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Causality violation in e^+e^- pair production by superluminal neutrinos

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Abstract – Pair emission by superluminal neutrinos is shown to be causality violating in the 10–50 GeV range covered by the OPERA experiment. Thus, the energy density of a freely propagating superluminal neutrino current is not affected by energy loss due to e^+e^- pair creation, in accordance with the unperturbed energy profile observed by ICARUS. Interaction processes involving sub- and superluminal particles give rise to time inversions in the rest frames of the subluminal constituents, resulting in causality violating predetermination. Therefore, kinematic causality constraints in addition to energy-momentum conservation are necessary to exclude causality violation outside the lightcone. Electron-positron pair production $\nu_\mu \to \nu_\mu + e^+ + e^-$ by superluminal muon neutrinos is forbidden in the OPERA energy range, as the kinematic constraints on the neutrino frequencies and wave vectors required by causality and energy-momentum conservation cannot simultaneously be satisfied.

Introduction. – OPERA reported a preliminary measurement of a superluminal neutrino speed $v/c - 1 \approx 2.4 \pm 0.6 \times 10^{-5}$ at an average energy of 17 GeV, inferred from a muon neutrino beam over a baseline of 730 km between the source at CERN and the OPERA detector at Gran Sasso [1]. Doubts regarding this excess speed have been raised by the ICARUS collaboration, who confirmed an undistorted energy profile of the freely propagating neutrino current [2] and found no evidence for a superluminal neutrino velocity [3]. Their objections were based on an untested energy loss rate, assuming pair emission $\nu_\mu \to \nu_\mu + e^+ + e^-$, and predicting a cutoff in the energy distribution at 13 GeV. This pair creation requires a superluminal neutrino speed, otherwise the kinematic constraints due to energy-momentum conservation cannot be met. Since there was no cutoff observable in the energy distribution of the CERN neutrino beam arriving at Gran Sasso, nor any direct evidence of e^+e^- pair events in the ICARUS detector, it was argued that the neutrinos cannot be superluminal [2].

Here, we show that pair production by superluminal neutrinos is forbidden in the OPERA energy range by causality violation. Energy-momentum conservation does not suffice to guarantee causal connections in an interaction process with sub- and superluminal constituents. Outside the lightcone, additional kinematic constraints have to be satisfied for an interaction to be causal.

We discuss superluminal signal transfer in an absolute spacetime conception, which allows to unambiguously distinguish causal from acausal connections by virtue of a universal cosmic time order. The absolute spacetime is manifested by a dispersive aether, the medium of wave propagation [4–6], and wave equations couple by a permeability tensor to the aether. The rest frame of the aether (aether frame) constitutes a distinguished frame of reference in which the permeability tensor is isotropic. The aether frame coincides with the rest frame of the cosmic microwave background radiation defined by vanishing temperature dipole anisotropy [7]. The Solar system barycenter is moving in the aether frame with a speed of $v/c \approx 1.23 \times 10^{-3}$ [8–11].

Inertial rest frames of subluminal particles have a well-defined relative speed $\boldsymbol{v}_r$ in the aether. The permeability tensor is anisotropic in moving inertial frames, depending on this relative speed. Superluminal particles propagating with speed $\boldsymbol{v}_s$ in the aether frame ($v_s > 1$, $\hbar = c = 1$ from now on) establish spacelike connections, whose time order can be overturned by Lorentz boosts. As a consequence,

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the cosmic time order of the aether frame appears inverted in the proper time of subluminal particles, resulting in causality violation [12–14]. To maintain causality in an interaction process involving sub- and superluminal particles, kinematic causality constraints are required in addition to energy-momentum conservation to preserve the time order of the aether frame in the rest frames of the subluminal constituents. These causality constraints are a set of inequalities, $\boldsymbol{v}_{r,i}\boldsymbol{v}_{s,j}<1$, where $\boldsymbol{v}_r$ and $\boldsymbol{v}_s$ are the velocities (in the aether frame) of the interacting sub- and superluminal particles labeled by i and j, respectively. If one of these inequalities is violated, a time inversion will occur in the rest frame of the respective subluminal particle. The pair production $\nu_\mu\to\nu_\mu+e^++e^-$ requires four inequalities $\boldsymbol{v}_{r,i}\boldsymbol{v}_{s,j}<1$, where index $i=e,p$ labels the subluminal speed of electron and positron, and index $j=1,2$ the superluminal velocity of the in- and outgoing neutrino.

Neutrinos freely propagating in the aether are described by the Dirac equation $\gamma_\mu g^{\mu\nu}\psi_{,\nu}+m\psi=0$ coupled to a real symmetric permeability tensor $g^{\mu\nu}(\omega)$, which depends on the frequency of the spinor modes $\psi\propto \mathrm{e}^{\mathrm{i}k_\mu x^\mu}$ [6]. The sign convention for the Minkowski metric is $\eta_{\mu\nu}=\mathrm{diag}(-1,1,1,1)$, so that the Dirac matrices satisfy $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\eta_{\mu\nu}$, where γ_0 is anti-Hermitian and the γ_i are Hermitian. Indices are raised and lowered with the Minkowski metric. In the aether frame, the permeability tensor is isotropic, $g^{00}=-\varepsilon(\omega)$, $g^{ik}=\delta^{ik}/\mu(\omega)$, $g^{0k}=0$, where ε and μ are real positive permeabilities. To obtain the dispersion relation for the wave vector of the spinorial plane waves, we square the Dirac equation,

$$(h^{\mu\nu}\partial_\mu\partial_\nu-m^2)\psi=0. \tag{1}$$

The squared permeability tensor $h^{\mu\nu}=g^{\mu\alpha}\eta_{\alpha\beta}g^{\beta\nu}$ is likewise isotropic,

$$h^{00}(\omega)=-\varepsilon^2(\omega),\quad h^{ik}(\omega)=\frac{\delta^{ik}}{\mu^2(\omega)},\quad h^{0k}=0. \tag{2}$$

