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CHAOS IN THE GALACTIC DYNAMICS

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Abstract

An elementary survey of cosmic chaos, its origins, and its physical impact in an open and multiply connected universe is given. A new type of cosmic evolution by global metrical deformations, unpredicted by Einstein's equations, is pointed out. The uniformity and the inhomogeneities of the galactic background are discussed in this context. There is a finite region in the open 3-space in which the galactic world-lines are chaotic, and the mixing taking place in this chaotic nucleus of the universe provides a mechanism to create the galactic equidistribution.

We review in a completely untechnical way how global metrical deformations of space-time cause particle creation in quantum fields, and induce angular fluctuations in the temperature of the cosmic microwave background radiation. They can generate topology changes without distorting the constant curvature of the 3-space. The micro-topology causes parity violation (i.e., a violation of the space-reflection symmetry) by self-interference.

One of the basic problems one faces today in cosmology is that Einstein's equations do not give any hint of the global topological structure of the Universe. This fact, namely that we do not know the boundary conditions to be imposed on cosmological solutions, was pointed out by Felix Klein soon after Einstein proposed his first cosmological model, in which the spacelike sections are closed and have constant positive curvature. Einstein assumed that the topology of the 3-space is that of a 3-sphere, but it can as well have the topology of projective 3-space, without being at odds with the field equations or the homogeneity and isotropy postulates. It was, however, not until much later that the global behaviour of world-lines in these cosmologies was studied,[1] and in Refs. 2 and 3 it was suggested to consider more general topologies for the space sections.

The actual question is not so much what is the topology, but rather *how does it evolve,* because otherwise it is difficult to answer the question why the 3-space should have acquired, once and for all, a particular type of topology and metric. What we will advocate here is a cosmology with an open 3-space that evolves by global metrical deformations, which can give rise to transitions from one topology to another (extended Robertson-Walker (RW) cosmology[4–6]). We do not know the present topology of the Universe, nor the laws which determined it, and so we have to content ourselves to figure out possible effects of a multiply connected topology.

There are three conditions to be satisfied, in order that the 3-space admits a dynamical evolution by global metrical deformations, and that space-time itself provides a mechanism to generate chaos, which can account for the uniformity of the galactic background. The 3-space must be open, multiply connected, and locally hyperbolic (i.e., negatively curved). Hyperbolicity is necessary to generate the instability of geodesics, and an infinite volume and a multiply connected topology are needed to allow evolution by global metrical deformations. These deformations do not *locally* affect the metric of the constantly curved 3-space. Finally, the multiple connectivity is necessary to confine the unstable world-lines to a finite region, the *center* of the open 3-space,[4,6] so that they can get chaotic there.

