accordingly, if $u_{A,\text{rec}}\tilde{\kappa}_A^{-1} < \hat{u}_\infty < u_B\tilde{\kappa}_A^{-1}$, with $\hat{u}_\infty$ as defined in (2.24), then a double image of the tachyon emerges in the $(t'', x'')$-frame. In this geodesic frame, the energy of $T_B$ at absorption is positive, $E(t''_{A,\mathrm{rec}}, x''_{A,\mathrm{rec}}) > 0$, cf. (2.7) and (2.9) (with $\delta \to \hat{\delta}$), and we find $E(t''_B, x''_B) < 0$ at emission. If $\hat{u}_\infty < u_{A,\mathrm{rec}}/\tilde{\kappa}_A$, then the energy of $T_B$ is negative along its trajectory connecting the observers, and positive if $\hat{u}_\infty > u_B/\tilde{\kappa}_A$. Since tachyon $T_B$ is not emitted by observer $O_A$, there are no constraints on its energy in his rest frame.

A similar reasoning, though with very different consequences, applies with regard to tachyon $T_B$ in the geodesic rest frame $(t', x')$ of observer $O_B$. Emission and absorption events for tachyon $T_B$ there read as

$$\begin{pmatrix} t'_B \\ x'_B \end{pmatrix} = \frac{\tau_B}{2}\left(\frac{u_B}{\tilde{\kappa}_B} \pm \frac{\tilde{\kappa}_B}{u_B}\right), \qquad \begin{pmatrix} t'_{A,\mathrm{rec}} \\ x'_{A,\mathrm{rec}} \end{pmatrix} = \frac{\tau_{A,\mathrm{rec}}}{2}\left(\frac{u_{A,\mathrm{rec}}}{\tilde{\kappa}_B} \pm \frac{\tilde{\kappa}_B}{u_{A,\mathrm{rec}}}\right), \tag{3.8}$$

analogous to (3.4) and (3.7). In the comoving $(\tau, u')$-frame, the tachyon moves from $u' = u_B/\tilde{\kappa}_B$ to $u' = u_{A,\mathrm{rec}}/\tilde{\kappa}_B$. If $u_{A,\mathrm{rec}}\tilde{\kappa}_B^{-1} < \hat{u}_\infty < u_B\tilde{\kappa}_B^{-1}$, then a double image appears in the $(t', x')$-frame, so that $E(t'_{A,\mathrm{rec}}, x'_{A,\mathrm{rec}}) > 0$ and $E(t'_B, x'_B) < 0$. If $\hat{u}_\infty < u_{A,\mathrm{rec}}/\tilde{\kappa}_B$, then the energy of $T_B$ is negative along its track connecting the observers, and positive if

$$\hat{u}_\infty > \frac{u_B}{\tilde{\kappa}_B}\,. \tag{3.9}$$

As tachyon $T_B$ happens to be emitted by observer $O_B$, its energy is positive in his rest frame, and hence condition (3.9) is a necessary constraint on the velocity of $T_B$, i.e. on the integration constant $\hat{\delta}$ in (2.22). This is quite analogous to the constraint (3.5) on the velocity of tachyon $T_A$.

**Remark.** The velocities of the tachyons $T_{A,B}$ in the comoving frame read $\mathbf{v}_{\mathrm{tach},A} = \delta > 1$ and $\mathbf{v}_{\mathrm{tach},B} = \hat{\delta} < -1$, respectively, cf. (2.5). The velocities of the observers $O_{A,B}$ at emission time read $\mathbf{v}_{\mathrm{obs},A,B} = \mathrm{sign}(\nu_{A,B})(1 + \tau_{A,B}^2/\nu_{A,B}^2)^{-1/2}$, see after (3.1). Condition (3.5) is equivalent to $\mathbf{v}_{\mathrm{tach},A}\mathbf{v}_{\mathrm{obs},A} < 1$, cf. (3.3) and (2.11), and condition (3.9) to $\mathbf{v}_{\mathrm{tach},B}\mathbf{v}_{\mathrm{obs},B} < 1$. If a geodesic observer moves with speed $\mathbf{v}_{\mathrm{obs}}$ in the comoving reference frame, then he can only emit tachyons whose speed satisfy

$$\mathbf{v}_{\mathrm{tach}}(\tau_{\mathrm{em}})\mathbf{v}_{\mathrm{obs}}(\tau_{\mathrm{em}}) < 1 \tag{3.10}$$

at emission time. It is easy to see, by virtue of locally geodesic coordinates, cf. the appendix, that this condition for tachyon emission also holds in any other Robertson–Walker cosmology, the product being taken in the three-space metric of the comoving reference frame. Condition (3.10) is equivalent to the positivity of the tachyon energy in the locally geodesic rest frame of the emitting observer. (The observer may be a decaying particle, and energy–momentum conservation holds.[11]) Evidently, condition (3.10) does not give a bound on $|\mathbf{v}_{\mathrm{tach}}|$ if tachyon and observer head in sufficiently opposite directions.