By substituting the plane waves $\psi\propto \mathrm{e}^{\mathrm{i}k_\mu x^\mu}$, $k_\mu=(-\omega,\mathbf{k})$, into the Klein-Gordon equation (1), we find the dispersion relation $h^{\alpha\beta}k_\alpha k_\beta+m^2=0$. This relation is equivalent to the Hamilton-Jacobi equation, if we identify $k_\mu=S_{,\mu}$, so that the classical action $S=k_\mu x^\mu$ coincides with the phase of the elementary waves. Employing the diagonal permeability tensor $h^{\mu\nu}$ in (2), we find $\mathbf{k}(\omega)=k(\omega)\mathbf{k}_0$, with a constant unit vector $\mathbf{k}_0$ and a positive wave number $k(\omega)$ defined by the dispersion relation $k^2=\mu^2(\varepsilon^2\omega^2-m^2)$ [6].

In this article, we study dispersive superluminal wave propagation in moving inertial frames, deriving anisotropic dispersion relations and neutrino Doppler shifts depending on the refractive index of the aether. The focus is on pair creation by superluminal neutrinos, illustrating causality violation outside the lightcone.

Pair creation $\nu_\mu\to\nu_\mu+e^++e^-$ by superluminal neutrinos? Energy-momentum and causality constraints. – The electron and positron wave vectors are denoted by $\mathbf{k}_i$, $i=e,p$, the wave numbers by k_i, and the frequencies by ω_i. The neutrino variables are $\mathbf{k}_{\nu j}$, $k_{\nu j}$, and $\omega_{\nu j}$, where the index $j=1,2$ refers to the in- and out-states. The dispersion relations defining the frequency dependence of the wave numbers in the aether frame read, cf. (2),

$$k_i=\sqrt{\omega_i^2-m_e^2},\quad k_{\nu j}=\mu_j\sqrt{\varepsilon_j^2\omega_{\nu j}^2-m_\nu^2}, \tag{3}$$

where m_e is the electron mass and m_ν a possible neutrino mass of a few eV. The index $i=e,p$ labels electron and positron; $k_{\nu1}$ and $k_{\nu2}$ are the wave numbers of the neutrino in- and out-states, respectively. The permeabilities $\varepsilon_j=\varepsilon(\omega_{\nu j})$ and $\mu_j=\mu(\omega_{\nu j})$, $j=1,2$, depend on the neutrino frequencies.

Energy and momentum conservation implies the constraints

$$\omega_{\nu1}=\omega_{\nu2}+\omega_e+\omega_p,\quad \mathbf{k}_{\nu1}=\mathbf{k}_{\nu2}+\mathbf{k}_e+\mathbf{k}_p. \tag{4}$$

We consider aligned in- and outgoing neutrino momenta, $\mathbf{k}_{\nu j}=k_{\nu j}\mathbf{k}_{\nu,0}$, where $\mathbf{k}_{\nu,0}$ is the neutrino unit wave vector. We also assume that electron and positron have the same emission angle $\mathbf{k}_{i,0}\mathbf{k}_{\nu,0}=\cos\theta$, $0\leqslant\theta\leqslant\pi/2$, and thus the same energy $\omega_i=\delta\omega_\nu/2$, $\delta\omega_\nu=\omega_{\nu1}-\omega_{\nu2}$. On multiplying the momentum conservation (4) by $\mathbf{k}_{\nu,0}$, we obtain

$$\cos\theta=\frac{\delta k_\nu}{\sqrt{\delta\omega_\nu^2-4m_e^2}}, \tag{5}$$

where $\delta k_\nu=k_{\nu1}-k_{\nu2}$. The same identity is found by squaring the momentum relation.

The four kinematic causality constraints mentioned in the introduction read

$$\boldsymbol{v}_i\boldsymbol{v}_{\nu j}<1,\quad i=e,p,\quad j=1,2, \tag{6}$$

where $\boldsymbol{v}_{i=e,p}=\mathbf{k}_i/\omega_i$ denotes the speed of electron and positron, and $\boldsymbol{v}_{\nu1,\nu2}$ are the superluminal velocities of the incoming ($\nu1$) and outgoing ($\nu2$) neutrino, defined by the group velocity of the wave modes, cf. (7). All velocities refer to the aether frame. The constraints (6) are necessary and sufficient to prevent causality-violating time inversions in the proper time of electron and positron, and will be derived in (26)–(33). In this section, we show that inequalities (6) cannot be satisfied in the energy range of the OPERA experiment.