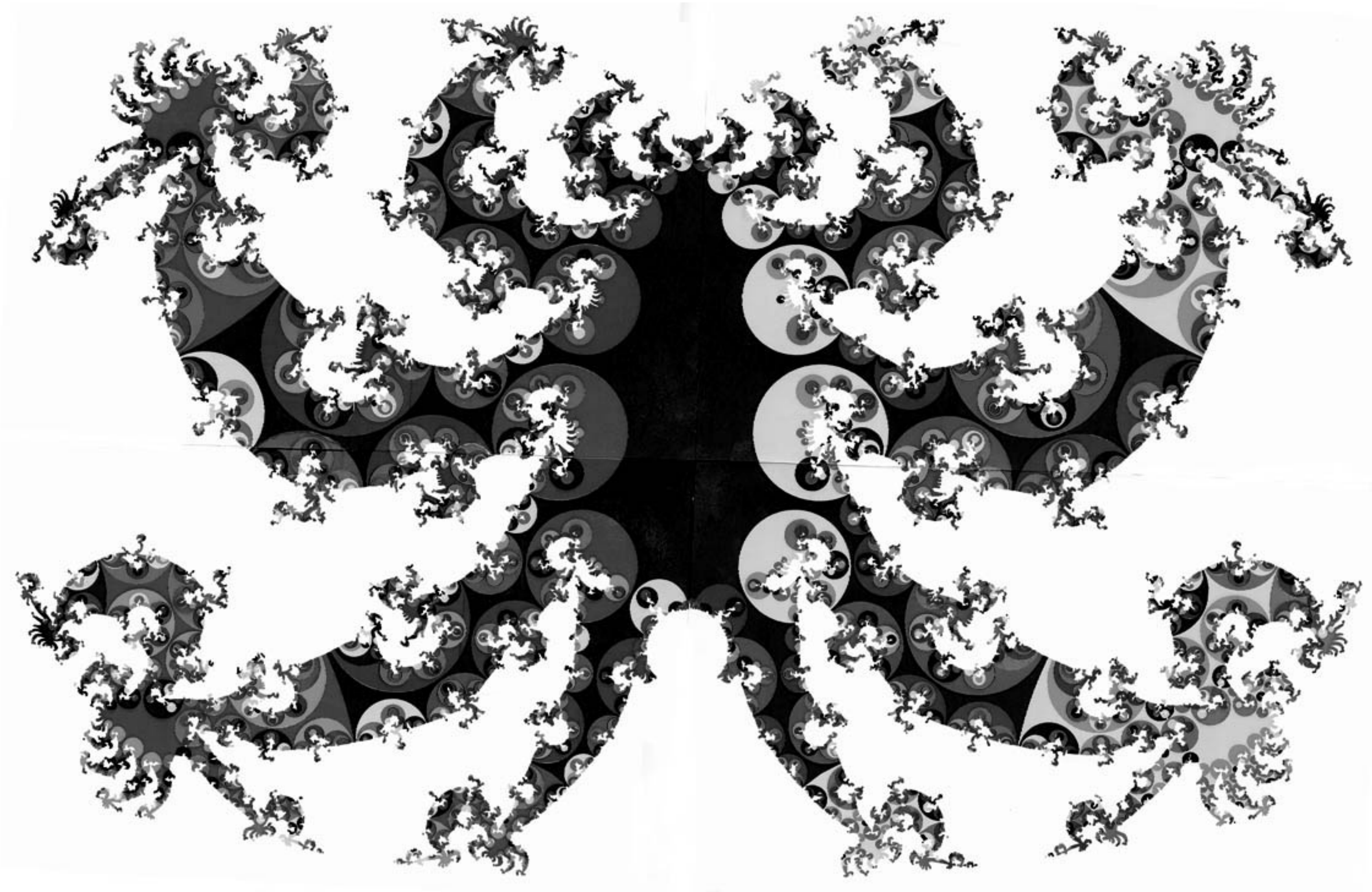
Chaoticity is an efficient mechanism to create an equidistribution, but the actual problem is to explain the inhomogeneities in the galactic distribution. The time evolution of the world-lines depends on the expansion and global deformations that the 3-space undergoes. Concepts like mixing and ergodicity, which are commonly used in Hamiltonian dynamics to describe the degree of chaoticity, do not reflect the time evolution of the system. They are based on the geometric shapes of trajectories of infinite length. Here the question is what happens in finite times, and with world-lines of finite length.[7] There are also regular trajectories which enter the chaotic center of the 3-space, and which are shadowed by chaotic ones, before they ultimately diffuse out of the center into the open 3-space.[8] The mixing in the center tends to create a uniform distribution, but inhomogeneities will always remain in a finite time.

The center of the 3-slices is geometrically determined by the convex hull of the limit set of the covering group, its explicit construction is a matter of elementary geometry.[5] The center is a finite domain with an infinitely pleated surface, due to the fractal nature of the limit set. Trajectories are shadowed by chaotic ones whenever their covering geodesics have initial or terminal points close to the limit set; cf. the caption of Fig. 1.

Objections against open cosmologies are sometimes based on the argument, that we will not be able to look at infinity and to verify what is happening there. What is overlooked here is that the topological structure of the 3-space can manifest itself locally. On a microscopic level a multiply connected structure of space-time has already been suggested by Weyl in the twenties.[9] Recently, a possible fractal microstructure has been pointed out by El Naschie.[10,11]

Particle creation in quantum fields and *backscattering of electromagnetic radiation*[4]: In simply connected Robertson-Walker cosmologies, it is known for a long time,[12] that the variation of the expansion factor can lead to particle creation. However, this cannot happen in conformally coupled neutrino or electromagnetic fields, because in the solutions of the corresponding wave equations the expansion factor scales out with a simple power law. But global metrical deformations of the 3-space do create particles and backscattering even in conformally coupled fields. The point here is that during a deformation the wave equation is not time-separable, and a wave initially composed of positive frequencies will acquire negative frequency modes. So antiparticles emerge in quantum fields, and backscattered wave trains in classical fields.