Finally, observer $O_A$ cannot receive the response $T_B$ prior to the emission of $T_A$; no predetermination can occur, because in the rest frame $(t'', x'')$ of observer $O_A$ the cosmic time order $\tau_A < \tau_{A,\mathrm{rec}}$ is preserved: $t''_A < t''_{A,\mathrm{rec}}$, which is an obvious consequence of (3.4) and (3.7).

## 4. Conclusion

Cosmic time defines a distinguished time order to which every observer can relate by connecting his geodesic rest frame to the comoving galaxy frame. The high isotropy of the microwave background makes it in practice possible for every observer to determine his movement in the galaxy background, and in this way to infer the cosmic time order of events connected by tachyons. The time order in the proper time of galactic or uniformly moving observers may well be inverted as compared to the cosmic time order of the comoving reference frame, but all observers can arrive at the same conclusion on the causality of the observed process. The causality of superluminal signal transfer is unambiguously defined by the cosmic time order, so that the traditional causality principle as stated in the Introduction is adhered to.

Cosmic space is generated by the galaxy grid, which provides a natural reference frame. The state of rest can be defined with respect to the galaxy background, and uniform motion and rest become easily distinguishable states. Whether an observer is at rest or in uniform motion with respect to the microwave background, this can really be unambiguously decided, quantitatively, by measuring the dipole anisotropy of the background temperature, caused by a Doppler shift. To figure out the causal connections in an experiment involving tachyons, one has to determine the motion of the laboratory relative to the galaxy background. The solar barycenter is moving with some 370 km/s, cf. Ref. 18, fast enough to even neglect the relative motions of the Earth in a first approximation. The introduction of the galaxy frame as reference frame suggests an absolute cosmic space–time and constitutes a fundamental departure from the relativity principle, in particular from the relativistic interpretation of Lorentz transformations and the relativistic definition of tachyonic energy.

Like the causality concept, the energy concept for tachyons is based on cosmic time and the comoving galaxy frame. Tachyonic energy and momentum are defined in this reference frame analogous to the energy of subluminal particles, and in geodesic rest frames by means of coordinate transformations. In this way the sign of tachyonic energy is unambiguously defined, cf. Sec. 2. Whenever the energy of a tachyon is negative in a geodesic rest frame, this indicates time inversion to the observer, the cosmic time order of events connected by the tachyon is interchanged in his proper time. Hence an observer can infer the cosmic time order either from the energy of the tachyon relating the respective events, or from his own movement relative to the background radiation, as pointed out above. In this context we demonstrated that no signals can be sent into the past of observers by means of conformally coupled tachyons.

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## Appendix A. Causality and Tachyonic Energy in a Minkowski Universe

We consider a static galaxy distribution; the underlying space–time geometry is Minkowskian with the line element $ds^2 = -dt^2 + d\mathbf{x}^2$. The frame in which the galaxies have constant space coordinates is denoted by $(t, \mathbf{x})$. We consider two observers $P_1$ and $P_2$ uniformly moving along the $x$-axis

$$\begin{aligned} P_1: &\quad x = \mathbf{v}_1^P t, && |\mathbf{v}_1^P| < 1, \\ P_2: &\quad x = \mathbf{v}_2^P t + x_2, && |\mathbf{v}_2^P| < 1; \end{aligned} \tag{A.1}$$