We write the neutrino velocity as $\boldsymbol{v}_\nu=v_{\mathrm{gr}}\mathbf{k}_{\nu,0}$, where $\mathbf{k}_{\nu,0}$ is the unit wave vector of the neutrino. The group velocity v_{gr} is defined by the frequency derivative of the wave number (3) [6,15,16],

$$\frac{1}{v_{\mathrm{gr}}(\omega_\nu)}=\frac{\mathrm{d}k_\nu}{\mathrm{d}\omega_\nu}=\frac{\omega_\nu}{k_\nu}\mu^2\varepsilon^2\kappa_{\mathrm{dis}}, \tag{7}$$

where $\kappa_{\rm dis}$ is the dispersion measure

$$\kappa_{\rm dis}(\omega_\nu) = 1 + \frac{\varepsilon'}{\varepsilon}\omega_\nu + \frac{\mu'}{\mu}\omega_\nu - \frac{\mu'}{\mu}\frac{m_\nu^2}{\omega_\nu \varepsilon^2}. \quad (8)$$

The primes denote frequency derivatives of the permeabilities $\varepsilon(\omega_\nu)$ and $\mu(\omega_\nu)$. (We here drop the index j labeling in- and out-states.) If the permeabilities are constant, we find $\kappa_{\rm dis} = 1$. We assume a positive group velocity, which means a positive dispersion measure $\kappa_{\rm dis} > 0$, as the permeabilities are nearly constant in the OPERA energy range [6]. Negative group velocities are possible [17,18], but require substantial logarithmic derivatives of the permeabilities to change the sign of $\kappa_{\rm dis}$. The neutrino velocity thus reads

$$\boldsymbol{\upsilon}_\nu = \frac{1}{\mu^2\varepsilon^2\kappa_{\rm dis}}\frac{\mathbf{k}_\nu}{\omega_\nu} = \upsilon_\nu \mathbf{k}_{\nu,0}, \quad \upsilon_\nu = \frac{\sqrt{\varepsilon^2\omega_\nu^2 - m_\nu^2}}{\mu\varepsilon^2\kappa_{\rm dis}\omega_\nu}, \quad (9)$$

where $\upsilon_\nu = \upsilon_{\rm gr}$ denotes the absolute value; we will also use $\upsilon_{\nu j}$ to label the in- and outgoing neutrino. Clearly, $\upsilon_\nu \leqslant 1/(\mu\varepsilon\kappa_{\rm dis})$, where equality is attained at zero neutrino mass. Thus, a superluminal neutrino speed can only be attained if $\mu\varepsilon\kappa_{\rm dis} < 1$, which is better seen by writing the neutrino energy and wave number as

$$\omega_\nu = \frac{m_\nu}{\varepsilon\sqrt{1 - \upsilon_\nu^2\mu^2\varepsilon^2\kappa_{\rm dis}^2}}, \quad k_\nu = \frac{m_\nu \upsilon_\nu \mu^2 \varepsilon \kappa_{\rm dis}}{\sqrt{1 - \upsilon_\nu^2\mu^2\varepsilon^2\kappa_{\rm dis}^2}}. \quad (10)$$

The refractive index [4,6]

$$n_{\rm r}(\omega_\nu) = \frac{k_\nu}{\omega_\nu} = \mu\varepsilon\sqrt{1 - \frac{m_\nu^2}{\varepsilon^2\omega_\nu^2}} \quad (11)$$

and its frequency derivative completely determine the neutrino velocity (9) at any given frequency,

$$\upsilon_\nu = \frac{n_{\rm r}(\omega_\nu)}{\mu^2\varepsilon^2\kappa_{\rm dis}} = \frac{1}{(\omega_\nu n_{\rm r})'} = \frac{1}{n_{\rm r}(1 + \omega_\nu n_{\rm r}'/n_{\rm r})}, \quad (12)$$

as well as the wave number $k_\nu = \omega_\nu n_{\rm r}(\omega_\nu)$. In the massless case, $m_\nu = 0$, we have $n_{\rm r} = \mu\varepsilon$.

The causality conditions $\boldsymbol{\upsilon}_i \boldsymbol{\upsilon}_{\nu j} < 1$ in (6) can be expressed in terms of the refractive index. The neutrino velocity is $\boldsymbol{\upsilon}_{\nu j} = \upsilon_{\nu j}\mathbf{k}_{\nu,0}$, with $\upsilon_{\nu j}$ in (12). The electron and positron velocities are $\boldsymbol{\upsilon}_i = k_i \mathbf{k}_{i,0}/\omega_i$, with the wave numbers k_i in (3). On substituting these velocities into the causality constraints (6), we obtain

$$\frac{k_i}{\omega_i}\frac{\mathbf{k}_{i,0}\mathbf{k}_{\nu j,0}}{(\omega_{\nu j} n_{{\rm r},j})'} < 1, \quad (13)$$

where $n_{{\rm r},j} = n_{\rm r}(\omega_{\nu j})$. The in- and outgoing neutrino variables are labeled by $j = 1, 2$, and the electron and positron variables by $i = e, p$. Since the electron and positron velocities give identical inequalities, we end up with two constraints ($j = 1, 2$), cf. after (4),

$$\frac{\sqrt{\delta\omega_\nu^2 - 4m_e^2}}{\delta\omega_\nu}\frac{\cos\theta}{(\omega_{\nu j} n_{{\rm r},j})'} < 1. \quad (14)$$

By substituting the emission angle (5), we obtain $\delta k_\nu/\delta\omega_\nu < (\omega_{\nu j} n_{\rm r,j})'$, or more explicitly,

$$\frac{\omega_{\nu 1} n_{\rm r}(\omega_{\nu 1}) - \omega_{\nu 2} n_{\rm r}(\omega_{\nu 2})}{\omega_{\nu 1} - \omega_{\nu 2}} < (\omega_{\nu j} n_{\rm r}(\omega_{\nu j}))'. \quad (15)$$