Angular anisotropy in the temperature of the cosmic microwave background radiation: The observed angular fluctuations of the temperature in the Planck distribution of the microwave background[13] are a possible consequence of global metrical deformations of the spacelike slices of extended RW cosmologies.[4,6] If a deformation is adiabatically switched on, this leads to a distortion of the wave vectors, which results in slightly angular dependent frequency shifts. These shifts can be absorbed in the Planck distribution by introducing an angular dependent temperature variable.

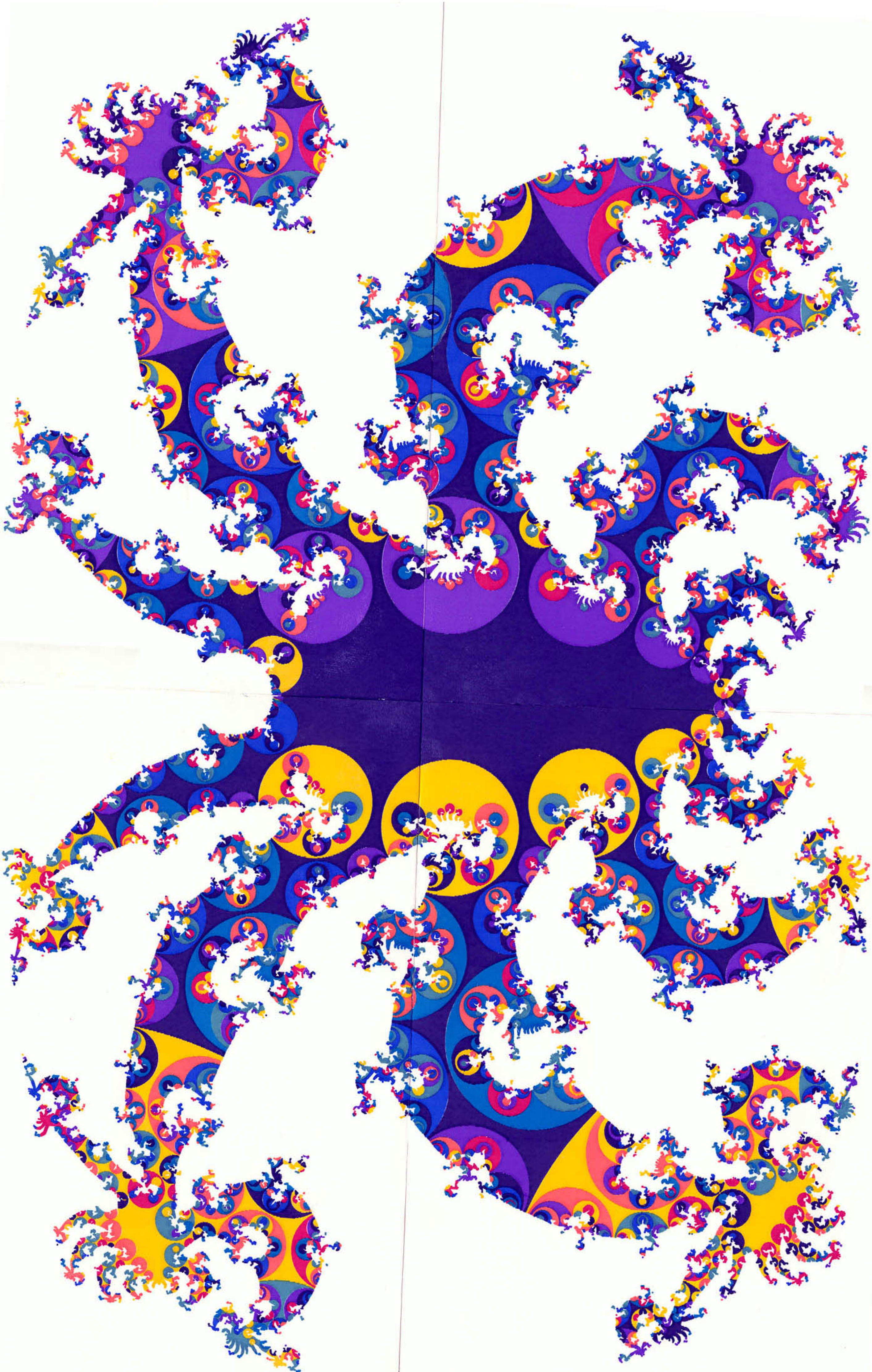


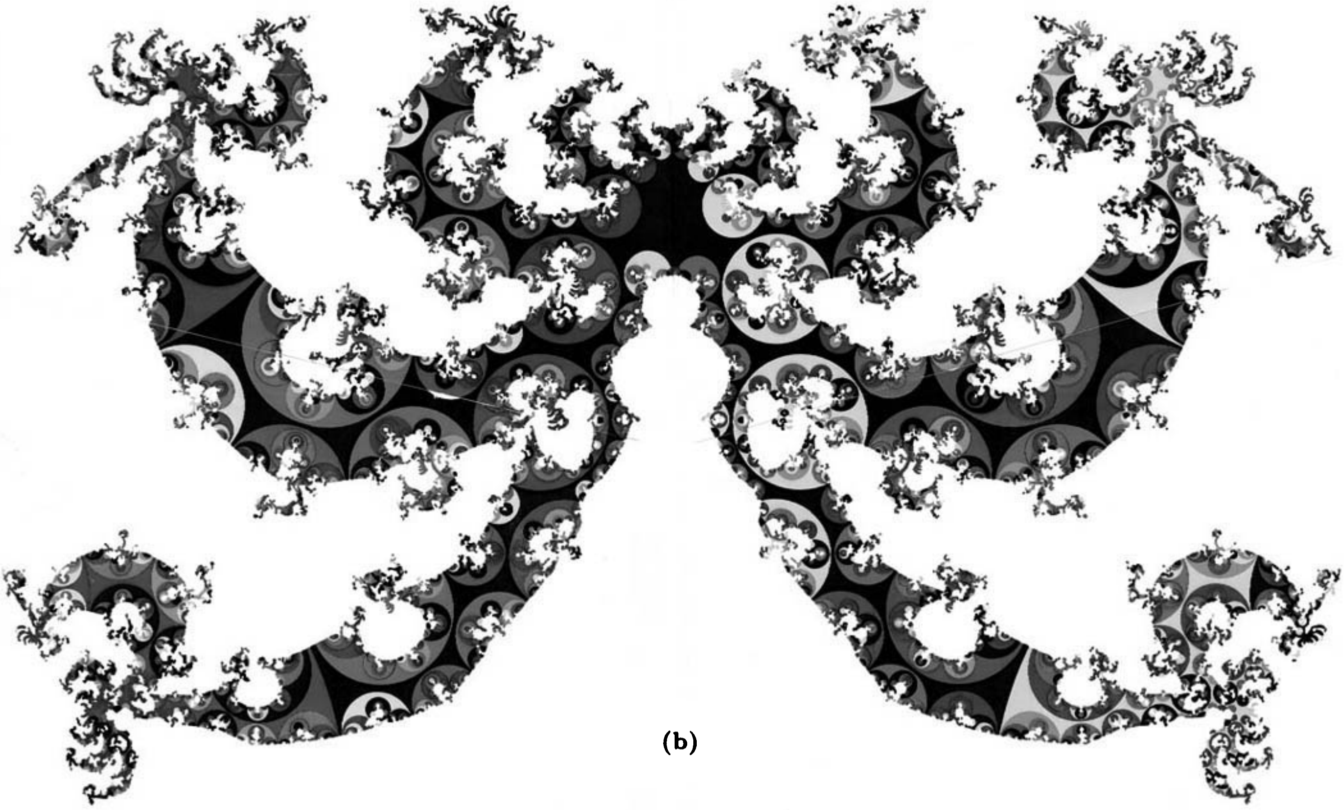
(a)