$x_2$ is an arbitrary constant. All velocities have zero $y$ and $z$-components. At $(t_0, x_0 = \mathbf{v}_1^P t_0)$ observer $P_1$ emits a tachyon $T_1$ of velocity $\mathbf{v}_1^T$, $|\mathbf{v}_1^T| > 1$, which moves according to

$$x = \mathbf{v}_1^T t + (\mathbf{v}_1^P - \mathbf{v}_1^T) t_0 . \tag{A.2}$$

The velocity $\mathbf{v}_1^T$ is chosen in a way that the tachyon collides with observer $P_2$. This collision takes place at

$$t_{\text{coll}} = (\mathbf{v}_2^P - \mathbf{v}_1^T)^{-1} [(\mathbf{v}_1^P - \mathbf{v}_1^T) t_0 - x_2], \qquad x_{\text{coll}} = \mathbf{v}_2^P t_{\text{coll}} + x_2 . \tag{A.3}$$

For the collision to take place at all,

$$t_0 < t_{\text{coll}} \tag{A.4}$$

must be satisfied, which we henceforth assume as condition on $\mathbf{v}_1^T$. Upon receipt of $T_1$, observer $P_2$ emits a tachyon $T_2$ at $(t_{\text{coll}}, x_{\text{coll}})$, carrying his response. We obtain for the trajectory of this tachyon

$$x = \mathbf{v}_2^T (t - t_{\text{coll}}) + x_{\text{coll}} . \tag{A.5}$$

Tachyon $T_2$ hits $P_1$ at

$$\begin{gathered} t_{\text{term}} = (\mathbf{v}_1^P - \mathbf{v}_2^T)^{-1} (\mathbf{v}_2^P - \mathbf{v}_1^T)^{-1} [t_0 (\mathbf{v}_1^P - \mathbf{v}_1^T)(\mathbf{v}_2^P - \mathbf{v}_2^T) + x_2 (\mathbf{v}_2^T - \mathbf{v}_1^T)], \\ x_{\text{term}} = \mathbf{v}_1^P t_{\text{term}} , \end{gathered} \tag{A.6}$$

provided

$$t_{\text{coll}} < t_{\text{term}} . \tag{A.7}$$

This condition means a restriction on $\mathbf{v}_2^T$.

We denote by $(t', x')$ the rest frame of observer $P_1$, which is related to the galaxy frame $(t, x)$ by the Lorentz boost

$$t' = \gamma_1 (t - \mathbf{v}_1^P x), \qquad x' = \gamma_1 (x - \mathbf{v}_1^P t), \qquad \gamma_1 := (1 - |\mathbf{v}_1^P|^2)^{-1/2} . \tag{A.8}$$

The rest frame $(t'', x'')$ of observer $P_2$ is connected to $(t, x)$ by the same transformation with $\mathbf{v}_1^P$ replaced by $\mathbf{v}_2^P$. In the rest frame $(t', x')$ of $P_1$, the time coordinates of the events $(t_0, x_0)$, $(t_{\rm coll}, x_{\rm coll})$, and $(t_{\rm term}, x_{\rm term})$ read respectively

$$t'_0 = \gamma_1^{-1} t_0 \,, \tag{A.9}$$

$$t'_{\rm coll} = \gamma_1 (\mathbf{v}_2^P - \mathbf{v}_1^T)^{-1} [(\mathbf{v}_1^P - \mathbf{v}_1^T)(1 - \mathbf{v}_1^P \mathbf{v}_2^P) t_0 - x_2 (1 - \mathbf{v}_1^T \mathbf{v}_1^P)] \,, \tag{A.10}$$

$$t'_{\rm term} = \gamma_1^{-1} t_{\rm term} \,. \tag{A.11}$$

Evidently,

$$t'_0 < t'_{\rm term} \tag{A.12}$$

holds; in the proper time of observer $P_1$, the response $T_2$ of observer $P_2$ does not arrive prior to the emission of $T_1$. No predetermination arises in this communication process; the response $T_2$ cannot be sent into the past of observer $P_1$ and influence his emission of $T_1$. Conditions (A.4) and (A.7) ensure that in the galaxy frame $(t, x)$ tachyon $T_1$ moves from $P_1$ to $P_2$ (and not vice versa), and that $T_2$ moves from $P_2$ to $P_1$. Since observer $P_1$ emits tachyon $T_1$, relation

$$t'_0 < t'_{\rm coll} \tag{A.13}$$

holds in his rest frame, which gives a further restriction on the velocity $\mathbf{v}_1^T$ of tachyon $T_1$, cf. (A.20). As mentioned in the Introduction, we use the terms emission and absorption in a geometric sense without reference to energy transfer. If a tachyon is emitted by a moving observer, then this emission appears as such in his own rest frame. Inequality (A.13) [which boils down to (A.20)] gives a restriction on the velocity by which a tachyon can be emitted in a moving frame.