The left-hand side can be replaced by the derivative $(\tilde\omega n_{\rm r}(\tilde\omega))'$, where $\tilde\omega$ is a frequency in the interval $\omega_{\nu 2} < \tilde\omega < \omega_{\nu 1}$ (mean value theorem), so that the causality conditions read $(\tilde\omega n_{\rm r}(\tilde\omega))' < (\omega_{\nu j} n_{\rm r}(\omega_{\nu j}))'$, $j = 1, 2$. In terms of the neutrino group velocity (12), this means $\upsilon_\nu(\tilde\omega) > \upsilon_\nu(\omega_{\nu j})$. As $\upsilon_\nu(\omega)$ is monotonically increasing in the OPERA energy range, the first of these inequalities, $j = 1$, is violated. At 13.8 GeV, 28.2 GeV, and 40.7 GeV, the preliminary result for the excess speed $\upsilon_\nu - 1$ is 2.25×10^{-5}, 2.5×10^{-5} and 2.8×10^{-5}, respectively [1]. In an energy range where the excess speed is decreasing, the second condition $\upsilon_\nu(\tilde\omega) > \upsilon_\nu(\omega_{\nu 2})$ is violated. Only if the group velocity $\upsilon_\nu(\omega)$ has a peak between the in- and outgoing neutrino frequencies, both causality constraints can be met.

Anisotropic permeability tensor and Doppler-shifted neutrino frequencies in moving inertial frames. – We consider an inertial frame moving in the aether with constant relative velocity $\boldsymbol{\upsilon}_r = \upsilon_r \boldsymbol{\upsilon}_{r,0}$. Zero subscripts denote unit vectors. The spacetime coordinates of the aether frame are denoted by $x^\mu = (t, \mathbf{x})$, and of the inertial frame by $x'^\mu = (t', \mathbf{x}')$. The proper orthochronous Lorentz boost $x'^\mu = \Lambda_\nu^{(-1)\mu} x^\nu$ relating the frames reads

$$t' = \gamma_r t - \gamma_r \boldsymbol{\upsilon}_r \mathbf{x}, \ \mathbf{x}' = -\gamma_r \boldsymbol{\upsilon}_r t + \mathbf{x} + (\gamma_r - 1)\boldsymbol{\upsilon}_{r,0}(\boldsymbol{\upsilon}_{r,0}\mathbf{x}), \quad (16)$$

with Lorentz factor $\gamma_r = (1 - \upsilon_r^2)^{-1/2}$, so that $\upsilon_r\gamma_r = \sqrt{\gamma_r^2 - 1}$. We can thus identify

$$\begin{aligned} \Lambda_0^{(-1)0} &= \gamma_r, \quad \Lambda_k^{(-1)0} = \Lambda_0^{(-1)k} = -\gamma_r \boldsymbol{\upsilon}_r^k, \\ \Lambda_k^{(-1)i} &= \delta_{ik} + (\gamma_r - 1)\boldsymbol{\upsilon}_{r,0}^i \boldsymbol{\upsilon}_{r,0}^k. \end{aligned} \quad (17)$$

The inverse transformation, $x^\mu = \Lambda_\nu^\mu x'^\nu$, is obtained by changing the sign of the (always subluminal) relative velocity $\boldsymbol{\upsilon}_r$. The speed $\boldsymbol{\upsilon}$ of a uniformly moving sub- or superluminal particle in the aether frame transforms as, cf. [6] and (33),

$$\boldsymbol{\upsilon}' = \frac{\boldsymbol{\upsilon} - \gamma_r \boldsymbol{\upsilon}_r + (\gamma_r - 1)\boldsymbol{\upsilon}_{r,0}(\boldsymbol{\upsilon}_{r,0}\boldsymbol{\upsilon})}{\gamma_r(1 - \boldsymbol{\upsilon}_r \boldsymbol{\upsilon})}. \quad (18)$$

Primed quantities refer to the inertial frame. This transformation preserves sub- and superluminal velocities, cf. (25).

The wave 4-vectors defining the phase of plane waves are denoted by $k_\mu = (-\omega, \mathbf{k})$ in the aether frame, and by $k'_\mu = (-\omega', \mathbf{k}')$ in the inertial frame. They transform covariantly, $k'_\mu = k_\alpha \Lambda_\mu^\alpha$, as

$$\omega' = \gamma_r\omega - \gamma_r \boldsymbol{\upsilon}_r \mathbf{k}, \ \mathbf{k}' = -\gamma_r \omega \boldsymbol{\upsilon}_r + (\gamma_r - 1)\boldsymbol{\upsilon}_{r,0}(\boldsymbol{\upsilon}_{r,0}\mathbf{k}) + \mathbf{k}. \quad (19)$$

The permeability tensor $h^{\mu\nu}$, cf. (1) and (2), transforms contravariantly, $h'^{\mu\nu} = h^{\alpha\beta}\Lambda_\alpha^{(-1)\mu}\Lambda_\beta^{(-1)\nu}$, and is anisotropic in the inertial frame,

$$
\begin{aligned}
h'^{00}(\omega) &= -\frac{1}{\mu^2(\omega)} - \left(\varepsilon^2(\omega) - \frac{1}{\mu^2(\omega)}\right)\gamma_r^2, \\
h'^{0n}(\omega) &= \left(\varepsilon^2(\omega) - \frac{1}{\mu^2(\omega)}\right)\gamma_r^2 \boldsymbol{\upsilon}_r^n, \\
h'^{mn}(\omega) &= \frac{1}{\mu^2(\omega)}\delta_{mn} - \left(\varepsilon^2(\omega) - \frac{1}{\mu^2(\omega)}\right)\gamma_r^2 \boldsymbol{\upsilon}_r^m \boldsymbol{\upsilon}_r^n.
\end{aligned} \tag{20}
$$

Relativistic covariance is abandoned, as this tensor depends on the frequency of the wave modes in the aether frame. The dispersion relation $h'^{\mu\nu}k'_\mu k'_\nu + m^2 = 0$ is satisfied by $k'_\mu = (-\omega', \mathbf{k}')$ in (19), and the classical action coinciding with the phase stays invariant, $S = S' = k'_\mu x'^\mu$, cf. after (2). As the wave 4-vectors transform linearly, the energy-momentum constraints (4) also hold in primed variables ω' and $\mathbf{k}'$.