Fig. 1(a)–(c) The horizon at infinity of the Poincaré half-space H^3, the universal covering space of the spacelike 3-sections of the extended RW cosmology. A spacelike slice (F, Γ) is realized in H^3 as a polyhedron F with face-identification. The face-pairing transformations generate a discrete group Γ which gives, if applied to the polyhedron, a tessellation $\Gamma(F)$ of H^3 with polyhedral images. This tessellation induces by continuity also a tiling on the boundary of H^3, which is depicted here (for three different slices), and which can easily be extended to three dimensions; the complete tiling of H^3 is simply obtained by placing hemispheres onto the circular arcs. This polyhedral tiling of hyperbolic space H^3 is the covering space construction for the RW geometry. So these two-dimensional tilings constitute a completely quantitative characterization of the three-dimensional spacelike slices.

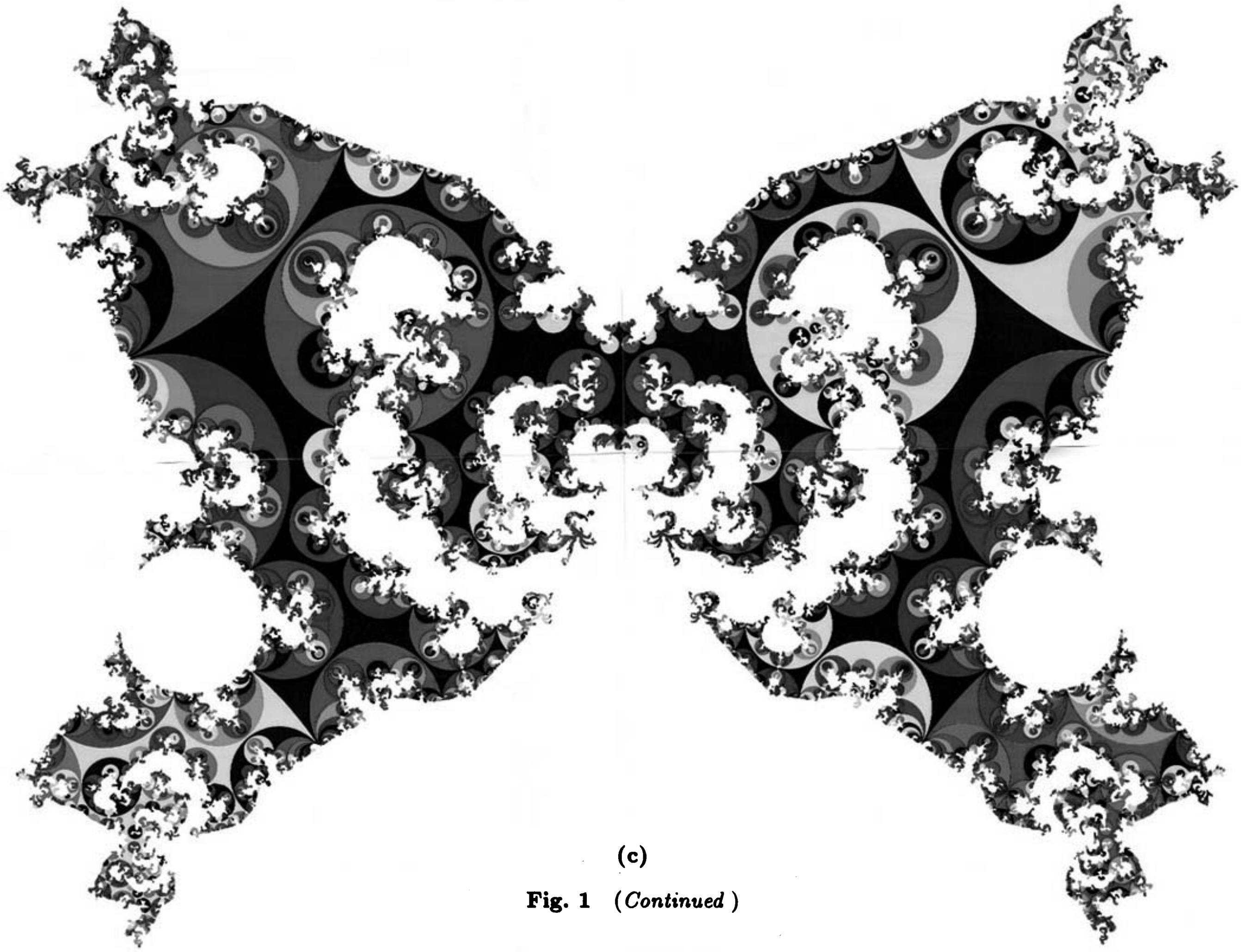
The qualitative structure of the fractal limit set $\Lambda(\Gamma)$ depends on the topology of the 3-slices, which is in turn determined by the covering group. For quasi-Fuchsian groups[15] such as in this case, the limit set is a Jordan curve which is *not* self-similar. The 3-space fibers over an open interval, with Riemann surfaces ($g = 49$) as fibers. The colors label generations in the tiling procedure, the algorithm being given in Ref. 16. The tilings correspond to 3-slices which are non-isometric, but have the same topology and curvature. They are three snapshots in a time parametrized sequence of continuous global deformations of the 3-space, realized here as deformations of the polyhedron F and the covering group Γ. The Hausdorff dimensions of the depicted limit sets vary only slightly, between 1.49 ± 0.03.