In the rest frame $(t'', x'')$ of $P_2$, we find

$$t''_0 = \gamma_2 (1 - \mathbf{v}_1^P \mathbf{v}_2^P) t_0 \,, \qquad \gamma_2 := (1 - |\mathbf{v}_2^P|^2)^{-1/2} \,, \tag{A.14}$$

$$t''_{\rm coll} = \gamma_2 (\mathbf{v}_2^P - \mathbf{v}_1^T)^{-1} [(\mathbf{v}_1^P - \mathbf{v}_1^T)(1 - |\mathbf{v}_2^P|^2) t_0 - x_2 (1 - \mathbf{v}_1^T \mathbf{v}_2^P)] \,, \tag{A.15}$$

$$t''_{\rm term} = \gamma_2 (1 - \mathbf{v}_1^P \mathbf{v}_2^P) t_{\rm term} \,. \tag{A.16}$$

As soon as observer $P_2$ absorbs tachyon $T_1$, he emits as response a tachyon $T_2$. Analogous to (A.13),

$$t''_{\rm coll} < t''_{\rm term} \,, \tag{A.17}$$

holds in the rest frame of $P_2$, because $T_2$ is emitted by this observer. (In the rest frame of observer $P_1$, however, $t'_{\rm coll}$ may well exceed $t'_{\rm term}$.)

The restrictions imposed on the tachyon velocities by inequalities (A.4), (A.7), (A.13) and (A.17) can be made more explicit. Inequality (A.4) may be written as

$$(\mathbf{v}_1^P - \mathbf{v}_2^P) t_0 \, {\rm sign}(\mathbf{v}_2^P - \mathbf{v}_1^T) > x_2 \, {\rm sign}(\mathbf{v}_2^P - \mathbf{v}_1^T) \,. \tag{A.18}$$

Inequality (A.13) reads as

$$(\mathbf{v}_1^P - \mathbf{v}_2^P) t_0 \, {\rm sign}[(\mathbf{v}_2^P - \mathbf{v}_1^T)(1 - \mathbf{v}_1^P \mathbf{v}_1^T)] > x_2 \, {\rm sign}[(\mathbf{v}_2^P - \mathbf{v}_1^T)(1 - \mathbf{v}_1^P \mathbf{v}_1^T)] \,, \tag{A.19}$$

and if combined with (A.18) it gives

$$1 - \mathbf{v}_1^P \mathbf{v}_1^T > 0\,. \tag{A.20}$$

Inequality (A.7) reads

$$\begin{aligned}(\mathbf{v}_1^P - \mathbf{v}_2^P) t_0 \,\mathrm{sign}[(\mathbf{v}_2^P - \mathbf{v}_1^T)(\mathbf{v}_1^P - \mathbf{v}_1^T)(\mathbf{v}_2^T - \mathbf{v}_1^P)] \\ > x_2 \,\mathrm{sign}[(\mathbf{v}_2^P - \mathbf{v}_1^T)(\mathbf{v}_1^P - \mathbf{v}_1^T)(\mathbf{v}_2^T - \mathbf{v}_1^P)]\,,\end{aligned} \tag{A.21}$$

and (A.17) gives

$$\begin{aligned}(\mathbf{v}_1^P - \mathbf{v}_2^P) t_0 \,\mathrm{sign}[(\mathbf{v}_2^P - \mathbf{v}_1^T)(\mathbf{v}_1^P - \mathbf{v}_1^T)(\mathbf{v}_2^T - \mathbf{v}_1^P)(1 - \mathbf{v}_2^P \mathbf{v}_2^T)] \\ > x_2 \,\mathrm{sign}[(\mathbf{v}_2^P - \mathbf{v}_1^T)(\mathbf{v}_1^P - \mathbf{v}_1^T)(\mathbf{v}_2^T - \mathbf{v}_1^P)(1 - \mathbf{v}_2^P \mathbf{v}_2^T)]\,.\end{aligned} \tag{A.22}$$

Combining (A.21) and (A.22), we obtain, analogous to (A.20),

$$1 - \mathbf{v}_2^P \mathbf{v}_2^T > 0\,. \tag{A.23}$$

Inequalities (A.18), (A.20), (A.21) and (A.23) are equivalent to conditions (A.4), (A.7), (A.13) and (A.17) on the velocities $\mathbf{v}_{1,2}^T$. Inequalities (A.18) and (A.21) just make sure that the tachyons reach the respective observers, and analogous conditions hold for subluminal signal transfer. Conditions (A.20) and (A.23), however, do not have a subluminal analog; if an observer moves with speed $\mathbf{v}_{\mathrm{obs}}$ in the comoving reference frame, then he can only emit tachyons whose speed satisfy $\mathbf{v}_{\mathrm{tach}}\mathbf{v}_{\mathrm{obs}} < 1$, as discussed in Sec. 3.