For instance, we may consider the pair production discussed in the previous section in the $(e^+, e^-, \nu 2)$ center-of-mass frame, $\nu 2$ being the outgoing neutrino. This frame is obtained via the Lorentz boost (16) with relative speed $\boldsymbol{\upsilon}_r = \mathbf{k}_{\nu 1}/\omega_{\nu 1}$, so that $\upsilon_r = n_{\mathrm{r},1} < 1$, where $n_{\mathrm{r},1} = n_\mathrm{r}(\omega_{\nu 1})$ is the refractive index at the frequency of the incoming neutrino $\nu 1$, cf. (11). The neutrino momentum $\mathbf{k}'_{\nu 1}$ vanishes, $\mathbf{k}'_{\nu 1} = \mathbf{k}'_{\nu 2} + \mathbf{k}'_e + \mathbf{k}'_p = 0$, cf. (19), and we find the Doppler-shifted neutrino energy and speed in the center-of-mass frame as

$$
\omega'_{\nu 1} = \omega_{\nu 1}\sqrt{1 - n_{\mathrm{r},1}^2}, \quad \boldsymbol{\upsilon}'_{\nu 1} = \frac{\upsilon_{\nu 1} - n_{\mathrm{r},1}}{1 - \upsilon_{\nu 1} n_{\mathrm{r},1}}\mathbf{k}_{\nu 1,0}, \tag{21}
$$

where $\upsilon_{\nu 1} = \upsilon_\nu(\omega_{\nu 1})$ is the neutrino group velocity in the aether frame, cf. (9) and (12). In the center-of-mass frame, the incoming neutrino emerges without wave vector, $\mathbf{k}'_{\nu 1} = 0$, yet propagating at a well-defined superluminal speed and energy.

In the next section, we will study the kinematics of pair emission, $\nu_\mu \to \nu_\mu + e^+ + e^-$, in the rest frame of the electron, illustrating causality violation by time inversion. (The discussion of the positron rest frame is completely analogous, and the superluminal neutrino does not admit a rest frame, as Lorentz boosts are subluminal.) We identify the relative speed in eqs. (16)–(20) with the electron velocity in the aether frame, $\boldsymbol{\upsilon}_r = \mathbf{k}_e/\omega_e = \boldsymbol{\upsilon}_e$, so that the Lorentz factor reads $\gamma_r = \gamma_e = (1 - \upsilon_e^2)^{-1/2}$. As for the neutrino wave numbers, we put $k_{\nu j} = \omega_{\nu j} n_{\mathrm{r},j}$, $j = 1, 2$, cf. after (12) and (13), and substitute this into (19), to find the Doppler-shifted in- and outgoing neutrino frequencies in the electron rest frame,

$$
\omega'_{\nu j} = \gamma_e \omega_{\nu j}(1 - n_{\mathrm{r},j}\upsilon_e \cos\theta). \tag{22}
$$

Here, $\cos\theta = \boldsymbol{\upsilon}_{e,0}\boldsymbol{\upsilon}_{\nu j,0}$ is the emission angle (5) of the electron in the aether frame, and $n_{\mathrm{r},j} = n_\mathrm{r}(\omega_{\nu j})$ the refractive index. We also note $\omega'_e = m_e$ and $\omega'_p = \omega'_{\nu 1} - \omega'_{\nu 2} - m_e$, cf. (4). Since $n_\mathrm{r} < 1$ and $\upsilon_e < 1$, the neutrino frequencies $\omega'_{\nu j}$ stay positive in the electron's rest frame, cf. (22). The positron frequency ω'_p in the electron rest frame is positive as well, since the Lorentz boosts (16) preserve the sign of energy of subluminal particles with timelike 4-momenta.

The electronic wave vector vanishes in the electron rest frame. As $\mathbf{k}'_e = 0$, positron and neutrino wave vectors are related by $\mathbf{k}'_p = \mathbf{k}'_{\nu 1} - \mathbf{k}'_{\nu 2}$. The in- and outgoing neutrino wave vectors (labeled by $j = 1, 2$) in the electron's rest frame read, cf. (19),

$$
\mathbf{k}'_{\nu j} = \omega_{\nu j}[n_{\mathrm{r},j}\boldsymbol{\upsilon}_{\nu j,0} - \gamma_e \upsilon_e \boldsymbol{\upsilon}_{e,0} + (\gamma_e - 1)n_{\mathrm{r},j}\boldsymbol{\upsilon}_{e,0}\cos\theta], \tag{23}
$$

where we have identified the unit vectors of neutrino speed and momentum in the aether frame, $\mathbf{k}_{\nu j,0} = \boldsymbol{\upsilon}_{\nu j,0}$, cf. (9). θ is the electron emission angle as in (22). The ratio $\mathbf{k}'_{\nu j}/\omega'_{\nu j}$, cf. (22) and (23), can be recovered from the neutrino velocity in the electron rest frame, cf. (18),

$$
\boldsymbol{\upsilon}'_{\nu j} = \frac{\upsilon_{\nu j}\boldsymbol{\upsilon}_{\nu j,0} - \gamma_e \upsilon_e \boldsymbol{\upsilon}_{e,0} + (\gamma_e - 1)\upsilon_{\nu j}\boldsymbol{\upsilon}_{e,0}\cos\theta}{\gamma_e(1 - \upsilon_{\nu j}\upsilon_e\cos\theta)}, \tag{24}
$$