The chaotic trajectories have covering trajectories with initial and terminal points in $\Lambda(\Gamma)$. If the end points are not in $\Lambda(\Gamma)$ but close to it, then the trajectory is regular, but it is shadowed by chaotic trajectories over a long period.[8] The convex hull of $\Lambda(\Gamma)$ is the intersection of all hyperbolic half-spaces[17] which contain $\Lambda(\Gamma)$. Projected into (F, Γ) by means of the universal covering projection, it constitutes the chaotic center of the 3-space. Here again, the tilings on the boundary of H^3 are the key to the explicit construction of this nucleus of the open 3-space by methods of elementary geometry.[5,18]



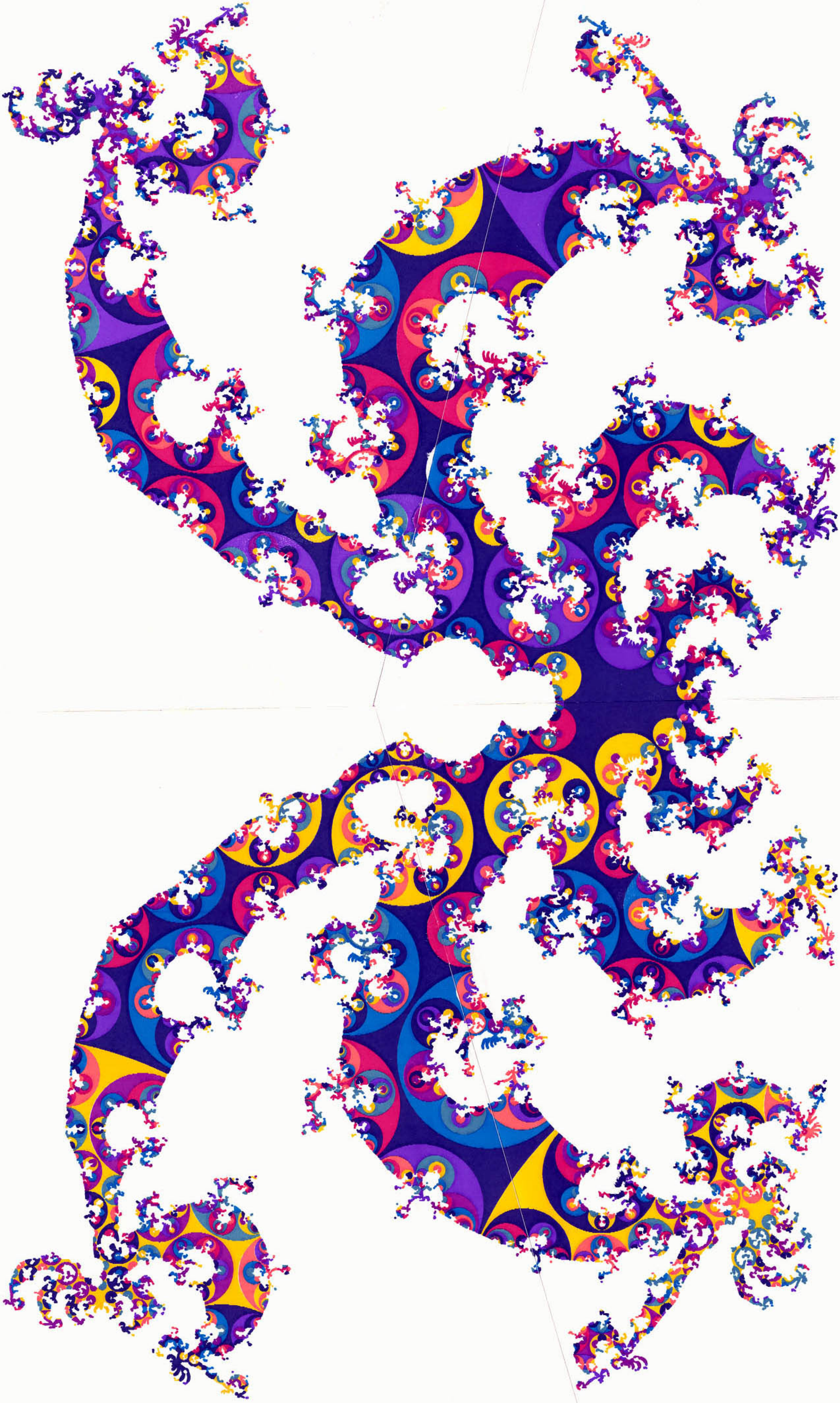


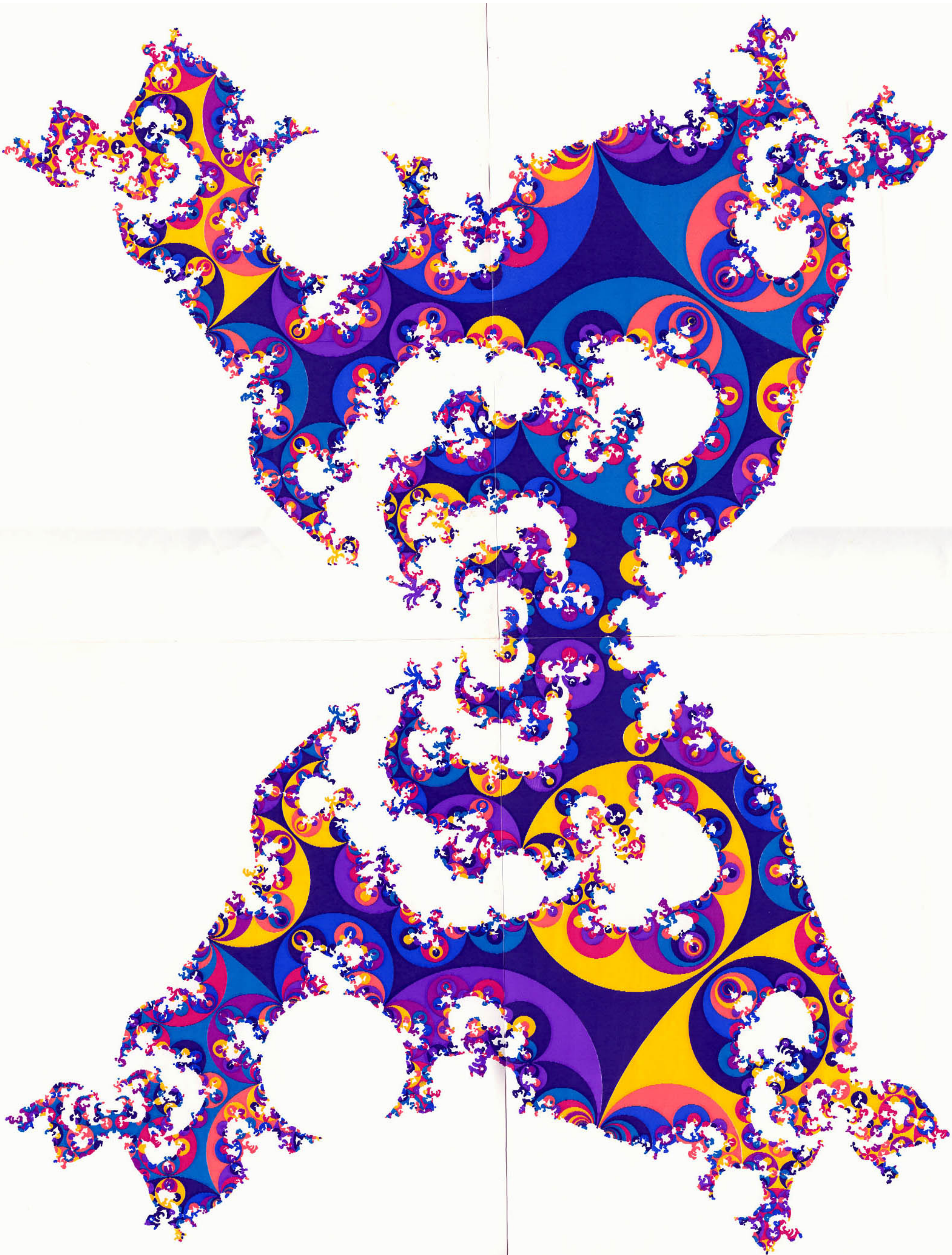
(b)