Finally, we turn to the energy concept for tachyons in a Minkowski universe, and demonstrate that there is no way to construct a tachyonic perpetuum mobile. We define in the galaxy frame tachyonic energy and momentum as $E = m\dot{t}(s)$ and $\mathbf{p} = m\dot{\mathbf{x}}(s)$, with positive mass. The Lagrangian reads $L = -m\sqrt{|\dot{t}^2 - \dot{\mathbf{x}}^2|}$. We choose the curve parameter $s$ in a way that $\dot{t}(s) > 0$ and $\dot{t}^2 - \dot{\mathbf{x}}^2 = \varepsilon$, where $\varepsilon = 1$ for particles and $\varepsilon = -1$ for tachyons. Hence, the energy of freely moving particles and tachyons is by definition positive in the galaxy frame. Energy and momentum can be parametrized by the 3-velocity,

$$E = \frac{m}{\sqrt{(1 - \mathbf{v}^2)\varepsilon}}\,, \qquad \mathbf{p} = \frac{m\mathbf{v}}{\sqrt{(1 - \mathbf{v}^2)\varepsilon}}\,. \tag{A.24}$$

In the limit of infinite speed, the energy of a tachyon is zero, but its momentum stays finite (mass times a unit vector); this gets important in elastic tachyon–particle collisions.[11] Next we define tachyonic energy in the rest frame $(t', \mathbf{x}')$ of an observer freely moving with speed $\mathbf{u}$, $|\mathbf{u}| < 1$, in the galaxy background. This frame is linked to the galaxy frame $(t, \mathbf{x})$ by the Lorentz boost

$$t' = \gamma(t - \mathbf{ux})\,, \quad \mathbf{x}' = \mathbf{x} - \gamma\mathbf{u}t + (\gamma - 1)(\mathbf{xu})\mathbf{u}|\mathbf{u}|^{-2}\,, \quad \gamma := (1 - \mathbf{u}^2)^{-1/2}\,. \tag{A.25}$$

Energy and momentum are defined in the moving frame by means of the differential version of (A.25),

$$E' := \gamma(E - \mathbf{up})\,, \qquad \mathbf{p}' := \mathbf{p} - \gamma\mathbf{u}E + (\gamma - 1)(\mathbf{pu})\mathbf{u}|\mathbf{u}|^{-2}\,. \tag{A.26}$$

For particles, $\varepsilon = 1$, this is just the transformation law for the four-vector $(E, \mathbf{p})$ in (A.24). However, in the case of tachyons Eqs. (A.24) is not a covariant definition of a four-vector, valid in all uniformly moving frames, because Lorentz transformations may change the sign of $E$ and $\mathbf{p}$ if $\varepsilon = -1$, cf. (A.30). Therefore, the galactic reference frame is necessary to unambiguously define energy and momentum in the rest frames of uniformly moving observers, unless one is willing to introduce an antiparticle concept for classical tachyons, cf. Sec. 1. This energy concept for tachyons is nonrelativistic, though it has a familiar relativistic look in a Minkowski universe.

The transformation law for velocities is readily obtained from (A.25),

$$\mathbf{v}' = \gamma^{-1}(1 - \mathbf{uv})^{-1}[\mathbf{v} - \gamma\mathbf{u} + (\gamma - 1)(\mathbf{vu})\mathbf{u}|\mathbf{u}|^{-2}] . \tag{A.27}$$