by replacing the neutrino group velocity $\upsilon_{\nu j}$ in (24) by the refractive index $n_{\mathrm{r},j}$. Group velocity and refractive index are related as stated in (12). We also note the electron variables $\gamma_e = \omega_e/m_e$ and $\upsilon_e = \sqrt{1 - m_e^2/\omega_e^2}$ in (24). By squaring eqs. (23) and (24), we find the absolute value $\upsilon'_{\nu j}$ of the neutrino velocities and the neutrino wave numbers $k'_{\nu j}$,

$$
\begin{aligned}
\upsilon'_{\nu j} &= \sqrt{1 + \frac{\upsilon_{\nu j}^2 - 1}{\gamma_e^2(1 - \upsilon_{\nu j}\upsilon_e\cos\theta)^2}}, \\
\frac{k'^2_{\nu j}}{\omega'^2_{\nu j}} &= 1 + \frac{n_{\mathrm{r},j}^2 - 1}{\gamma_e^2(1 - n_{\mathrm{r},j}\upsilon_e\cos\theta)^2},
\end{aligned} \tag{25}
$$

with $\omega'_{\nu j}$ in (22) and $n_{\mathrm{r},j} = n_\mathrm{r}(\omega_{\nu j})$. In the following, we discuss the real-space world lines of the in- and outgoing neutrino as they appear in the electron's proper time.

Causality violation by time inversion. – In the preceding sections, we have studied the energy-momentum variables of pair emission, $\nu_\mu \to \nu_\mu + e^+ + e^-$. Here, we analyze the real-time kinematics. The coordinates of the aether frame are denoted by $(t, \mathbf{x})$. The world lines of the in- and outgoing superluminal neutrino read $\mathbf{x}_{\nu j}(t) = \tilde{\mathbf{x}} + \boldsymbol{\upsilon}_{\nu j}(t - t_c)$, $j = 1, 2$. The world lines of electron and positron are $\mathbf{x}_e(t) = \tilde{\mathbf{x}} + \boldsymbol{\upsilon}_e(t - t_c)$ and $\mathbf{x}_p(t) = \tilde{\mathbf{x}} + \boldsymbol{\upsilon}_p(t - t_c)$, respectively. At $t = t_c$, these four world lines coalesce at $\tilde{\mathbf{x}}$. Neutrino $\nu 1$ is emitted by a source located at $(t_0, \mathbf{x}_{\nu 1}(t_0))$, $t_0 < t_c$. The electron-positron pair emission takes place at $(t_c, \tilde{\mathbf{x}})$. At a later instant $t_1 > t_c$, the positron is absorbed by a detector located at $(t_1, \mathbf{x}_p(t_1))$, the electron at $(t_1, \mathbf{x}_e(t_1))$, and neutrino $\nu 2$ at $(t_1, \mathbf{x}_{\nu 2}(t_1))$.

The electron rest frame is denoted by primed coordinates $(t', \mathbf{x}')$, and connected to the aether frame $(t, \mathbf{x})$ by the Lorentz boost (16), with relative velocity $\boldsymbol{\upsilon}_r = \boldsymbol{\upsilon}_e$ and

$\gamma_r = \gamma_e = (1-\upsilon_e^2)^{-1/2}$. We discuss only the rest frame of the electron; the positron rest frame can be dealt with in like manner. As mentioned, there is no rest frame for superluminal particles [19], as the relative speed defining Lorentz boosts is subluminal; a possible neutrino mass in wave equation (1) is not a rest mass if the neutrino is superluminal. The existence of a rest frame in a permeable spacetime exclusively depends on the particle speed, which has to be subluminal. If the particle is subluminal in the aether frame, it remains so in any other inertial frame, and analogously for superluminal velocities, cf. (18) and (25).

In the electron's rest frame $(t', \mathbf{x}')$, we parametrize the world lines with the reference time of the aether frame. The world line $(t'_{\nu 1}(t), \mathbf{x}'_{\nu 1}(t))$ of the incoming neutrino $\nu 1$ reads

$$\begin{aligned} t'_{\nu 1}(t) &= \gamma_e(1-\boldsymbol{\upsilon}_e\boldsymbol{\upsilon}_{\nu 1})t - \gamma_e\boldsymbol{\upsilon}_e\tilde{\mathbf{x}} + \gamma_e\boldsymbol{\upsilon}_e\boldsymbol{\upsilon}_{\nu 1}t_c, \\ \mathbf{x}'_{\nu 1}(t) &= [\boldsymbol{\upsilon}_{\nu 1} - \gamma_e\boldsymbol{\upsilon}_e + (\gamma_e-1)\boldsymbol{\upsilon}_{e,0}(\boldsymbol{\upsilon}_{e,0}\boldsymbol{\upsilon}_{\nu 1})]t \\ &\quad + \tilde{\mathbf{x}} + (\gamma_e-1)\boldsymbol{\upsilon}_{e,0}(\boldsymbol{\upsilon}_{e,0}\tilde{\mathbf{x}}) - \boldsymbol{\upsilon}_{\nu 1}t_c \\ &\quad -(\gamma_e-1)\boldsymbol{\upsilon}_{e,0}(\boldsymbol{\upsilon}_{e,0}\boldsymbol{\upsilon}_{\nu 1})t_c. \end{aligned} \tag{26}$$