(c)

Fig. 1 (*Continued*)





If one studies the impact of global metrical deformations on the free photon gas, one has to take two time scales into account.[4] Cosmic time as defined by the expansion, which is the relevant time of an observer co-moving with the galactic background. The second time scale is that on which metrical deformations take place. Only if this second scale is much larger, in periods where the deformations are sufficiently adiabatic, has one a uniform expansion. If these scales approach each other, the expansion of the 3-space will become inhomogeneous, though the 3-space stays constantly curved. The mixing, which occurs in the center because the world-lines are unstable and confined, will always tend to create an equidistribution, but in this case not any more a statistical equilibrium.

Topological CP violation by self-interference: The reasons for CP violation (C: charge conjugation, P: space reflection symmetry, cf., e.g., Ref. 14) are as yet not really understood. This is so despite, or perhaps because this extraordinary phenomenon could be easily incorporated into particle physics by adding symmetry breaking interactions to the Lagrangians. The space-reflection symmetry is of course always present in classical mechanics; its violation is a pure quantum effect. In Ref. 4, it was therefore suggested to explain parity violation as a topological interference phenomenon, by adopting Weyl's idea to associate elementary particles with topological excitations of the 3-space.[9] A space-reflected wave can wrap around a microscopic geodesic loop, and this causes self-interference[5], which destroys the anti-unitarity of the parity operator. Thus parity is a broken symmetry already on the level of free wave equations.

Global metrical deformations, topology changes and Einstein's equations: The spacelike slices in extended RW cosmology constitute a time-parametrized sequence of *globally non-isometric* open hyperbolic 3-manifolds; cf. the caption of Fig. 1. (But if rescaled by the expansion factor, they are *locally* isometric, having the same constant curvature.) This deformation sequence is extended to a 4-manifold by attaching a time-axis. To do so, one has to keep the covering group time independent, and to simulate its variation by a time-parametrized coordinate change in the covering space.[4,6] Such deformations ultimately lead to topology changes, the 3-space can disintegrate, bubbles of infinite volume peel off, leaving behind topological cusp singularities. The topology, the dynamics induced by it via global deformations, and the resulting topology changes are not predictable by Einstein's equations, which nevertheless impose restrictions on the speed and size of global deformations, because the energy and pressure densities *defined* by the Einstein tensor must stay positive.

Gravitational waves and the graviton spectrum in a multiply connected space-time have been recently discussed in Ref. 19. An outline of several open research problems in extended Robertson-Walker cosmology can be found in Ref. 20.

The topology and the topological dynamics of an open universe may have very concrete physical manifestations. I have indicated here some reasons to treat the topology of space-time as a dynamical object, which is able to evolve like the metric in local problems of general relativity.

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