Here $\mathbf{v}$ and $\mathbf{v}'$ may be sub- or superluminal. If $\mathbf{uv} \to 1$, then $|\mathbf{v}'| \to \infty$; in this limit the tachyon approaches infinite speed and zero energy in the rest frame of observer $\mathbf{u}$. It follows from the differential version of (A.25) and its inverse that

$$(1 + \mathbf{uv}')(1 - \mathbf{uv}) = 1 - \mathbf{u}^2 . \tag{A.28}$$

Thus $1 - \mathbf{uv}$ and $1 + \mathbf{uv}'$ have equal sign. We obtain from (A.27)

$$1 - \mathbf{v}^2 = \gamma^{-2}(1 + \mathbf{uv}')^{-2}(1 - \mathbf{v}'^2) , \tag{A.29}$$

and from the preceding formulas we easily derive

$$E' = \frac{m\,\mathrm{sign}(1 - \mathbf{uv})}{\sqrt{(1 - \mathbf{v}'^2)\varepsilon}} , \qquad \mathbf{p}' = \frac{m\mathbf{v}'\,\mathrm{sign}(1 - \mathbf{uv})}{\sqrt{(1 - \mathbf{v}'^2)\varepsilon}} , \tag{A.30}$$

compare Eqs. (2.7) and (2.8). The energy of a tachyon is positive in a moving frame only if $1 - \mathbf{uv} > 0$ or, equivalently, if $1 + \mathbf{uv}' > 0$. We recover here conditions (A.20) or (A.23) for tachyon emission in moving frames. The energy of tachyons is positive in the respective rest frames in which they are emitted, cf. Sec. 3. This positivity means a restriction (3.10) on their (initial) velocities, which excludes predetermination. If $1 - \mathbf{uv} > 0$, no time inversion can occur: As $dt' = \gamma\, dt(1 - \mathbf{uv})$, cf. (A.25), the time intervals $dt$ and $dt'$ have the same sign.

It follows from (A.24) and (A.26) that

$$E' = m\gamma \frac{1 - |\mathbf{u}||\mathbf{v}| \cos\theta}{\sqrt{\mathbf{v}^2 - 1}} ; \tag{A.31}$$

$\mathbf{v}$ and $\mathbf{u}$ are the velocities of tachyon and observer in the galaxy frame, and $0 \le \theta \le \pi$. $E'$ is positive in the limit $|\mathbf{v}| \to 1$, and for $|\mathbf{v}| \to \infty$ it attains its minimum value, $E'_{\mathrm{min}} = -m\gamma|\mathbf{u}| \cos\theta$. Hence, only a limited amount of energy can be extracted from a tachyon, as the energy of tachyons is bounded from below in the rest frames of uniformly moving observers.

## References


1. S. Tanaka, *Prog. Theor. Phys.* **24**, 171 (1960).
2. Ya. P. Terletsky, *Sov. Phys. Dokl.* **5**, 782 (1961).
3. G. Feinberg, *Phys. Rev.* **159**, 1089 (1967).
4. F. A. E. Pirani, *Phys. Rev.* **D1**, 3224 (1970).
5. G. Feinberg, *Sci. Am.* **222 (2)**, 69 (1970).
6. G. Feinberg, *Phys. Rev.* **D17**, 1651 (1978).
7. R. Tomaschitz, in *Group Theoretical Methods in Physics, 20th Int. Colloquium*, eds. A. Arima *et al.* (World Scientific, Singapore, 1995).
8. R. Tomaschitz, *Chaos, Solitons & Fractals* **7**, 753 (1996); *ibid.* **8**, 761 (1997); *ibid.* **9**, 1199 (1998).
9. R. Tomaschitz, *Int. J. Mod. Phys.* **A14**, 4275 (1999); *ibid.* **A14**, 5137 (1999).
10. R. Tomaschitz, *Class. Quantum Grav.* **16**, 3349 (1999).
11. R. Tomaschitz, *Int. J. Mod. Phys.* **D7**, 279 (1998).
12. E. A. Milne, *Nature* **130**, 9 (1932).
13. K. Matsuzaki and M. Taniguchi, *Hyperbolic Manifolds and Kleinian Groups* (Clarendon, Oxford, 1998).
14. L. Infeld and A. Schild, *Phys. Rev.* **68**, 250 (1945).
15. P. A. M. Dirac, *Nature* **139**, 323 (1937).
16. R. Tomaschitz, *Int. J. Theor. Phys.* **37**, 1121 (1998).
17. R. Tomaschitz, *Astrophys. & Space Sci.* **259**, 255 (1998); *ibid.* **271**, 181 (2000).
18. G. F. Smoot and D. Scott, *Eur. Phys. J.* **C3**, 1 (1998).