This is obtained by applying the Lorentz boost (16) (with $\boldsymbol{\upsilon}_r = \boldsymbol{\upsilon}_e$) to the neutrino world line $(t, \mathbf{x}_{\nu 1}(t))$ in the aether frame as defined at the beginning of this section. The time interval between the emission of neutrino $\nu 1$ at t_0 and the subsequent e^+e^- pair emission at t_c is

$$t'_{\nu 1}(t_c) - t'_{\nu 1}(t_0) = \gamma_e(1-\boldsymbol{\upsilon}_e\boldsymbol{\upsilon}_{\nu 1})(t_c - t_0). \tag{27}$$

Thus, if the causality condition $\boldsymbol{\upsilon}_e\boldsymbol{\upsilon}_{\nu 1} < 1$ is violated, cf. (6), the time order of neutrino emission and pair production is inverted in the proper time of the emitted electron. This happens in the OPERA energy range, where $\boldsymbol{\upsilon}_e\boldsymbol{\upsilon}_{\nu 1} > 1$, cf. after (15). As a consequence, neutrino $\nu 1$ reemerges in the electron rest frame during the electron's proper lifetime, which is the interval between the pair creation at $t'_e(t_c)$ and the absorption of the electron by the electron detector at a later instant $t'_e(t_1)$. This is causality violating, as the incoming neutrino $\nu 1$ was annihilated at the time of pair creation by decay into the e^+e^- pair and neutrino $\nu 2$ of lower energy. In the universal reference time of the aether frame, neutrino $\nu 1$ exists in the interval $[t_0, t_c]$, and the e^+e^- pair coexists with the outgoing neutrino $\nu 2$ in the disjoint interval $[t_c, t_1]$, which is the electron's actual lifetime, cf. (29).

The world line of the outgoing neutrino $\nu 2$ and its lifetime in the electron rest frame are obtained by replacing the subscript index $\nu 1$ in eqs. (26) and (27) by $\nu 2$. In this case, the time order of the aether frame is preserved, since the causality condition $\boldsymbol{\upsilon}_e\boldsymbol{\upsilon}_{\nu 2} < 1$ preventing a time inversion in the electron rest frame is met, cf. after (15). Neutrino $\nu 2$ exists in the interval $[t'_{\nu 2}(t_c), t'_{\nu 2}(t_1)]$, $t'_{\nu 2}(t_c) < t'_{\nu 2}(t_1)$, covering part of or extending beyond the electron's proper lifetime $[t'_e(t_c) = t'_{\nu 2}(t_c), t'_e(t_1)]$. It covers the electron's full lifespan if $\boldsymbol{\upsilon}_e\boldsymbol{\upsilon}_{\nu 2} < \upsilon_e^2$, since $t'_{\nu 2}(t_1) - t'_e(t_1)$ factorizes as $\gamma_e(\upsilon_e^2 - \boldsymbol{\upsilon}_e\boldsymbol{\upsilon}_{\nu 2})(t_1 - t_c)$, cf. (28).

Accordingly, the incoming neutrino $\nu 1$ as well as the outgoing neutrino $\nu 2$ simultaneously emerge in the rest frame of the electron, during the electron's proper lifetime, in violation of causality. Energy and momentum are not conserved either, even though the constraints (4) (in primed variables) are satisfied, since the in- and outgoing neutrinos exist at the same time in the electron's rest frame, propagating along different trajectories.

We consider the world line $(t'_e(t), \mathbf{x}'_e)$ of the electron in its rest frame $(t', \mathbf{x}')$,

$$\begin{aligned} t'_e(t) &= \gamma_e(1-\upsilon_e^2)t - \gamma_e\boldsymbol{\upsilon}_e\tilde{\mathbf{x}} + \gamma_e\upsilon_e^2 t_c, \\ \mathbf{x}'_e &= \tilde{\mathbf{x}} + (\gamma_e-1)\boldsymbol{\upsilon}_{e,0}(\boldsymbol{\upsilon}_{e,0}\tilde{\mathbf{x}}) - \gamma_e\boldsymbol{\upsilon}_e t_c. \end{aligned} \tag{28}$$

The electron's proper lifetime between its creation and absorption in the electron's rest frame is thus

$$t'_e(t_1) - t'_e(t_c) = (t_1 - t_c)/\gamma_e, \tag{29}$$

contracted due to the relative motion in the aether [14]. We place the electron detector in a way that the electron is absorbed at a time t_1 defined by equating $t'_e(t_1) = t'_{\nu 1}(t_0)$, cf. (26) and (28), which means

$$t_1 - t_c = \gamma_e^2(\boldsymbol{\upsilon}_e\boldsymbol{\upsilon}_{\nu 1} - 1)(t_c - t_0). \tag{30}$$

This choice of t_1 is only possible if the causality condition $\boldsymbol{\upsilon}_e\boldsymbol{\upsilon}_{\nu 1} < 1$ is violated, as the time intervals $t_c - t_0$ and $t_1 - t_c$ are defined positive from the outset, cf. the beginning of this section. The electron located at $\mathbf{x}'_e$ will be absorbed by the approaching detector at $t'_e(t_1)$. Thus, in the electron rest frame, the incoming neutrino $\nu 1$ coexists with the electron throughout the electron's proper lifetime in the interval $[t'_e(t_c), t'_e(t_1)]$, since $t'_{\nu 1}(t_0) = t'_e(t_1)$ and

$$t'_e(t_c) = t'_{\nu 1}(t_c) = \gamma_e(t_c - \boldsymbol{\upsilon}_e\tilde{\mathbf{x}}). \tag{31}$$

In this time interval $[t'_e(t_c), t'_e(t_1)]$, neutrino $\nu 1$ moves from $\mathbf{x}'_{\nu 1}(t_c) = \mathbf{x}'_e$ to

$$\mathbf{x}'_{\nu 1}(t_0) = \mathbf{x}'_e - [\boldsymbol{\upsilon}_{\nu 1} - \gamma_e\boldsymbol{\upsilon}_e + (\gamma_e-1)\boldsymbol{\upsilon}_{e,0}(\boldsymbol{\upsilon}_{e,0}\boldsymbol{\upsilon}_{\nu 1})](t_c - t_0). \tag{32}$$

Combining this with the time inversion (27), we may write

$$\mathbf{x}'_{\nu 1}(t_0) - \mathbf{x}'_{\nu 1}(t_c) = \boldsymbol{\upsilon}'_{\nu 1}(t'_{\nu 1}(t_0) - t'_{\nu 1}(t_c)), \tag{33}$$

where $\boldsymbol{\upsilon}'_{\nu 1}$ is the neutrino speed in the electron rest frame as stated in (24). In this way, we have also recovered the addition law (18).

To summarize, the Lorentz boost relating the aether frame to the electron's rest frame maps the world line of the incoming neutrino $\nu 1$ into the proper lifetime of the electron. The reason for this is the time inversion (27), which occurs since the causality constraint $\boldsymbol{\upsilon}_e\boldsymbol{\upsilon}_{\nu 1} < 1$ is violated. Analogous reasoning holds for the positron rest frame. The causality conditions preventing a time inversion in the electron and positron rest frames are $\boldsymbol{\upsilon}_e\boldsymbol{\upsilon}_{\nu j} < 1$ and $\boldsymbol{\upsilon}_p\boldsymbol{\upsilon}_{\nu j} < 1$, where $\boldsymbol{\upsilon}_{\nu 1,\nu 2}$ are the velocities of the in- and outgoing neutrino; the electron, positron and neutrino velocities all refer to the aether frame.

More generally, in an interaction process involving sub- and superluminal particles, the causality conditions $\boldsymbol{v}_i \boldsymbol{v}_j < 1$ have to be met, where index i labels the velocities of the subluminal particles in the aether frame, and index j the superluminal velocities. These conditions are necessary and sufficient to avoid time inversion and thus causality violation in the rest frames of the subluminal particles taking part in the interaction. Pion and kaon decay $\pi \to \mu + \nu_\mu$ into subluminal muons and superluminal neutrinos [20,21], and the causality constraints on photon emission by superluminal charges, $\pi \to \pi + \gamma$, will be discussed elsewhere.

Conclusion. – Causality implies that 1) every effect has a cause; that 2) the cause precedes the effect; and 3) the distinction of cause and effect is unambiguous. Superluminal energy transfer establishes spacelike connections, whose time order can be altered by Lorentz boosts, so that different inertial observers see a different time order. Thus superluminal signal transfer violates either condition 2) or 3) in a relativistic context, as observations made in different inertial frames have to be regarded as equally real. There have been suggestions that acausal relativistic signal transfer is not logically inconsistent [22]. However, the predetermination that goes with effects preceding their causes lends itself better to science fiction than to an empirical science.

Causality violation in superluminal signal transfer is conceptually different from the acausal advanced wave fields of electrodynamics. So long as the signal transfer happens at the speed of light or is subluminal, acausal solutions of evolution equations can unambiguously be identified. Maxwell's equations admit acausal solutions generated by advanced Green functions. If the signal is (sub)luminal, Lorentz boosts cannot alter the time order of events connected by the signal. Thus a retarded or advanced solution remains so in all inertial frames, and one can discard advanced solutions on the grounds of causality violation.

Signal transfer by superluminal neutrinos requires a universal frame of reference to discern causal from acausal connections, as Lorentz boosts can change the time order of emission and absorption. There are two groups (equivalence classes) of inertial observers; one sees the emission prior to absorption in their proper time, and the second observes a time-inverted neutrino current propagating from the detector toward the source. The reference frame allows both groups to unambiguously discern cause and effect. All observers can compare their proper time to the universal time order defined by the reference frame, and thus determine whether a time inversion (27) occurs in their rest frames due to their motion in the aether [5,7,14]. In contrast, superluminal signal transfer in conjunction with a relativistic interpretation of the Lorentz symmetry conflicts with causality, as cause and effect cannot invariantly be identified: what appears causal to one observer, is acausal for another. Thus, unlike in electrodynamics, a reference frame is necessary to discriminate causal and acausal solutions of evolution equations. Here, we have identified the emission of an e^+e^- pair as an acausal solution of the energy-momentum conditions.

The speed of signal transfer (7) is determined by a dispersion relation, which depends on the particle mass and the permeability tensor by which the wave equation is coupled to the aether. Different particles couple with different permeability tensors. The universal frame of reference is identified as the aether frame in which the permeability tensors are isotropic, cf. (2) and (20), and is manifested as the rest frame of the cosmic microwave background. Inertial frames are connected to the aether frame by Lorentz boosts and characterized by a subluminal relative velocity $\boldsymbol{v}_r$ in the aether. When calculating cross-sections of interactions outside the lightcone, involving sub- and superluminal particles, one has to take account of kinematic causality conditions. The constraints necessary and sufficient to exclude causality-violating time inversions in the rest frames of the subluminal constituents are a set of inequalities $\boldsymbol{v}_{r,i} \boldsymbol{v}_{s,j} < 1$. The $\boldsymbol{v}_{s,j}$ denote the group velocities of the superluminal modes, and the $\boldsymbol{v}_{r,i}$ are the velocities of the subluminal particles in the interaction process with rest frames defined by the Lorentz boosts (16); all velocities refer to asymptotic in- and out-states in the aether frame. These causality constraints prohibit pair emission by superluminal neutrinos in the OPERA energy range.

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